Ringdown Gravitational Waves from Close Scattering of Two Black Holes [gr-qc 2310.18686]

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GW Observation

1. **Current Status**:
   1. **First Detection** by LIGO in 2015
   2. **Advanced Detectors:** improvements in LIGO, Virgo, KAGRA
   3. **Catalog of Events:** growing catalog of GW events (BBH, BNS, BH-NS mergers)
2. **Significance of GW Observations from BHs**:
   1. **Testing Relativity:** Provide a testbed for Einstein's theory of GR
   2. **Black Hole Properties** (BH parameters, BH dynamics..)
   3. **Astrophysical Processes** (Binary evolution, stellar collapses..)
   4. **Universe's History** (BH formation, Cosmic expansion, Dark matter..)

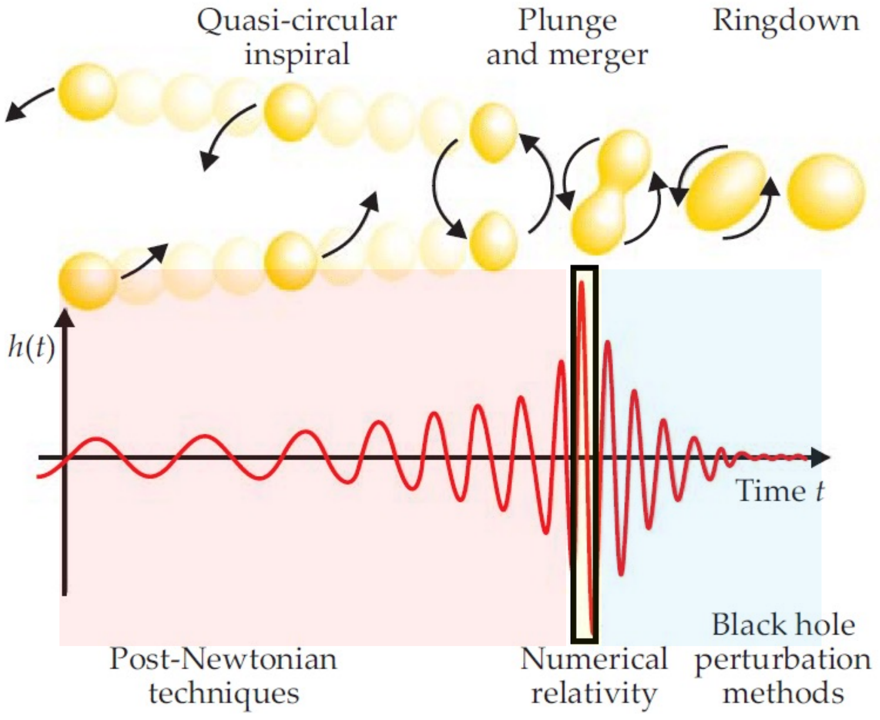
GW Observation Enhancement

1. **Enhanced Detection Capabilities**:
   - **Increased Sensitivity:** Enhanced sensitivity of LIGO and other detectors leading to more frequent detections.
   - **Collaborative Observations:** Global network of GW observatories
2. **Expanded Catalog of GW Events with Diverse Sources**:
   - Not only BBH but also BNS, .., Dynamical capture, hyperbolic encounters
3. **Advancements in Multi-Messenger Astronomy**:
   - **Electromagnetic Counterparts:** Detecting electromagnetic signals alongside GWs (GW170817).
4. **Next-Generation Detectors:**
   - LIGO-India, LISA, Einstein Telescope, Cosmic Explorer
     for observing GWs from a broader range of sources.

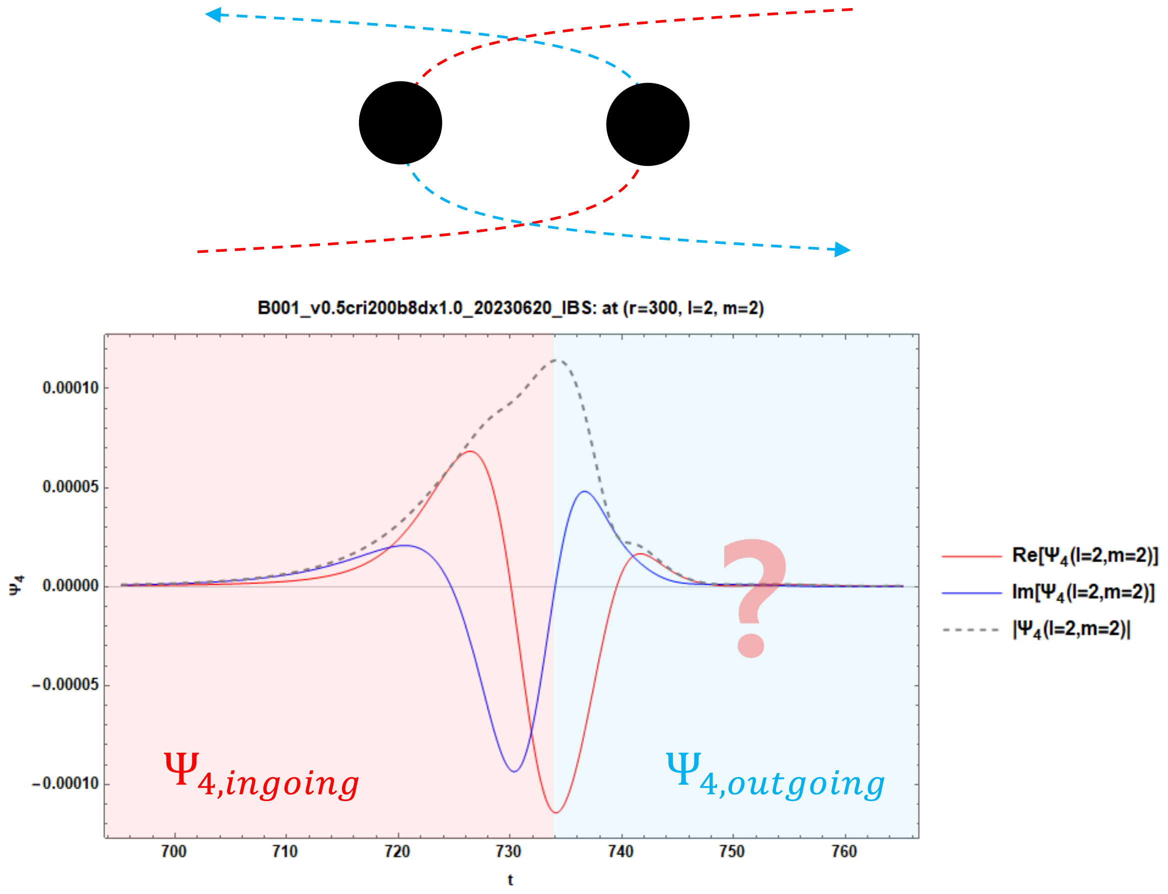
Close encounter for strong gravity test

- **Simplicity and Control:**
  - BH do not merge, allowing for a simpler system where the individual properties of each BH (like mass and spin) remain relatively unchanged.
- **Isolation of Effects:**
  - No merger, so the strong gravitational interactions can be studied more cleanly.
    The tidal deformation and the resulting effects can be examined clearly.
- **Controllable Parameters:**
  - By adjusting initial parameters such as impact parameter and velocity, we can systematically study the gravitational interaction
    without the complicated effects of a merger.

Study of waveforms for BBH



$\Psi_{4,inspiral}$ $\Psi_{4,merging}$ $\Psi_{4,ringdown}$



Tidal Deformation and Ringdown Possibility in Close Encounters

1. **Tidal Deformation Studies**:
   - **Schwarzschild Black Holes**: Tidal Love numbers vanish for Schwarzschild BHs in four dimensions [PRD80.084018,2009]
   - **Spinning (Kerr) Black Holes**: Tidal Love numbers may not vanish for certain types of perturbations [PRL.126,131102(2021)]
2. **Ringdown Possibility in Close Encounters of BHs**
   1. **Non-vanishing Love number in BBH Coalescence** [PRD105,044019(2022)]
   2. **Ringdown and echoes in test particle study around SBH** [PRD.94.084031(2016)]
3. **Despite these possibilities, there had been no waveform studies on the ringdown signal of the BBH system that take into account the effects of the non-perturbative regime.**

QNM in Close Encounter?

- **Perturbative study in Spherical/Axisymmetric spacetime with an isolated single black hole**

$$\frac{d^2\psi_{\ell m}(r)}{dr^{*2}} + \left[\omega^2 - V(r)\right]\psi_{\ell m}(r) = 0$$ **b.c. at horizon and infinity**

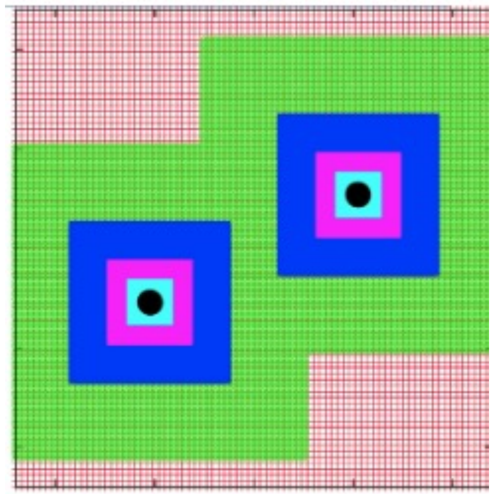
- $\boldsymbol{\omega_{nlm} = \omega_R + i\,\omega_i}$ **: oscillatory and damping frequencies**

$$\begin{cases} {}_sS_{\ell m} \xrightarrow{s=0} S_{\ell m} & \text{(scalar case)} \\ {}_sS_{\ell m} \xrightarrow{c=0\ (a=0)} {}_sY_{\ell m} & \text{(zero-rotation limit)} \\ {}_sS_{\ell m} \xrightarrow{s=0,\ c=0} Y_{\ell m} & \text{(zero-rotation and scalar)} \end{cases}$$

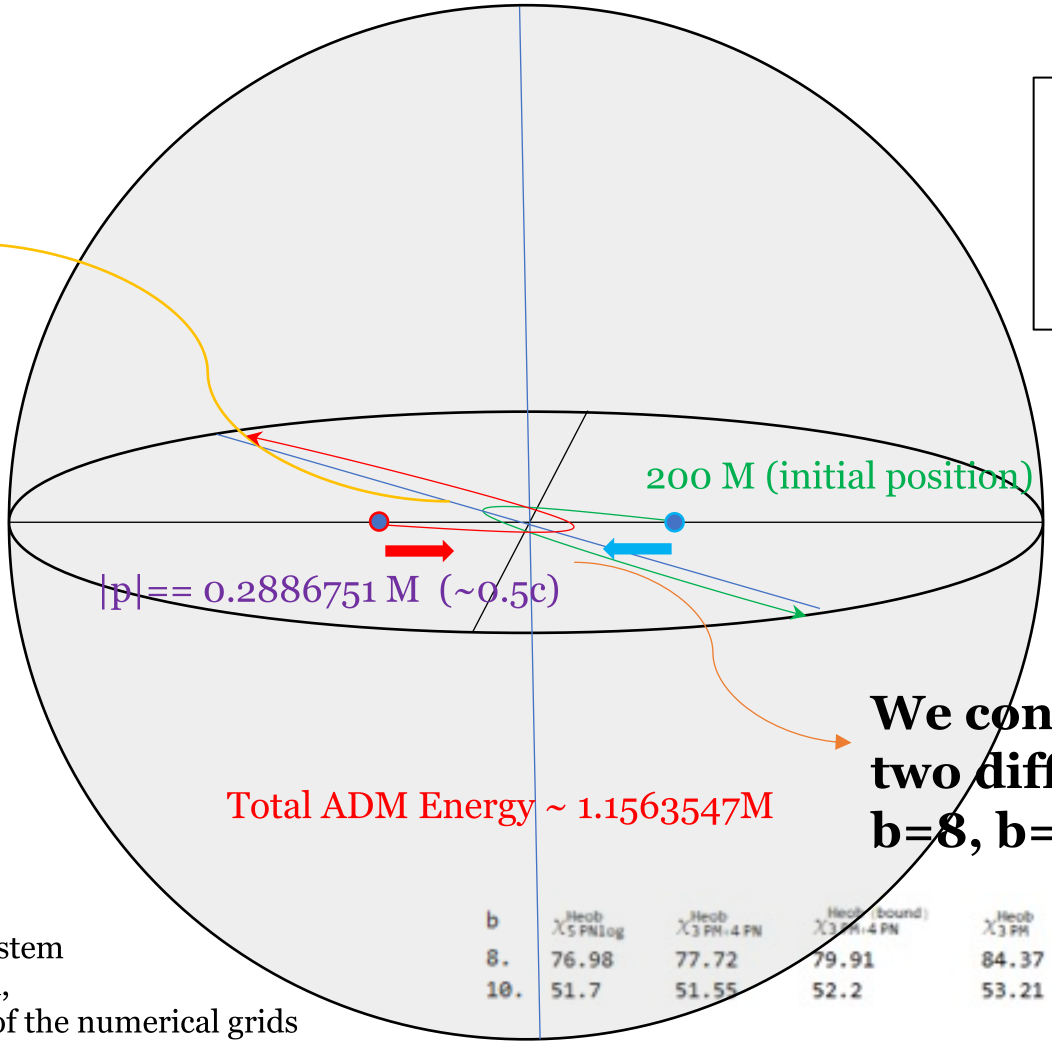
(scalar) spherical harmonics	Y
vector/tensor spherical harmonics	vY/tY
spin-weighted spherical harmonics	sY
spheroidal harmonics	S
spin-weighted spheroidal harmonics	sS

- **How about from two BHs?**

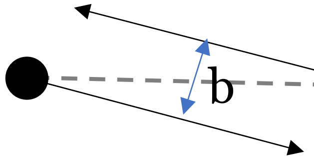
Numerical Study Setup



7^{th} mesh refinement levels
dx=0.1M, 0.875M
(finest grid: dx=0.0137M, 0.0156M)



We used EinsteinToolkit code, one of the most widely used open-source codes based on the Cactus framework



We considered two different impact parameters b=8, b=10.

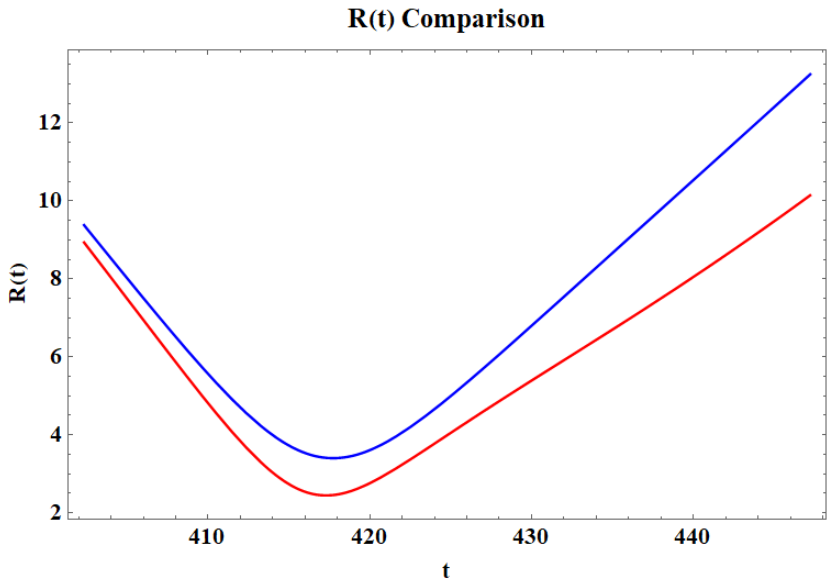
b	χ^{Heob}_{5PNlog}	$\chi^{Heob}_{3PM+4PN}$	$\chi^{Heob(bound)}_{3PM+4PN}$	χ^{Heob}_{3PM}	χ^{H}_{3PM}	χ_{4PN}	χ_{3PN}	χ^{Heob}_{4PN}	χ^{Heob}_{3PN}	χ^{Heob}_{2PN}	χ^{Heob}_{1PN}
8.	76.98	77.72	79.91	84.37	0	71.68	70.85	77.31	81.01	92.66	101.4
10.	51.7	51.55	52.2	53.21	0	50.51	50.06	51.76	52.54	55.43	58.14

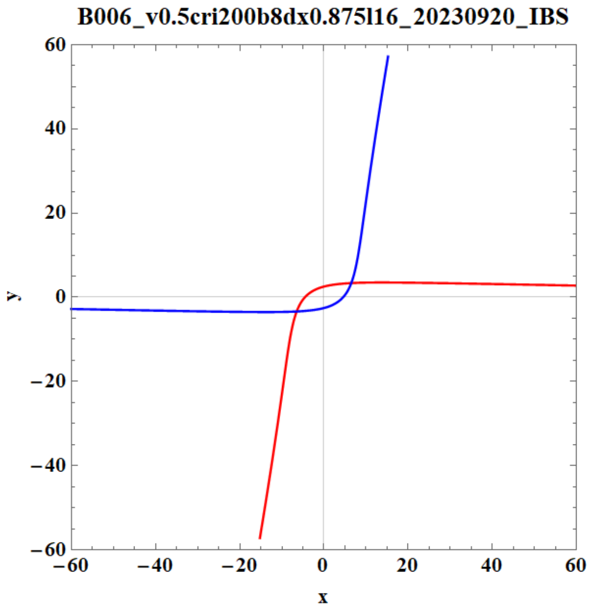
McLachlan is used for time evolution of the system
with 8th order spatial finite difference method,
and Carpet for the adaptive mesh refinement of the numerical grids

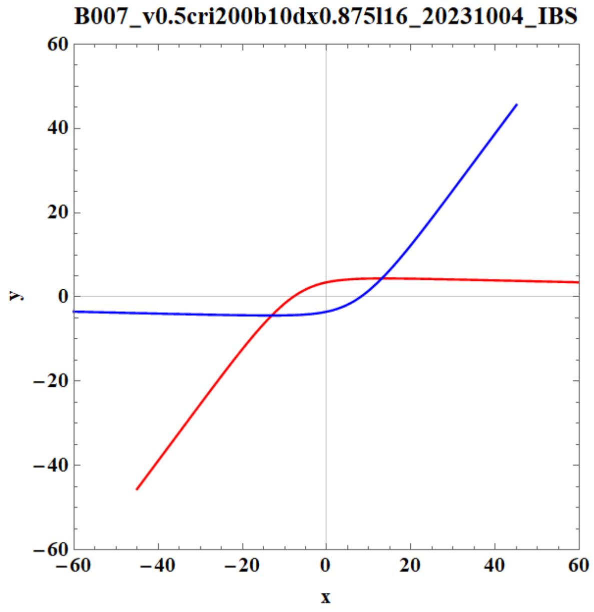
NR Result of Close Encounter: Strong Gravity Regime

Minimum coordinate relative distance:

B006_v0.5cri200b8dx0.875l16_20230920_IBS: R_{min}=2.44932
B007_v0.5cri200b10dx0.875l16_20231004_IBS: R_{min}=3.40454





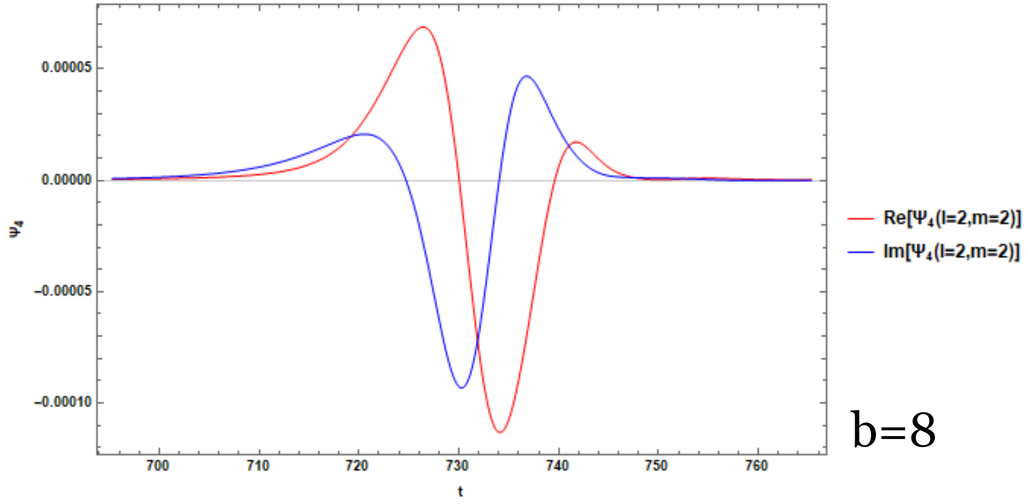


Ψ_4^{22} Mode

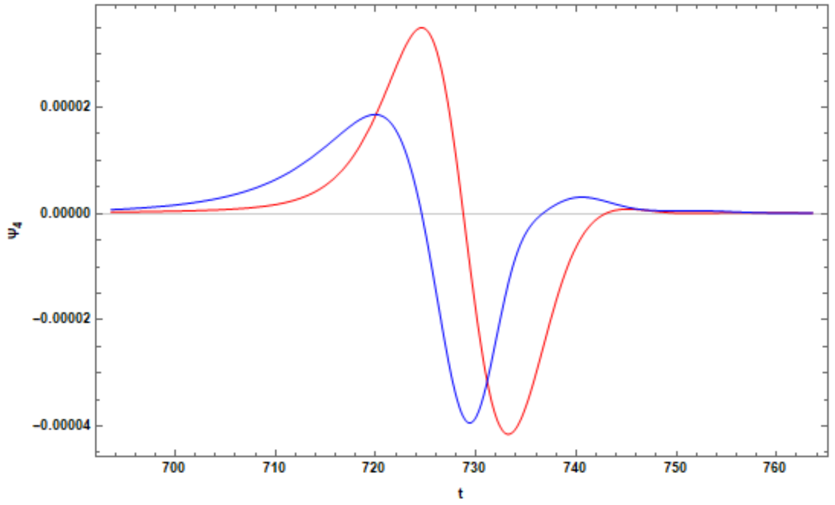
- **Spin-weighted spherical harmonics decomposition**

$$\Psi_4(t,r,\theta,\varphi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} {}_{-2}\Psi_4^{lm}(t,r)\,{}_{-2}Y_{lm}(\theta,\varphi)$$

- **The most dominant mode**
- **At $r_{extraction} = 300M$**
- **Two polarizations (red:+, blue:×)**
- **Burst-like waveform**
- **Ringdown?**



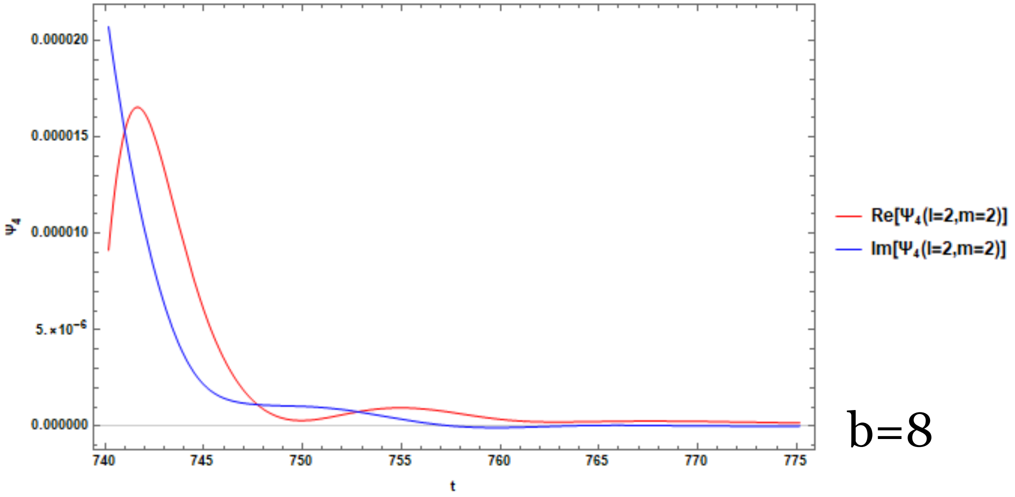
b=8



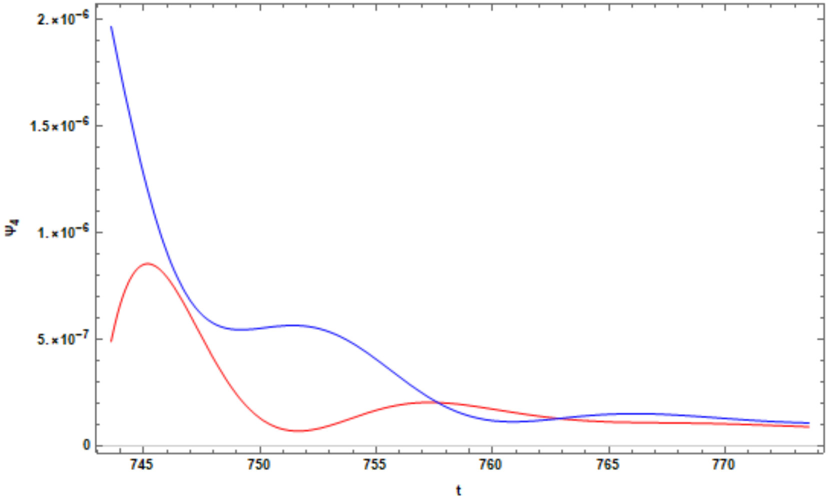
b=10

Ψ_4^{22} Mode – Outgoing Phase

- **Looks weak, modulated > ringdown?**



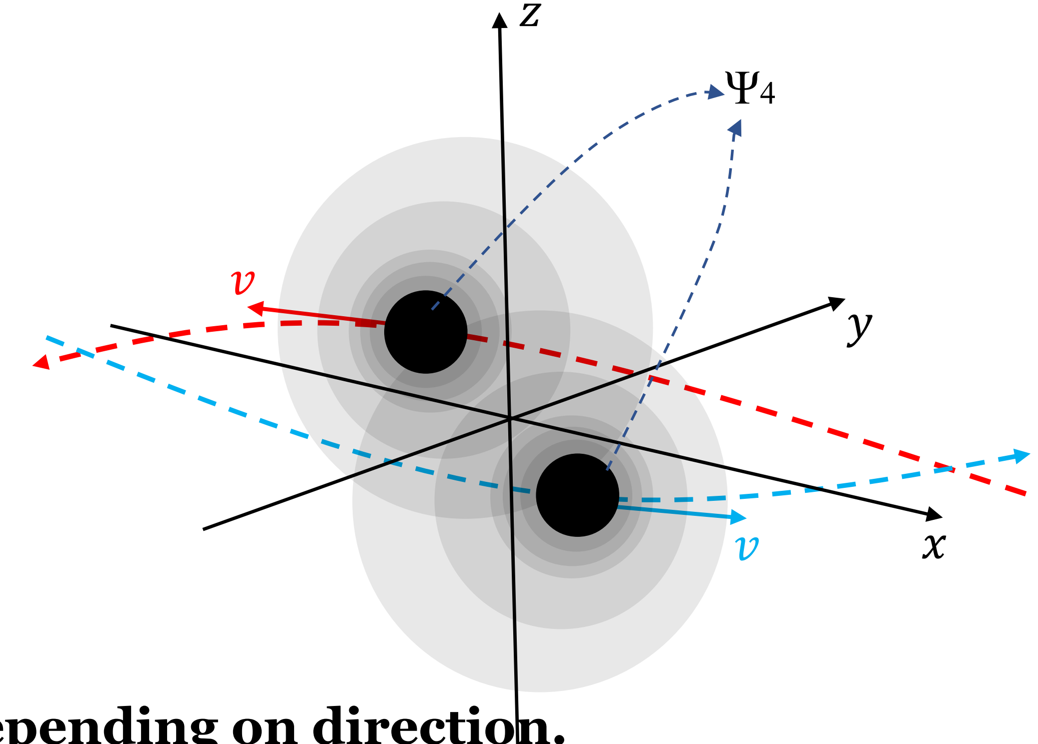
b=8



b=10

Understanding BH Scattering System

- **When each BH is tidally deformed and stabilized, its GW radiation….**
- **Possible modulation causes:**
  When radiating ringdown,
  - **BH is moving: red/blue Doppler shift**
    (One BH is red shifted,
    the other is blue shifted for any observer)
    (Relativistic Doppler shift including time dilation)
  - **BH speed is not constant.**
    (Doppler shift is a time-varying effect.)
  - **Different positions of two black hole**
    (phase difference between two ringdown signals)
  - **Not spherical or axisymmetric background due to the other BH**
    (gravitational lensing effect, absorption)
  - **BH's ringdown signal comes NOT from the origin**
    (mode mixing during wave extraction)



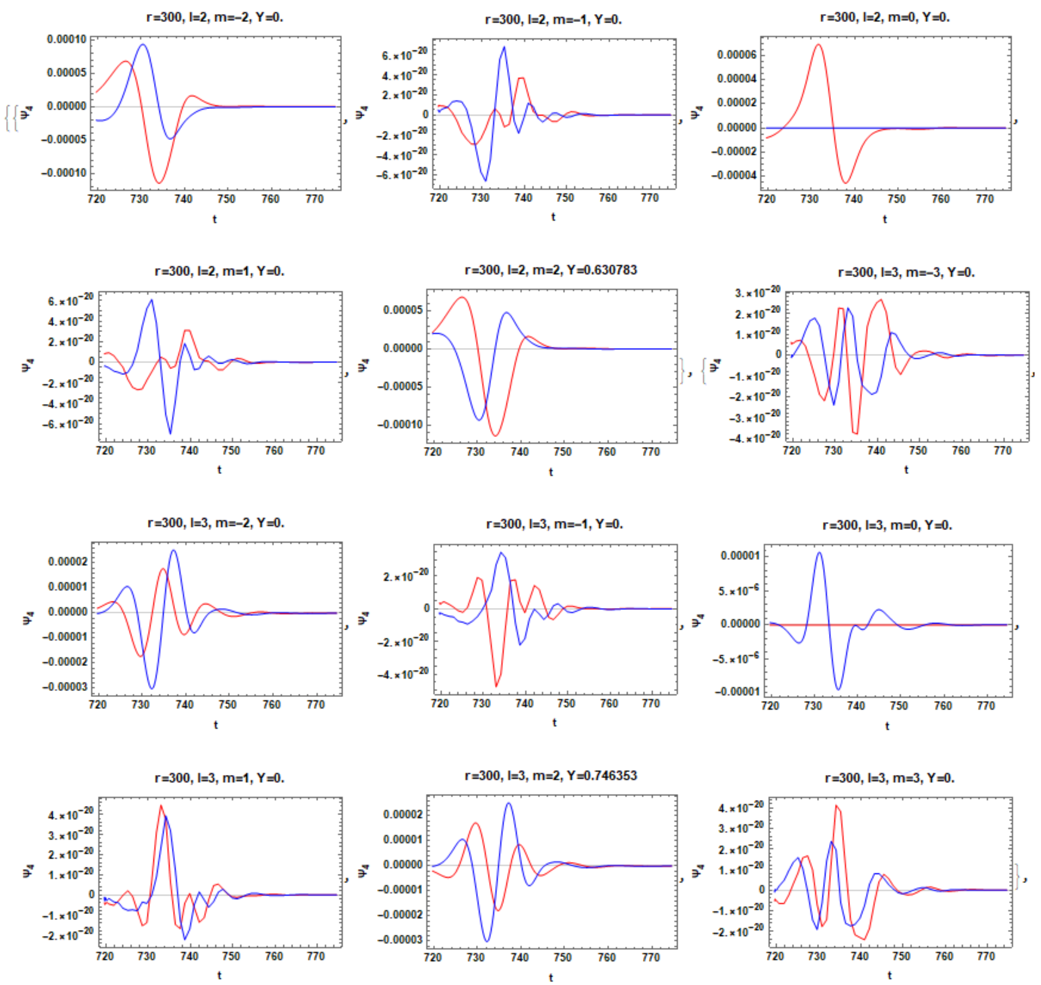
- **The above modulations appear differently depending on direction.**
  And multipole modes, which are obtained by integrating over whole direction, show weak, highly modulated ringdown signals.

Modulations of the Ringdown-like Signal

- **In the multipole modes, ringdown-like signal shows highly modulated behaviors as we discussed.**
- **Therefore, to check the signal, we consider a specific observer. This can be done by just reading Ψ_4.**

$$\Psi_4(t,r,\theta,\varphi) \approx \sum_{l=2}^{16} \sum_{m=-l}^{l} \Psi_4^{lm}(t,r)\,{}_{-2}Y_{lm}(\theta,\varphi)$$

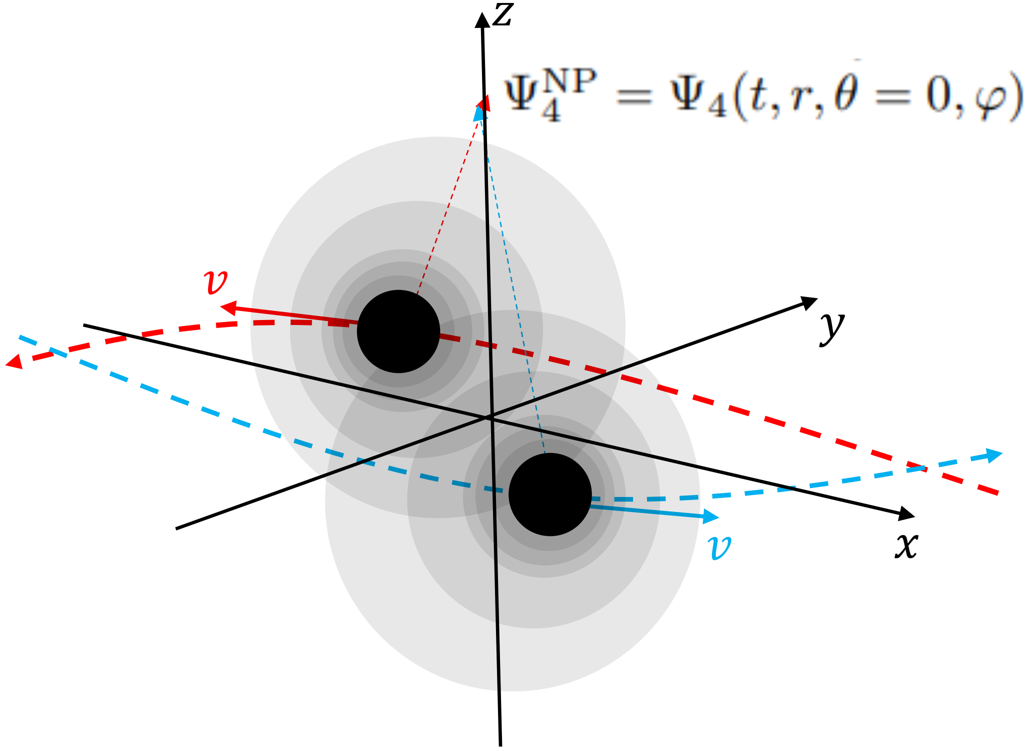
- **We used the multipole modes up to l=16.**



Symmetrical Observer and its GW

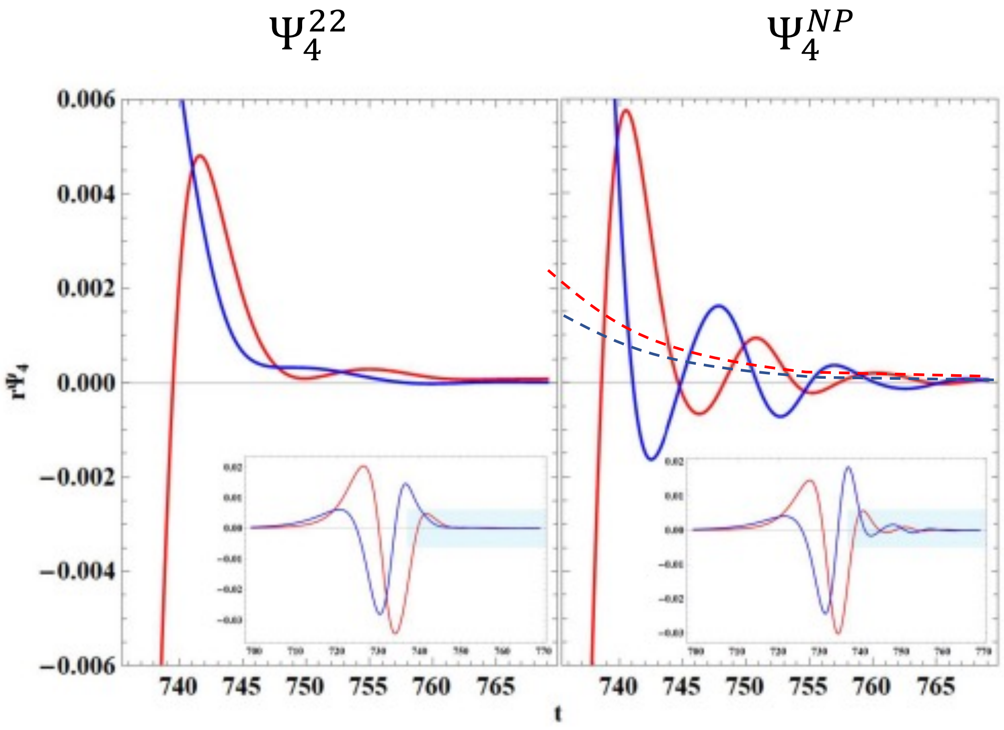
- **Among possible observer direction,**
  the most symmetric location is north/south pole.

- **Possible modulation causes:**
  When radiating ringdown,
    - ~~**BH is moving: red/blue Doppler shift**~~
      ~~(One BH is red shifted,~~
      ~~the other is blue shifted for any observer)~~
      (Relativistic Doppler shift including time dilation)
    - **BH speed is not constant.**
      (Doppler shift is a time-varying effect.)
    - ~~**Different positions of two black hole**~~
      ~~(phase difference between two ringdown signals)~~
    - **Not spherical or axisymmetric background due to the other BH**
      (gravitational lensing effect, absorption)
    - ~~**BH's ringdown signal comes NOT from the origin**~~
      ~~(mode mixing during wave extraction)~~



Ψ_4^{NP} Waveform (Including Ringdown)

- At $r_{extraction}$=300M
- At northpole, we observe a distinct ringdown signal.
- Only 2% strength of hidden ringdown in 22 mode is amplified to 10% of the maximum strength.
- In Ψ_4^{NP}, we see ringdown has QNM property (oscillating, damping), and find its dominant mode.
- We can see that the ringdown(RD) signals from each BH is superposed with trajectory-driven(TD) waveform, which is studied in approximation methods

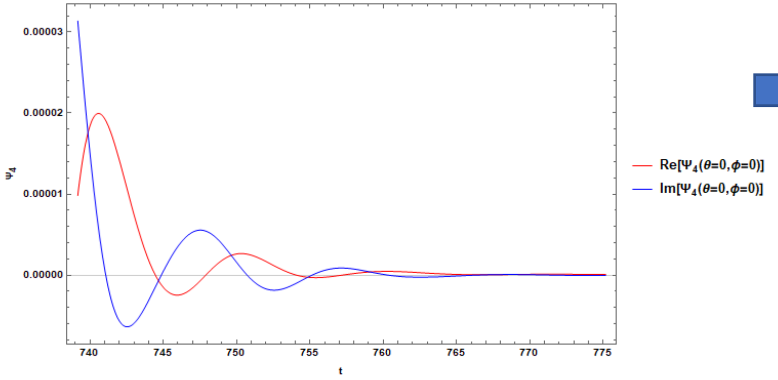


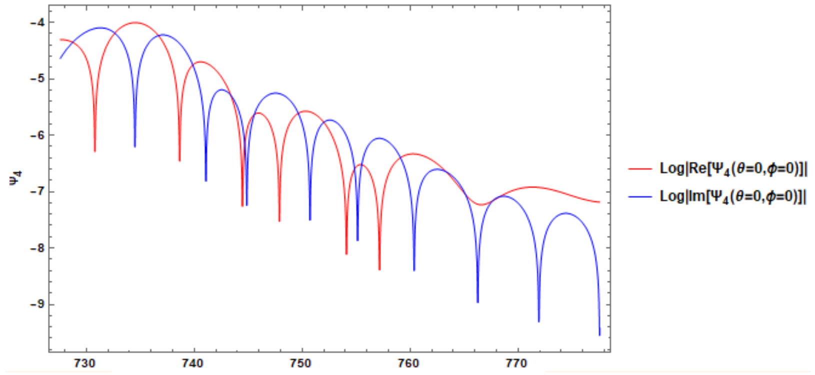
$$\Psi_4 = \Psi_4^{\mathrm{TD}} + \Psi_4^{\mathrm{RD}}$$

TD Wave Influence in Fitting

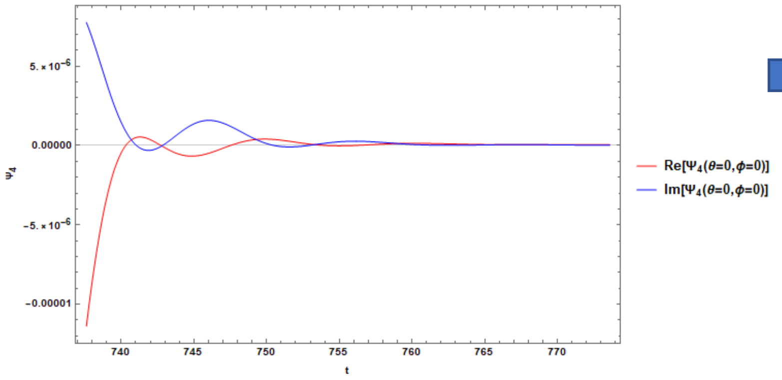
- **Since RD is superposed with TD,**
  its log plot shows different behaviors from merging ringdown cases:

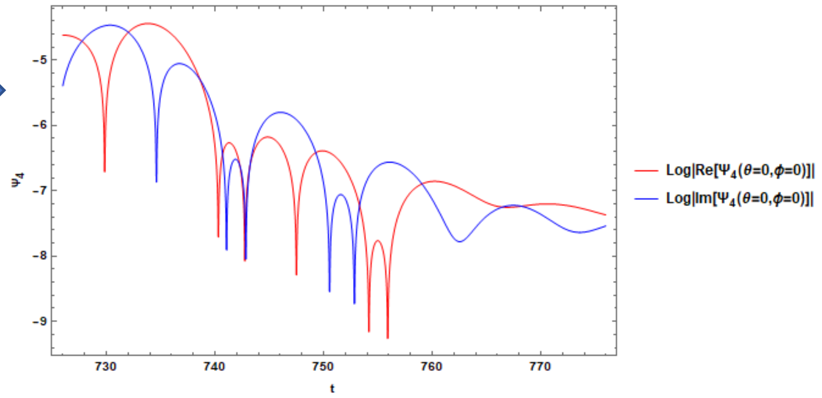
b=8





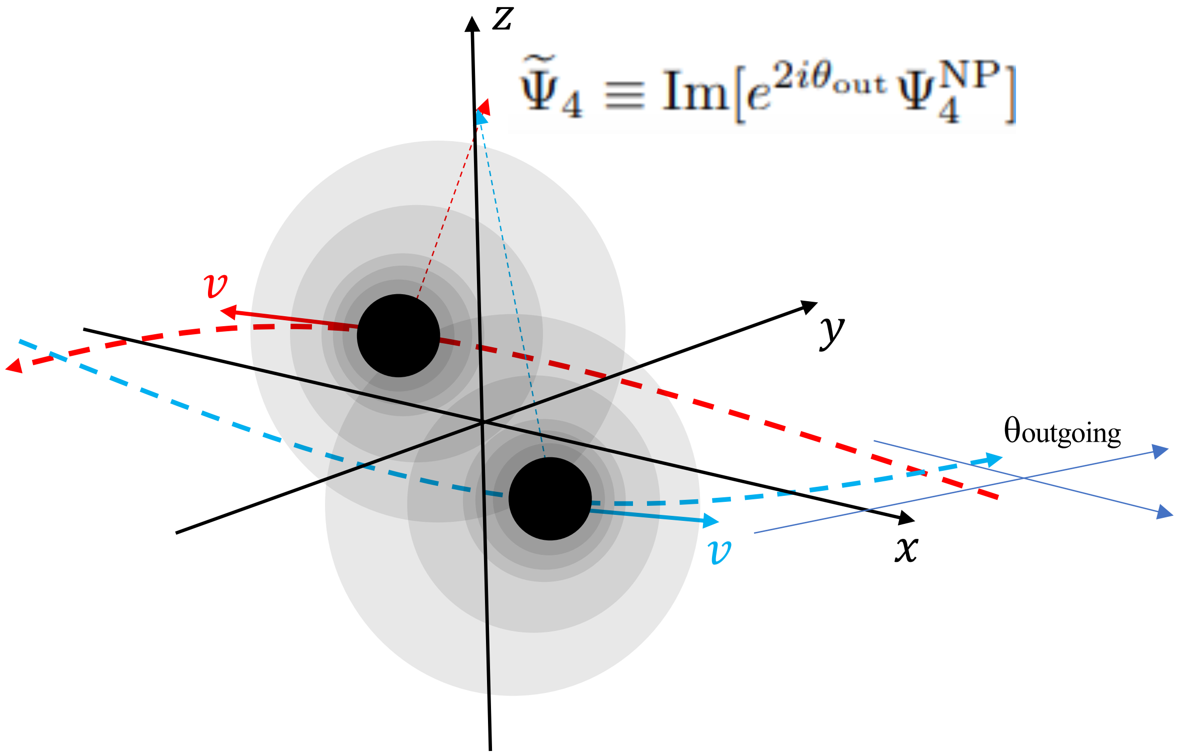
b=10

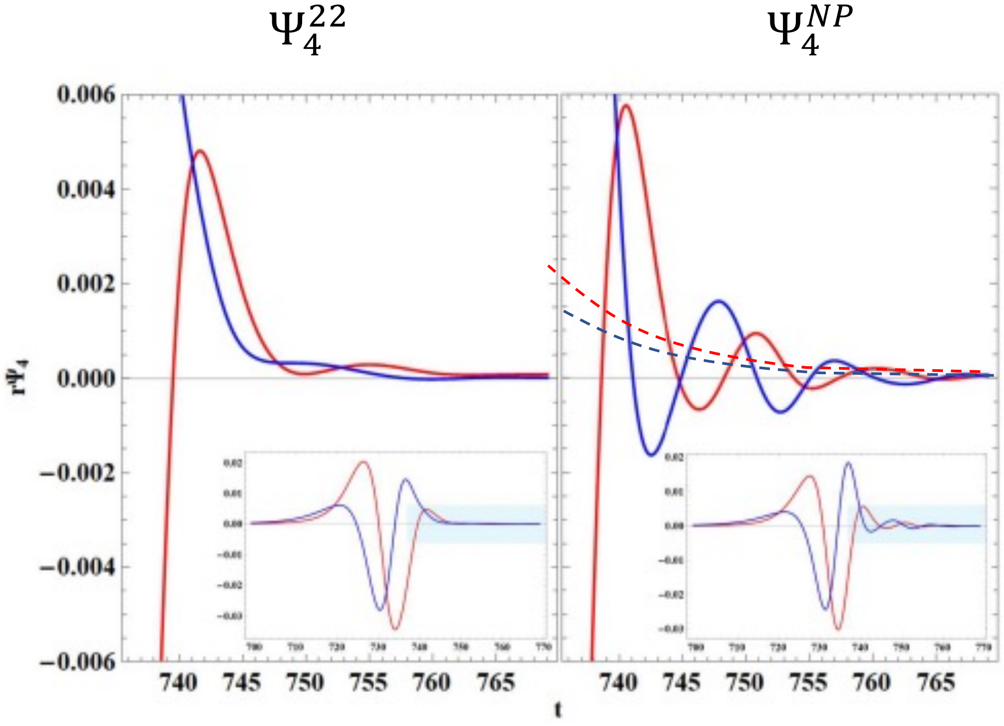




To identify ringdown as QNM

- **Frame-rotation to minimize TD influence**

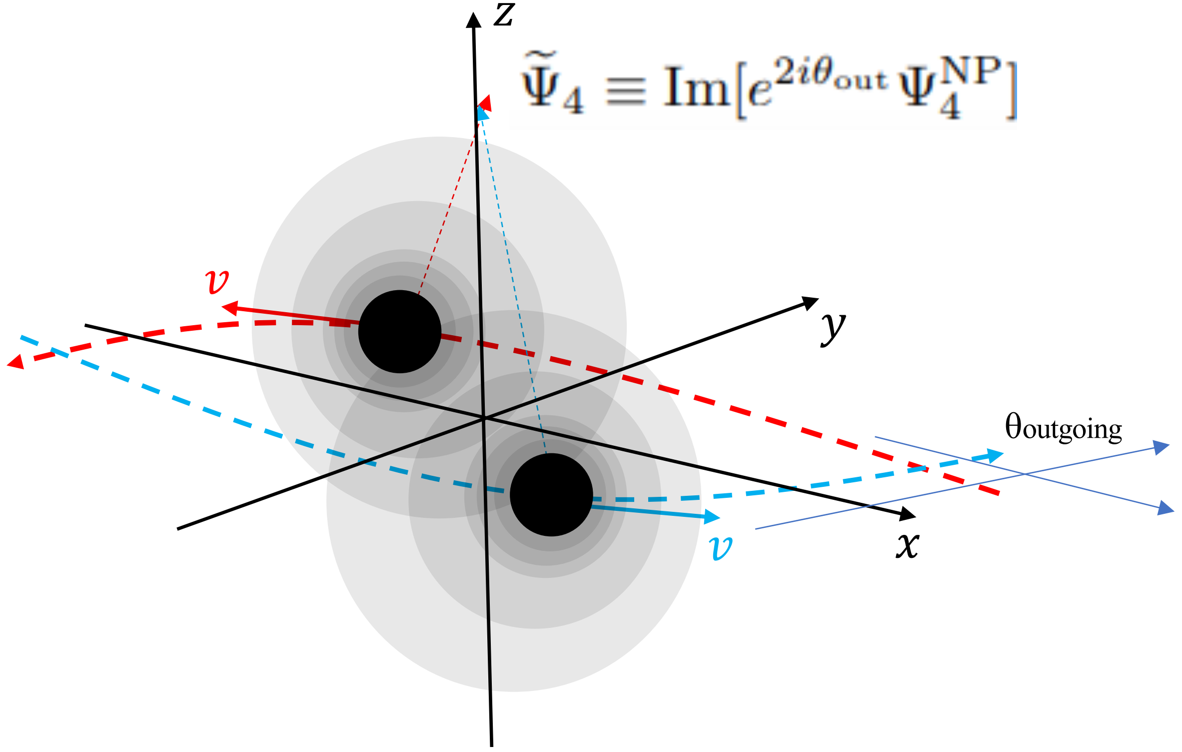




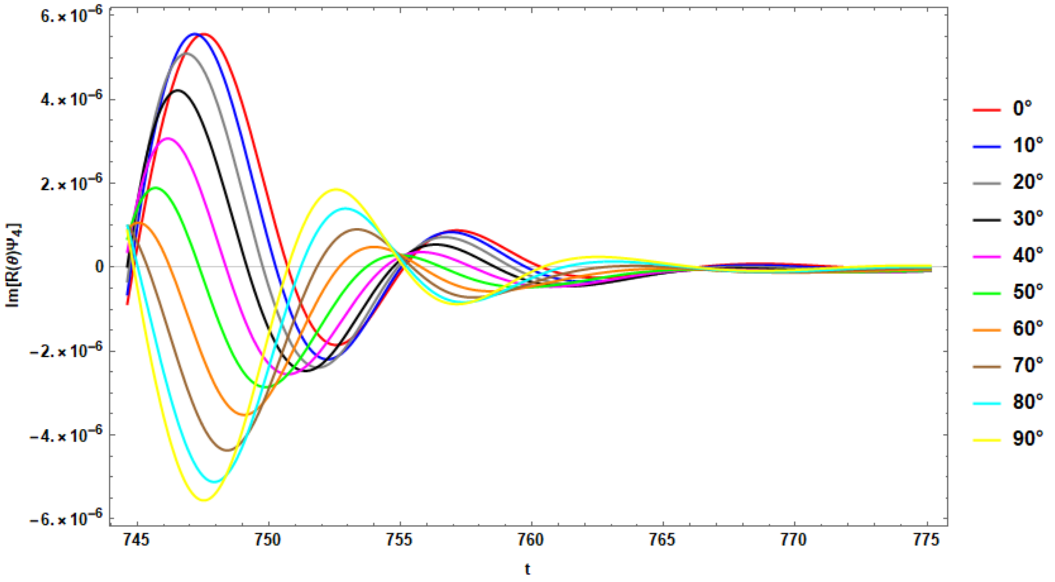
$$\Psi_4 = \Psi_4^{\rm TD} + \Psi_4^{\rm RD}$$

Frame-Rotation and TD Behavior

- **Frame-rotation to minimize TD influence**



b=8, r_{ext}=300



$\widetilde{\Psi}_4$ Waveform and QNM identification

- **In this waveform, TD is rapidly damping, we assume it follows a power-law damping form.**
- **Analysis domain**
  - **From the first peak of the $|\Psi_4|$**
  - **Until the peak point before numerical noise becomes comparable.**
- **Analysis time interval includes only ~2 cycles.**

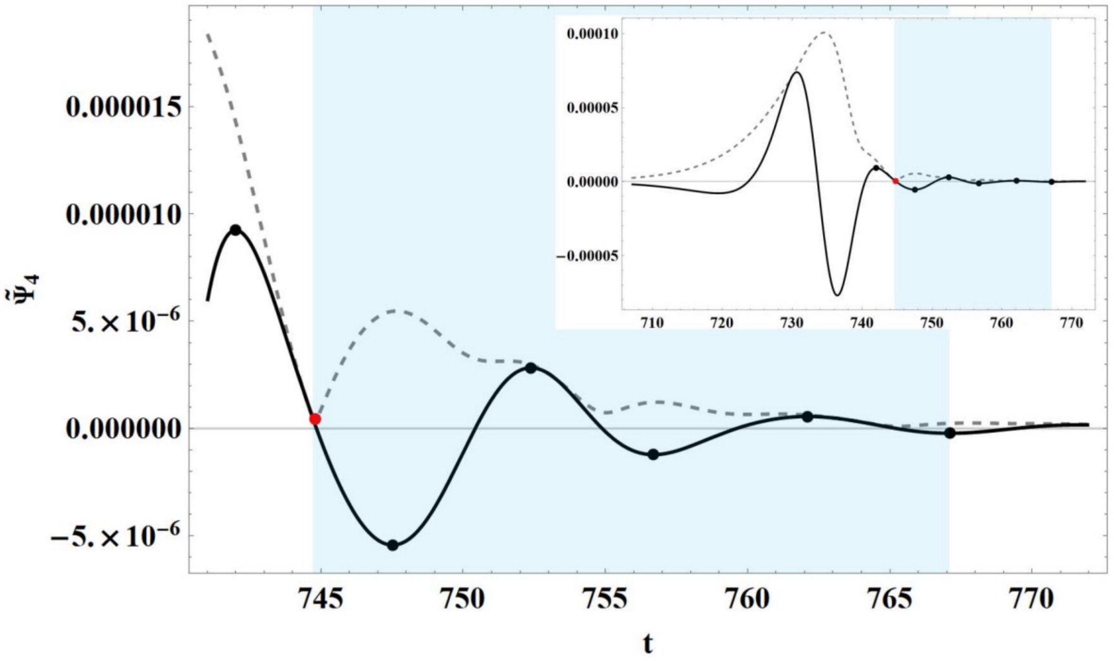
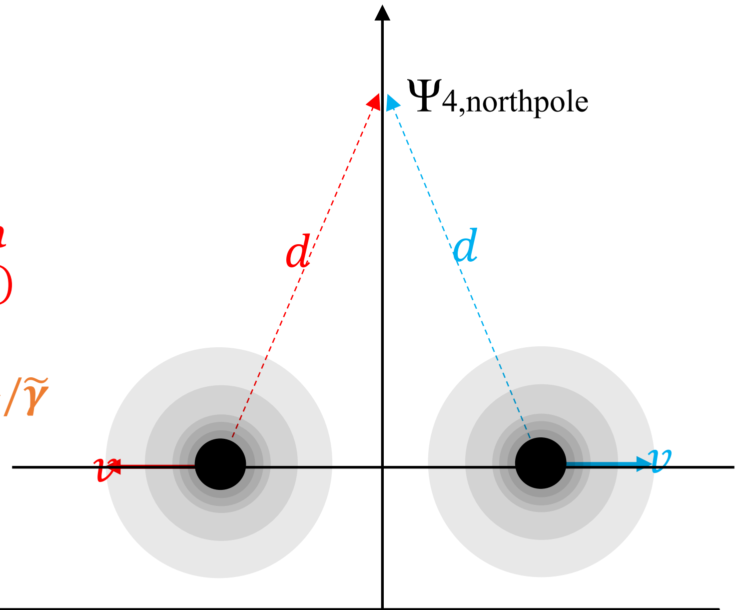


FIG. 3. Waveforms of $\widetilde{\Psi}_4$ (solid line) and $|\Psi_4^{\rm NP}|$ (dashed line) for $b = 8\,M$ at $r_{\rm ext} = 300\,M$ in the outgoing phase, respectively. Extremum points are denoted by bold dots. Ringdown waves used for QNM analysis is indicated by the sky-blue area.

$\widetilde{\Psi}_4$ Waveform Fitting Assumptions

- **Possible modulation causes:**
  When radiating ringdown,
  - ~~**BH is moving: red/blue Doppler shift**~~
    ~~(One BH is red shifted,~~
    ~~the other is blue shifted for any observer)~~ >> ***weak line-of-sight motion***
    (Relativistic Doppler shift including time dilation**)** >>> $\boldsymbol{\omega^{NP}(t) = \omega_{QNM}/\gamma(t)}$
  - **BH speed is not constant.**
    (Doppler shift is a time-varying effect.) >> **about 2% in** $\boldsymbol{\gamma}$ >> $\boldsymbol{\omega^{NP} \equiv \omega_{QNM}/\tilde{\gamma}}$
  - ~~**Different positions of two black hole**~~
    ~~(phase difference between two ringdown signals)~~
  - **Not spherical or axisymmetric background due to the other BH**
    (gravitational lensing effect, absorption) >> **assume weak**
  - ~~**BH's ringdown signal comes NOT from the origin**~~
    ~~(mode mixing during wave extraction)~~
- **Fitting functional form** >>>>>>>>>>>>>>>>>>>>



$$\widetilde{\Psi}_4 = \sum_i A_i e^{-\omega_{\mathrm{I}}^{(i)}(t-\Delta t_i)} \cos(\omega_{\mathrm{R}}^{(i)}(t-\Delta t_i)) + \frac{B}{(t-\Delta t_{\mathrm{T}})^p} + C$$

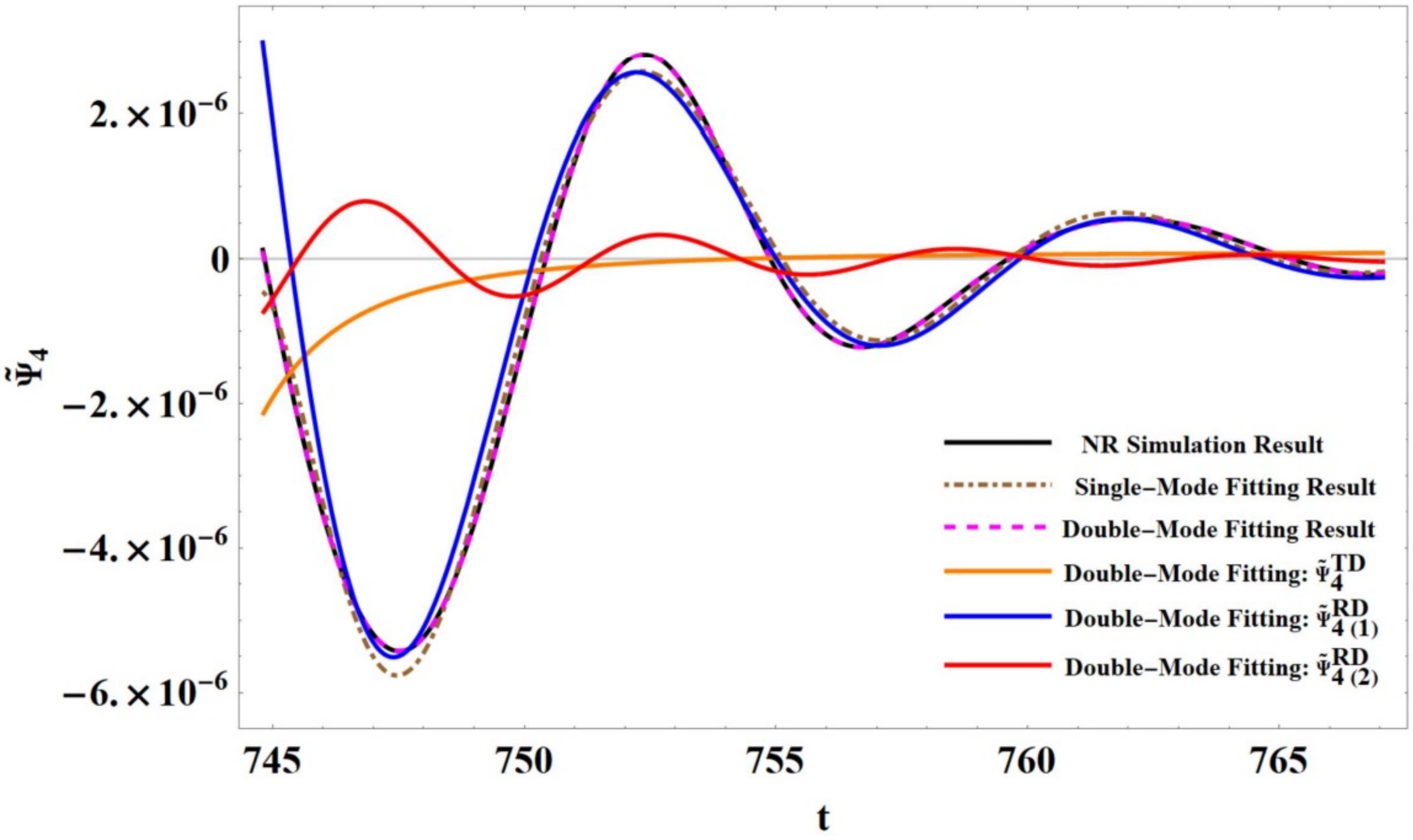
$\widetilde{\Psi}_4$ RD Fitting with Sch. BH. QNM

- **Dimensionless spin parameter ~ 10^{-4}**
  >> We assume the characteristic excitations of the close BH encounter
  to be like Schwarzschild BH QNM,
  since there's no study of QNM from two BHs.
- **For i=1 (single mode fitting), we only consider the dominant QNM.**
  For i=1,2 (double mode fitting), we consider up to the sub-dominant QNM.
- **We performed a nonlinear regression using Newton's method, which iteratively refines the parameter estimates to minimize the sum of squared residuals.**

$$\widetilde{\Psi}_4 = \sum_i A_i e^{-\omega_{\mathrm{I}}^{(i)}(t-\Delta t_i)} \cos(\omega_{\mathrm{R}}^{(i)}(t-\Delta t_i)) + \frac{B}{(t-\Delta t_{\mathrm{T}})^p} + C$$

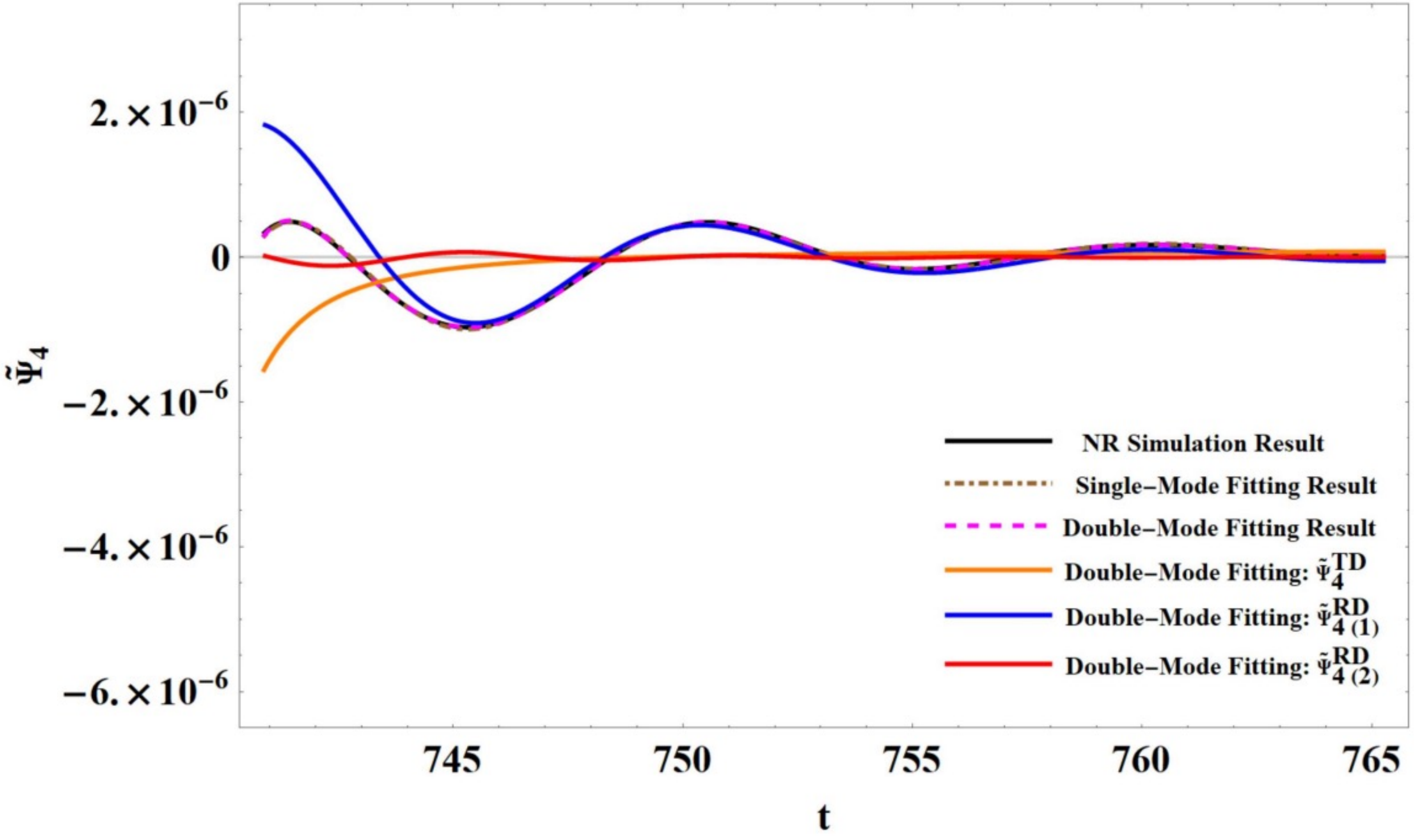
$\widetilde{\Psi}_4$ Fitting Result (r_{ext}=300, b=8)

- **For b=8 case,
One mode fitting result shows deviation
due to the fitting without non-negligible sub-dominant modes**
- **In double-mode fitting,
it shows good agreement.**
- **TD wave is minimized in $\widetilde{\Psi}_4$, but still gives uncertainties in fitting.**



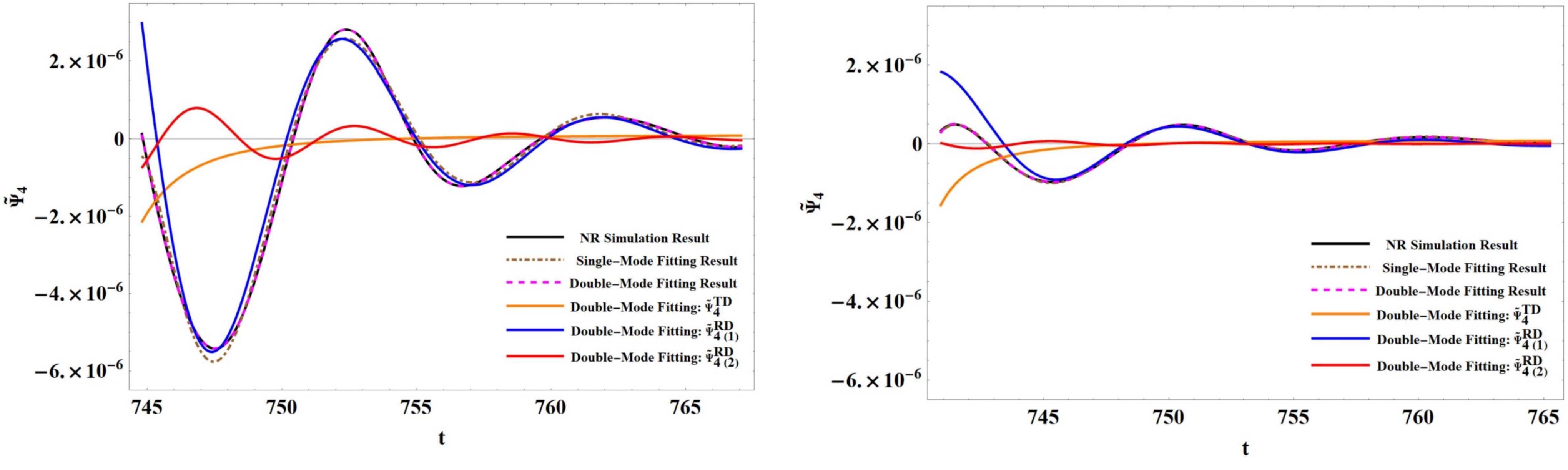
$\widetilde{\Psi}_4$ Fitting Result (r_{ext}=300, b=10)

- **For b=10 case,
  One mode fitting result
  already shows
  good agreement
  due to
  weak sub-dominant mode**
- **In double-mode fitting,
  it also shows
  good agreement.**
- **TD wave is minimized
  in $\widetilde{\Psi}_4$, but still gives
  uncertainties in fitting.**



$\tilde{\Psi}_4$ Fitting Comparison

- The strengths of both RD and TD contributions decrease, presumably, due to the less-strong interactions for the larger impact parameter.
- The weakening of the TD radiation is not severe relative to that of the RD one.
- The sub-dominant RD contributions are suppressed to about 13 % and 7 % of the dominant ones for the cases of b = 8 M and b = 10 M.

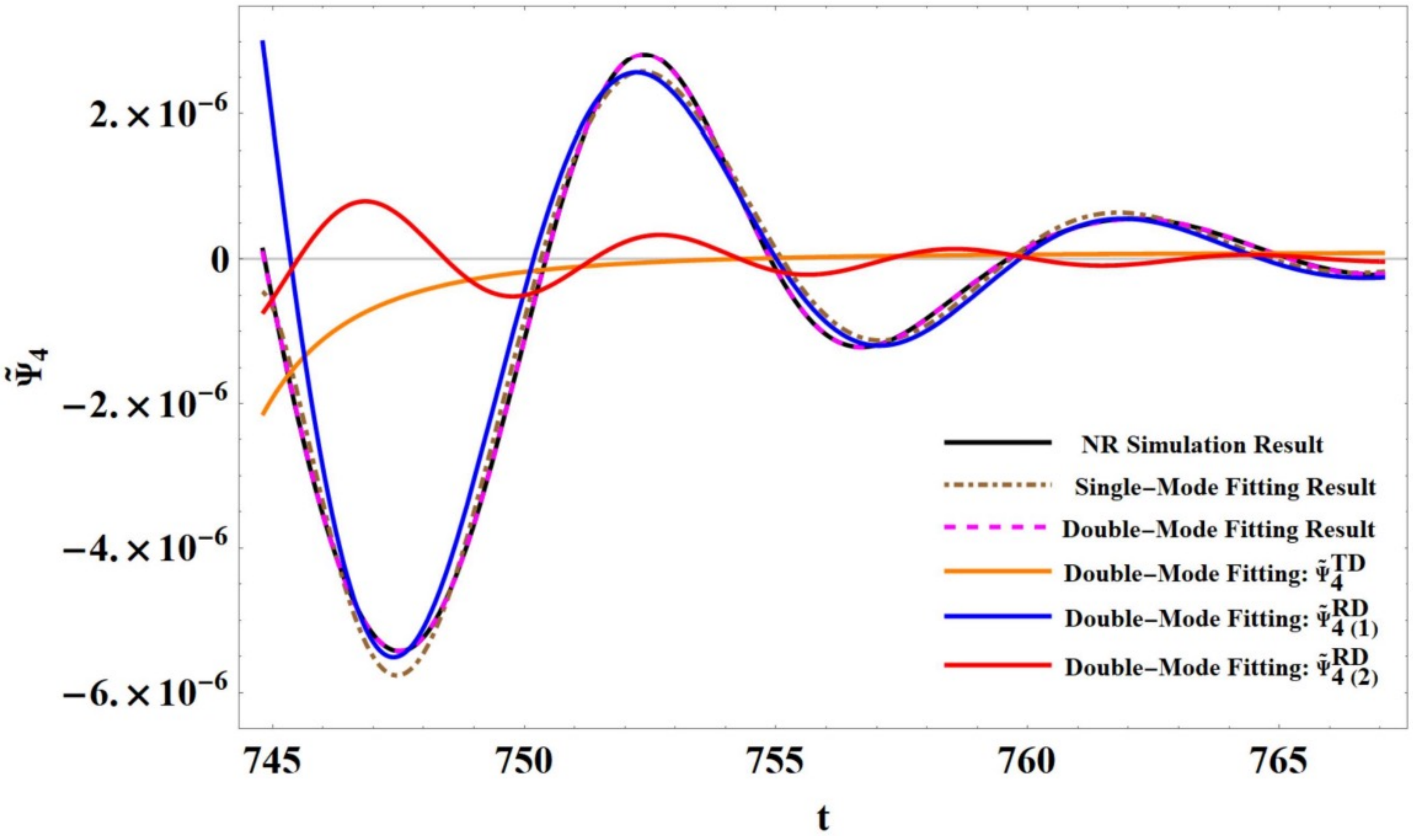


$\widetilde{\Psi}_4$ Fitting Frequencies (r_{ext}=300, b=8)

In double-mode fitting:

- $\omega_R^{(1)} = 0.650782 \pm 0.000039$
- $\omega_I^{(1)} = 0.158008 \pm 0.000040$
- $\omega_R^{(2)} = 1.0732 \pm 0.000253$
- $\omega_I^{(2)} = 0.148805 \pm 0.000254$

Now, we obtain the frequencies measured at infinity
by extrapolating (200~400M) fitting frequencies.



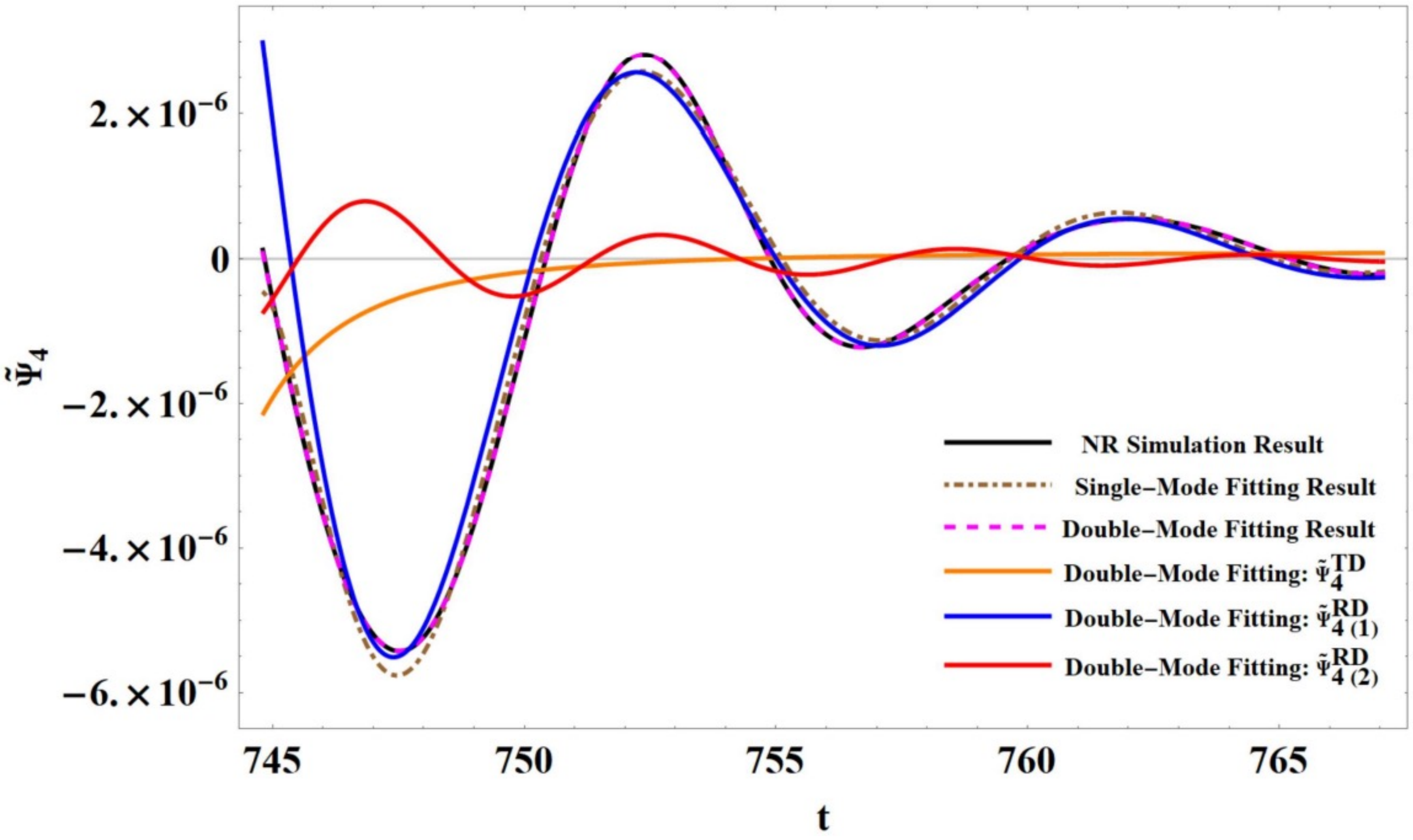
$\widetilde{\Psi}_4$ Fitting Frequencies at ∞ (b=8)

In double-mode fitting:

- $\omega_R^{(1)} = 0.662456$
- $\omega_I^{(1)} = 0.155667$
- $\omega_R^{(2)} = 1.08448$
- $\omega_I^{(2)} = 0.148994$

Note that, this is not ω_{QNM}. This is $\omega_{QNM}/\widetilde{\gamma}$.

And we don't know $\widetilde{\gamma}$.



Comparison with QNM theory values

In double-mode fitting:

- $\omega_R^{(1)} = 0.662456$
- $\omega_I^{(1)} = 0.155667$
- $\omega_R^{(2)} = 1.08448$
- $\omega_I^{(2)} = 0.148994$

  (Note that $\omega_{QNM}/\tilde{\gamma}$)

For l=2, 3 mode,

the QNM frequencies obtained in the perturbation method are given in terms of dimensionless $(M_{BH}\omega_{QNM})$.

The difference comes from $(M_{BH}, \tilde{\gamma})$

….

$l=2$, $n=0$					
j	$m=2$	$m=1$	$m=0$	$m=-1$	$m=-2$
0.00	.3737,.0890	.3737,.0890	.3737,.0890	.3737,.0890	.3737,.0890
0.10	.3870,.0887	.3804,.0888	.3740,.0889	.3678,.0890	.3618,.0891
0.20	.4021,.0883	.3882,.0885	.3751,.0887	.3627,.0889	.3511,.0892
0.30	.4195,.0877	.3973,.0880	.3770,.0884	.3584,.0888	.3413,.0892
0.40	.4398,.0869	.4080,.0873	.3797,.0878	.3546,.0885	.3325,.0891
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.98	.8254,.0386	.5642,.0516	.4223,.0735	.3439,.0837	.2927-.0881

$l=2$, $n=1$					
j	$m=2$	$m=1$	$m=0$	$m=-1$	$m=-2$
0.00	.3467,.2739	.3467,.2739	.3467,.2739	.3467,.2739	.3467,.2739
0.10	.3619,.2725	.3545,.2731	.3472,.2737	.3400,.2744	.3330,.2750
0.20	.3790,.2705	.3635,.2717	.3486,.2730	.3344,.2744	.3206,.2759
0.30	.3984,.2680	.3740,.2698	.3511,.2718	.3296,.2741	.3093,.2765
0.40	.4208,.2647	.3863,.2670	.3547,.2700	.3256,.2734	.2989,.2769
0.50	.4474,.2602	.4009,.2631	.3594,.2674	.3225,.2723	.2893,.2772
0.60	.4798,.2538	.4183,.2575	.3655,.2638	.3201,.2708	.2803,.2773
0.70	.5212,.2442	.4399,.2492	.3732,.2585	.3184,.2686	.2720,.2773
0.80	.5779,.2281	.4676,.2358	.3826,.2507	.3173,.2658	.2643,.2772
0.90	.6677,.1953	.5059,.2097	.3935,.2385	.3167,.2620	.2570,.2770
0.98	.8249,.1159	.5477,.1509	.4014,.2231	.3164,.2581	.2515,.2768

$l=2$, $n=2$					
j	$m=2$	$m=1$	$m=0$	$m=-1$	$m=-2$
0.00	.3011,.4783	.3011,.4783	.3011,.4783	.3011,.4783	.3011,.4783
0.10	.3192,.4735	.3104,.4756	.3017,.4778	.2932,.4801	.2846,.4825
0.20	.3393,.4679	.3214,.4719	.3038,.4764	.2866,.4811	.2697,.4862
0.30	.3619,.4613	.3342,.4671	.3074,.4739	.2813,.4814	.2559,.4895
0.40	.3878,.4533	.3492,.4607	.3124,.4701	.2772,.4808	.2433,.4925
0.50	.4179,.4433	.3669,.4522	.3190,.4647	.2741,.4794	.2316,.4952
0.60	.4542,.4303	.3878,.4407	.3273,.4571	.2721,.4768	.2207,.4977
0.70	.4999,.4123	.4133,.4241	.3374,.4464	.2709,.4729	.2107,.4999
0.80	.5622,.3839	.4451,.3984	.3488,.4307	.2703,.4674	.2013,.5019
0.90	.6598,.3275	.4867,.3502	.3591,.4067	.2697,.4600	.1925,.5038
0.98	.8238,.1933	.5201,.2331	.3599,.3808	.2686,.4527	.1858,.5051

TABLE III: First three overtones for $l=3$.

$l=3$, $n=0$							
j	$m=3$	$m=2$	$m=1$	$m=0$	$m=-1$	$m=-2$	$m=-3$
0.00	.5994,.0927	.5994,.0927	.5994,.0927	.5994,.0927	.5994,.0927	.5994,.0927	.5994,.0927
0.10	.6208,.0924	.6137,.0925	.6067,.0926	.5999,.0926	.5932,.0927	.5867,.0928	.5802,.0928
0.20	.6448,.0920	.6297,.0921	.6153,.0923	.6014,.0924	.5880,.0926	.5752,.0927	.5628,.0929
0.30	.6721,.0913	.6480,.0915	.6252,.0918	.6038,.0921	.5837,.0923	.5647,.0926	.5469,.0928
0.40	.7037,.0902	.6689,.0906	.6369,.0911	.6074,.0915	.5802,.0920	.5553,.0924	.5323,.0927
0.50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	467,.0920	.5188,.0925
0.60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	388,.0916	.5063,.0922
0.70	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	316,.0912	.4946,.0919
0.80	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	250,.0906	.4837,.0916
0.90	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	191,.0900	.4735,.0913
0.98	1.2602,.0387	.9769,.0453	.7833,.0643	.6615,.0782	.5773,.0856	.5146,.0894	.4657,.0910

$l=3$, $n=1$							
j	$m=3$	$m=2$	$m=1$	$m=0$	$m=-1$	$m=-2$	$m=-3$
0.00	.5826,.2813	.5826,.2813	.5826,.2813	.5826,.2813	.5826,.2813	.5826,.2813	.5826,.2813
0.10	.6053,.2802	.5978,.2805	.5904,.2808	.5832,.2811	.5761,.2814	.5691,.2817	.5623,.2820
0.20	.6306,.2785	.6148,.2791	.5995,.2798	.5848,.2804	.5706,.2811	.5569,.2817	.5437,.2823
0.30	.6593,.2761	.6341,.2771	.6102,.2782	.5876,.2793	.5662,.2804	.5459,.2814	.5268,.2823
0.40	.6924,.2726	.6563,.2741	.6227,.2758	.5916,.2776	.5627,.2793	.5360,.2808	.5112,.2822
0.50	.7312,.2676	.6821,.2698	.6375,.2724	.5969,.2751	.5602,.2777	.5270,.2800	.4967,.2818
0.60	.7782,.2604	.7130,.2634	.6550,.2674	.6038,.2717	.5587,.2756	.5188,.2789	.4833,.2813
0.70	.8375,.2494	.7510,.2538	.6762,.2600	.6124,.2668	.5580,.2728	.5113,.2775	.4708,.2807
0.80	.9176,.2315	.8004,.2379	.7025,.2484	.6230,.2598	.5582,.2692	.5045,.2758	.4590,.2800
0.90	1.0425,.1966	.8714,.2068	.7362,.2277	.6357,.2491	.5593,.2644	.4983,.2739	.4480,.2792
0.98	1.2599,.1161	.9706,.1341	.7679,.1930	.6468,.2359	.5606,.2593	.4938,.2720	.4396,.2784

$l=3$, $n=2$							
j	$m=3$	$m=2$	$m=1$	$m=0$	$m=-1$	$m=-2$	$m=-3$
0.00	.5517,.4791	.5517,.4791	.5517,.4791	.5517,.4791	.5517,.4791	.5517,.4791	.5517,.4791
0.10	.5766,.4763	.5684,.4771	.5603,.4779	.5523,.4787	.5445,.4795	.5368,.4803	.5292,.4812
0.20	.6043,.4725	.5872,.4741	.5705,.4758	.5543,.4775	.5386,.4792	.5234,.4809	.5086,.4826
0.30	.6356,.4674	.6084,.4699	.5825,.4725	.5577,.4753	.5340,.4781	.5114,.4809	.4897,.4836
0.40	.6714,.4605	.6328,.4640	.5966,.4679	.5625,.4721	.5305,.4763	.5005,.4804	.4723,.4842
0.50	.7131,.4511	.6612,.4558	.6131,.4614	.5689,.4675	.5282,.4736	.4907,.4793	.4561,.4845
0.60	.7632,.4379	.6948,.4441	.6327,.4522	.5769,.4611	.5270,.4698	.4819,.4778	.4410,.4846
0.70	.8258,.4185	.7358,.4268	.6561,.4387	.5868,.4521	.5267,.4648	.4739,.4757	.4269,.4843
0.80	.9096,.3874	.7884,.3988	.6844,.4177	.5984,.4391	.5272,.4581	.4667,.4730	.4136,.4839
0.90	1.0383,.3282	.8622,.3449	.7177,.3806	.6106,.4197	.5282,.4492	.4600,.4696	.4011,.4834
0.98	1.2592,.1935	.9605,.2181	.7349,.3224	.6176,.3976	.5286,.4403	.4550,.4665	.3916,.4828

.3737,.0890

.5994,.0927

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Boosted Mass Approximation

In double-mode fitting:

- $\omega_R^{(1)} = 0.662456$
- $\omega_I^{(1)} = 0.155667$
- $\omega_R^{(2)} = 1.08448$
- $\omega_I^{(2)} = 0.148994$

  (Note that $\omega_{QNM}/\tilde{\gamma}$)

$$M_{\mathrm{BH}}\omega_{\mathrm{QNM}} = (\tilde{\gamma} M_{\mathrm{BH}})\left(\frac{\omega_{\mathrm{QNM}}}{\tilde{\gamma}}\right) \equiv M_{\mathrm{boost}}\omega_{\infty}$$

Here $M_{\mathrm{boost}} \equiv \tilde{\gamma} M_{\mathrm{BH}}$ is the boosted mass

$$M_{\mathrm{boost}} \simeq \gamma_{\infty} M_{BH} = (M_{\mathrm{ADM}} - \Delta E)/2$$

Here γ_{∞} is the Lorentz factor of the individual BHs when they get separated away sufficiently, which differs from $\tilde{\gamma}$ about 1.5 %. ΔE is the radiated energy through GWs in total. Our numerical results give $\Delta E = 0.01312$, 0.00507 for $b = 8\,M$, $10\,M$, respectively.

With this approximation,

we can compare NR-obtained dimensionless frequencies with theoretically obtained one when we compare scattering RD frequencies with Schwarzschild one.

QNM Comparison (b=8, fine resolution)

Theory's $\{M\omega_R, M\omega_I\}_{1\text{-}2} = \{0.373672, 0.0889623\}$ $\{M\omega_R, M\omega_I\}_{1\text{-}3} = \{0.599443, 0.092703\}$

NR result $\{M\omega_R, M\omega_I\} = \{0.376596, 0.0857404, 0.595716, 0.132749\}$

NR deviation (%) $\left\{\frac{M\omega_R^{NR} - M\omega_R}{M\omega_R}, \frac{M\omega_I^{NR} - M\omega_I}{M\omega_I}\right\} = \{0.782512, -3.62169, -0.62183, 43.1986\}$

- **Deviations only a few % for the dominant mode.**
- **Using the Richardson extrapolation with a conservative assumption of 4th-order convergence for the results from the different resolutions, finally, we present the RD frequencies (see next slide table).**

Final Result: QNM Comparison (1)

RD Frequencies		$M\omega_R^{(l=2)}$	$M\omega_I^{(l=2)}$	$M\omega_R^{(l=3)}$	$M\omega_I^{(l=3)}$
Perturbation Theory		0.3737	0.0890	0.5994	0.0927
NR ($b = 8$)	*single-mode*	0.3915(91) (+4.8 %)	0.0906(45) (+1.8 %)	–	–
	double-mode	0.3798(11) (+1.6 %)	0.0894(4) (+0.5 %)	0.5965(234) (−0.5 %)	0.0617(234) (−33.4 %)
NR ($b = 10$)	*single-mode*	0.3738(14) (+0.0 %)	0.0737(68) (−17.2 %)	–	–
	double-mode	0.3741(25) (+0.1 %)	0.0826(31) (−7.2 %)	0.6498(541) (+8.4 %)	0.0549(778) (−40.7 %)

TABLE I. Ringdown frequencies numerically obtained from scattering BHs. The percentages represent deviations from the theoretical values, $(M\omega)_{\rm QNM}$, for a single Schwarzschild BH.

- For the case of b = 8 M, our single-mode fitting shows that the tidally-deformed RD wave could be due to the excitation of the fundamental l = 2 quasi-normal mode within about (4.8, 1.8) % for the oscillatory and damping parts, respectively.
- Actually, the QNM frequencies of the overtone mode or higher multipole mode give much larger disagreement, excluding such modes as the dominant excitation.

Final Result: QNM Comparison (2)

RD Frequencies		$M\omega_{\mathrm{R}}^{(l=2)}$	$M\omega_{\mathrm{I}}^{(l=2)}$	$M\omega_{\mathrm{R}}^{(l=3)}$	$M\omega_{\mathrm{I}}^{(l=3)}$
Perturbation Theory		0.3737	0.0890	0.5994	0.0927
NR ($b = 8$)	*single-mode*	0.3915(91) (+4.8 %)	0.0906(45) (+1.8 %)	–	–
	double-mode	0.3798(11) (+1.6 %)	0.0894(4) (+0.5 %)	0.5965(234) (−0.5 %)	0.0617(234) (−33.4 %)
NR ($b = 10$)	*single-mode*	0.3738(14) (+0.0 %)	0.0737(68) (−17.2 %)	–	–
	double-mode	0.3741(25) (+0.1 %)	0.0826(31) (−7.2 %)	0.6498(541) (+8.4 %)	0.0549(778) (−40.7 %)

TABLE I. Ringdown frequencies numerically obtained from scattering BHs. The percentages represent deviations from the theoretical values, $(M\omega)_{\mathrm{QNM}}$, for a single Schwarzschild BH.

- For the possibility of sub-dominant QNM excitations, our double-mode fitting shows that l = 2, 3 fundamental modes are excited in agreement within about (1.6, 0.5) % and (0.5, 33.4) %, respectively.
  Note that the agreement with the l = 2 mode is improved.

Final Result: QNM Comparison (3)

RD Frequencies		$M\omega_R^{(l=2)}$	$M\omega_I^{(l=2)}$	$M\omega_R^{(l=3)}$	$M\omega_I^{(l=3)}$
Perturbation Theory		0.3737	0.0890	0.5994	0.0927
NR ($b=8$)	single-mode	0.3915(91) (+4.8 %)	0.0906(45) (+1.8 %)	–	–
	double-mode	0.3798(11) (+1.6 %)	0.0894(4) (+0.5 %)	0.5965(234) (−0.5 %)	0.0617(234) (−33.4 %)
NR ($b=10$)	single-mode	0.3738(14) (+0.0 %)	0.0737(68) (−17.2 %)	–	–
	double-mode	0.3741(25) (+0.1 %)	0.0826(31) (−7.2 %)	0.6498(541) (+8.4 %)	0.0549(778) (−40.7 %)

TABLE I. Ringdown frequencies numerically obtained from scattering BHs. The percentages represent deviations from the theoretical values, $(M\omega)_{\rm QNM}$, for a single Schwarzschild BH.

- For the case of a less strong encounter with b = 10 M,
  slight enhancement in the agreements of real frequencies for the dominant mode.
  This enhancement might be because two BHs are in the linear regime more than the case of b = 8 M.
  Or, it could be simply because, the subdominant mode is too weak
  so that the dominant mode fitting is already good enough.
  Such behavior may also result in a larger deviation for the subdominant mode.

Final Result: QNM Comparison (4)

RD Frequencies		$M\omega_{\mathrm{R}}^{(l=2)}$	$M\omega_{\mathrm{I}}^{(l=2)}$	$M\omega_{\mathrm{R}}^{(l=3)}$	$M\omega_{\mathrm{I}}^{(l=3)}$
Perturbation Theory		0.3737	0.0890	0.5994	0.0927
NR $(b=8)$	*single-mode*	0.3915(91) (+4.8 %)	0.0906(45) (+1.8 %)	–	–
	double-mode	0.3798(11) (+1.6 %)	0.0894(4) (+0.5 %)	0.5965(234) (−0.5 %)	0.0617(234) (−33.4 %)
NR $(b=10)$	*single-mode*	0.3738(14) (+0.0 %)	0.0737(68) (−17.2 %)	–	–
	double-mode	0.3741(25) (+0.1 %)	0.0826(31) (−7.2 %)	0.6498(541) (+8.4 %)	0.0549(778) (−40.7 %)

TABLE I. Ringdown frequencies numerically obtained from scattering BHs. The percentages represent deviations from the theoretical values, $(M\omega)_{\mathrm{QNM}}$, for a single Schwarzschild BH.

- Note first that the agreement would depend on how accurately the TD wave can be subtracted from the whole ringdown signal.
- We speculate that the decaying behavior of the RD wave is more sensitive than its oscillatory one.
- Consequently, the agreements in imaginary frequencies would be worse than those in real frequencies.

Final Result: QNM Comparison (5)

RD Frequencies		$M\omega_R^{(l=2)}$	$M\omega_I^{(l=2)}$	$M\omega_R^{(l=3)}$	$M\omega_I^{(l=3)}$
Perturbation Theory		0.3737	0.0890	0.5994	0.0927
NR ($b = 8$)	*single-mode*	0.3915(91) (+4.8 %)	0.0906(45) (+1.8 %)	–	–
	double-mode	0.3798(11) (+1.6 %)	0.0894(4) (+0.5 %)	0.5965(234) (−0.5 %)	0.0617(234) (−33.4 %)
NR ($b = 10$)	*single-mode*	0.3738(14) (+0.0 %)	0.0737(68) (−17.2 %)	–	–
	double-mode	0.3741(25) (+0.1 %)	0.0826(31) (−7.2 %)	0.6498(541) (+8.4 %)	0.0549(778) (−40.7 %)

TABLE I. Ringdown frequencies numerically obtained from scattering BHs. The percentages represent deviations from the theoretical values, $(M\omega)_{\rm QNM}$, for a single Schwarzschild BH.

- The relative suppression of the TD wave is not severe in the case of b = 10 M, which likely leads to larger deviations in the imaginary frequencies.
- It is important to note that the SRD frequency we obtained matches well with the QNM, not that the SRD must match the QNM.

Conclusion

- First numerical evidence that hyperbolic BH encounters can produce non-merging ringdown gravitational waves from dynamic tidal deformations.
- Ringdown waves in Ψ_4 constitute about 10% of the strength compared to trajectory-driven radiation, matching fundamental quasi-normal modes.
- We offers a new method to test strong gravitational interactions, expanding the scope beyond binary BH coalescence.
- If observed, these waves could reveal critical system parameters, such as inclination angle and BH velocities.
- Our work paves the way for studying more complex scenarios, including encounters with unequal mass, spinning BHs, and other compact objects.
- Scattering ringdown enhances our understanding of strong gravitational interactions and the potential for observational tools in astrophysics.

Thank You!