Gravitational-wave Lensing

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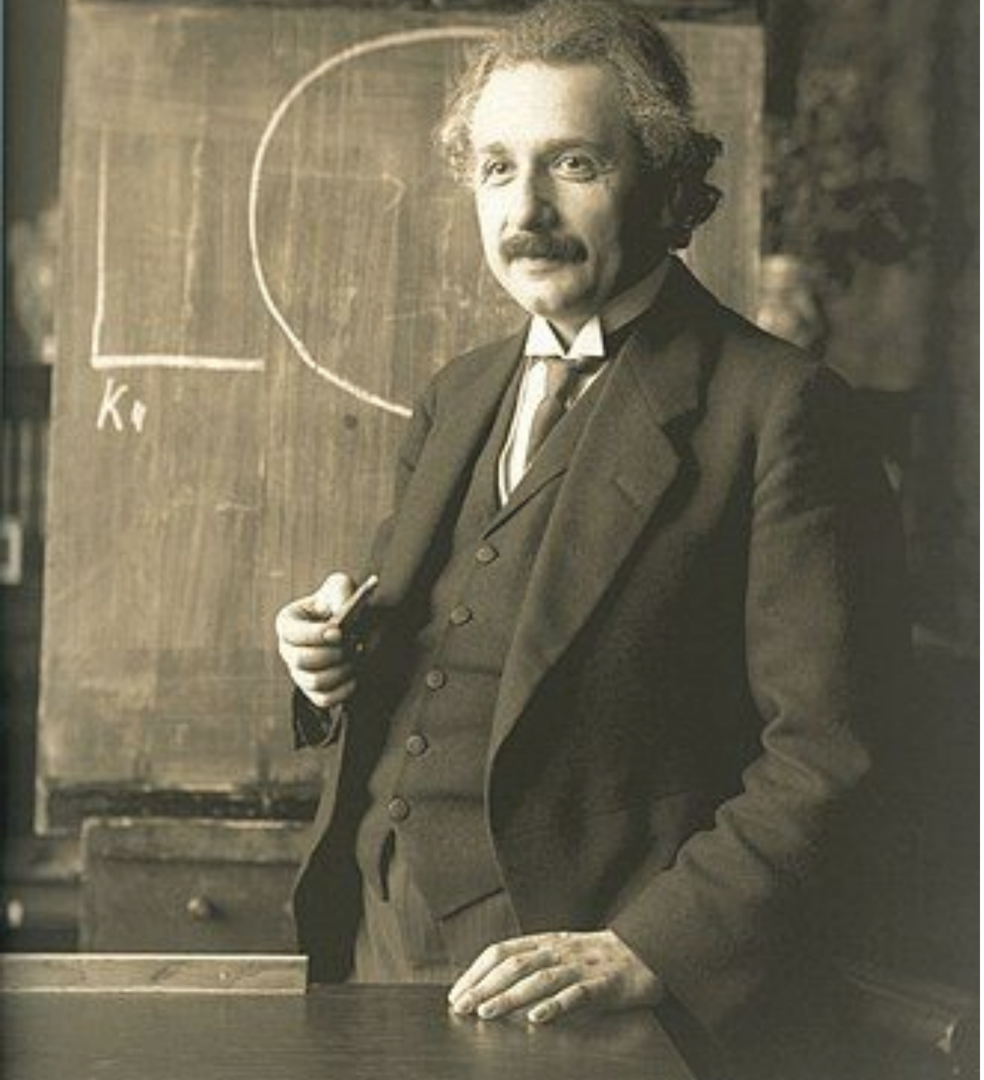
Credits: R. Buscicchio
University of Birmingham

Introduction



Gravitational Waves

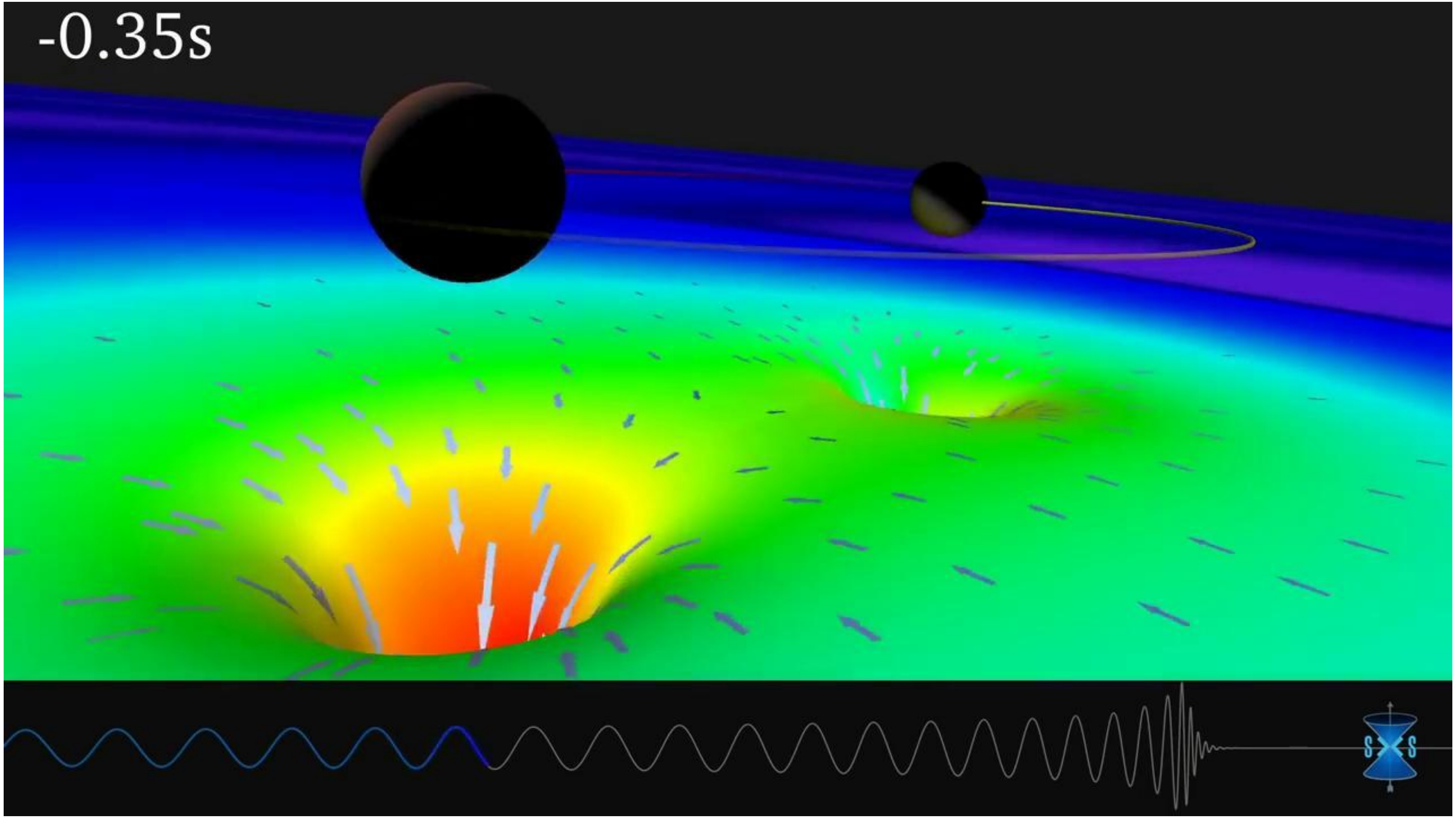
- Predicted by Einstein in 1916

Properties:

- Travels at the speed of light
- Two polarizations
- Passes through matter unimpeded

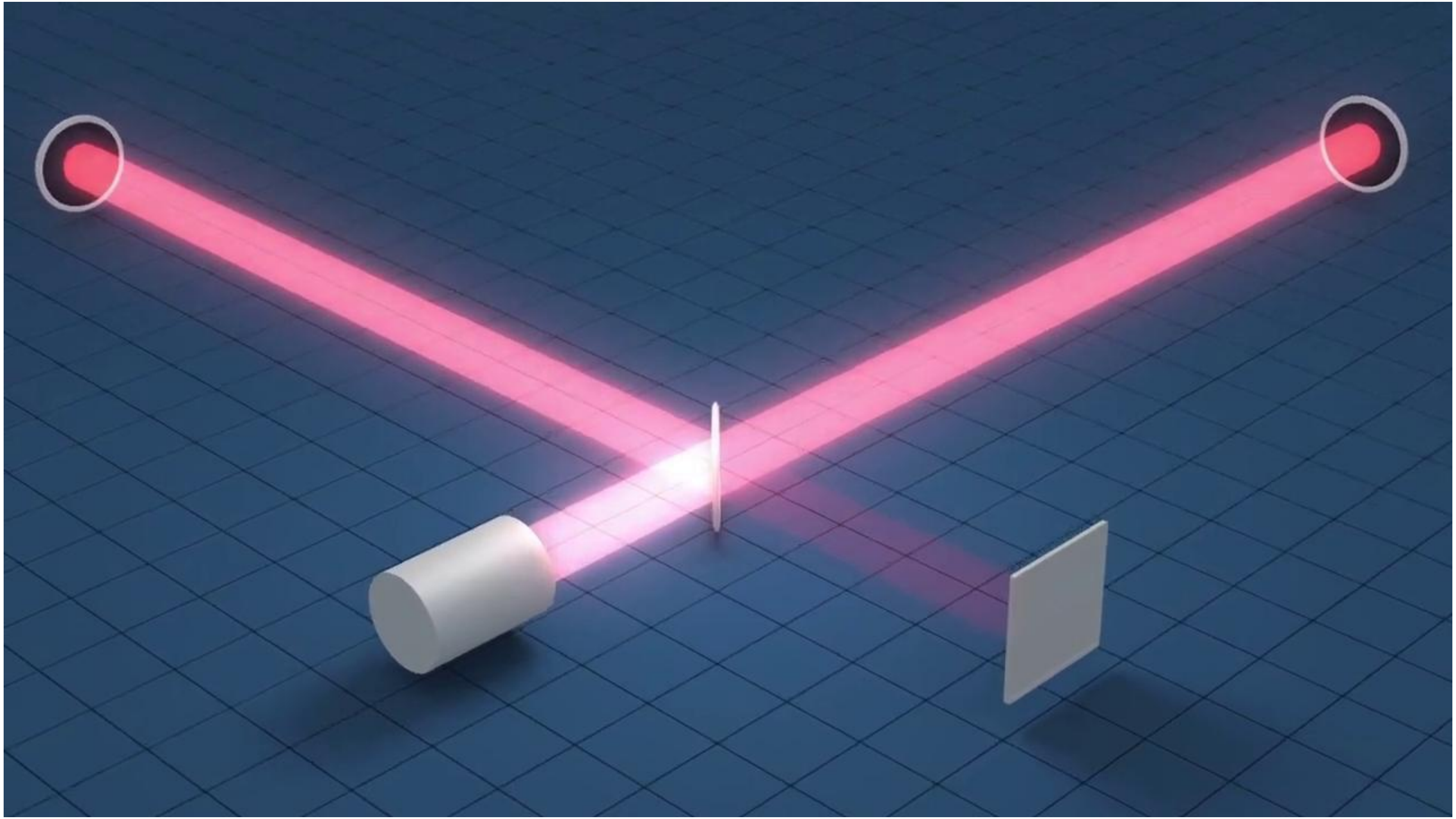
$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Gravitational-Wave Generation



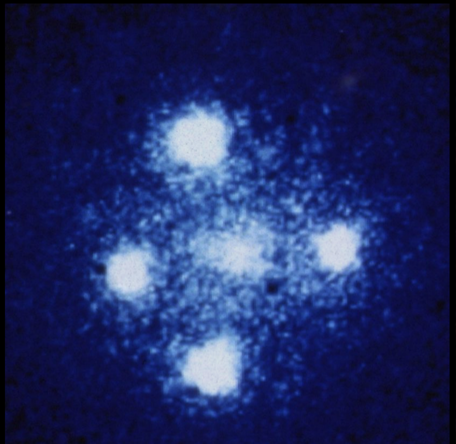
Credits: LIGO/Virgo, SXS

Gravitational-Wave Detection

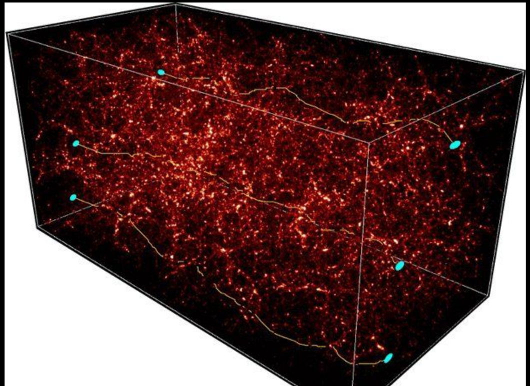


Credits: LIGO/Virgo, SXS

Gravitational lensing of light



Source: NASA, ESA & STScI

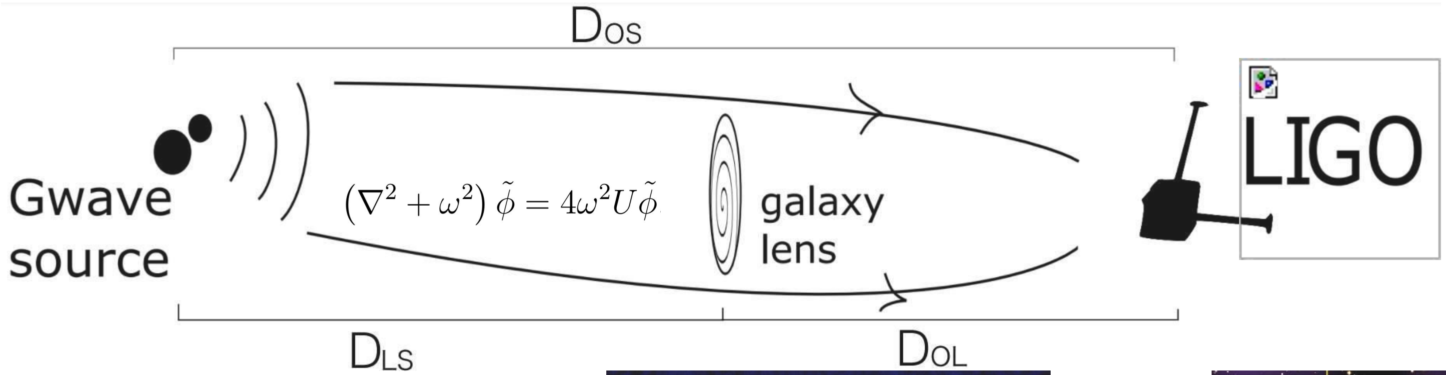


Source: Munshi et.al 2006

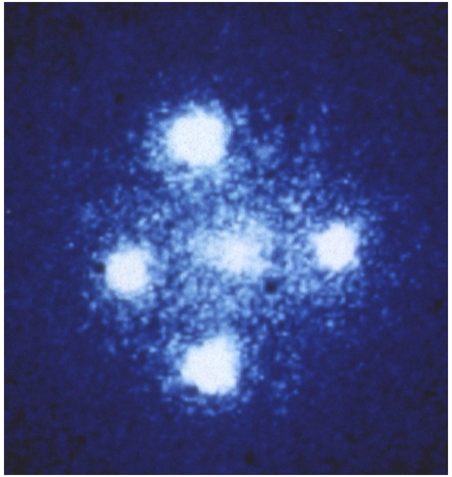
Source: NASA

Source: Chandra

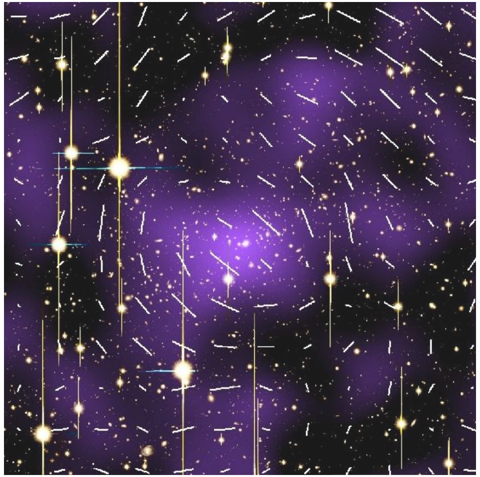
Gravitational lensing of gravitational waves



ESA/Hubble & NASA

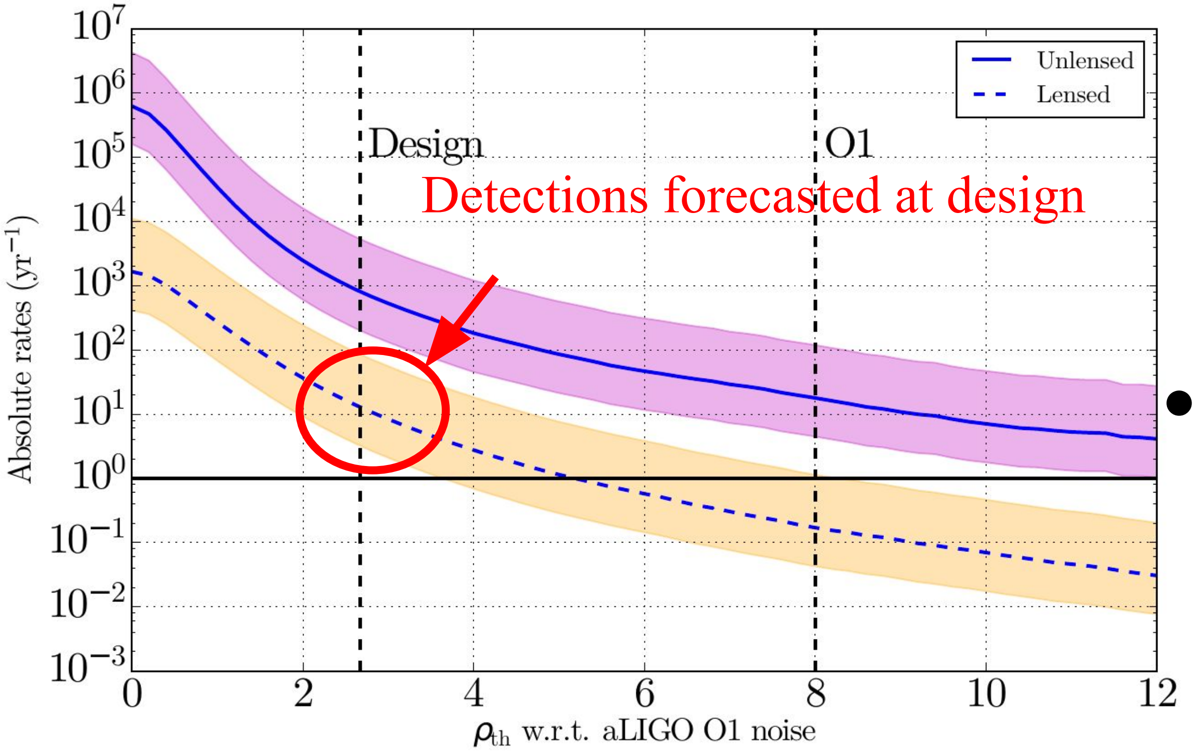


NASA, ESA & STScI



Oguri et al.

Why is gravitational lensing exciting now?



- Forecasts predict strong lensing at a reasonable rate

Ng et al. (2017); see also *Li et al. (2018)*, *Oguri (2018); Wierda et al. (2021); Xu et al. (2021)*

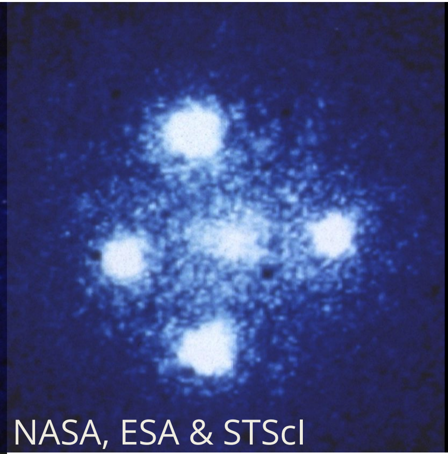
Why is gravitational lensing exciting now?

An entirely new avenue to probe gravitational lensing

→new studies of astrophysics, cosmology, and fundamental physics

- **Study the origin of black holes** *(Hannuksela et al. 2020)*
- **Tests of fundamental physics** *(Collett & Bacon 2017,Wong et al., in prep)*
- **Study the expansion of the Universe** *(Baker & Trodden 2017, Liao et al. 2017, Hannuksela et al. 2020)*
- **Study microlens populations** *(Lai et al. 2018, Jung et al., 2019)*
- **Study wave optics** *(Cheung et al., 2020)*


ESA/Hubble & NASA


NASA, ESA & STScI

Chandra


Oguri et al.

How does gravitational-wave lensing work?

Basics of gravitational-wave lensing

Described by the amplification factor
(Takahashi & Nakamura, 2002)

$$F(f) = \frac{D_S \, \xi_0^2 (1+z_L)}{D_L D_{LS}} \frac{f}{i} \int d^2\mathbf{x} \ \exp\left[\, 2\pi i f t_d(\mathbf{x}, \mathbf{y}) \right]$$

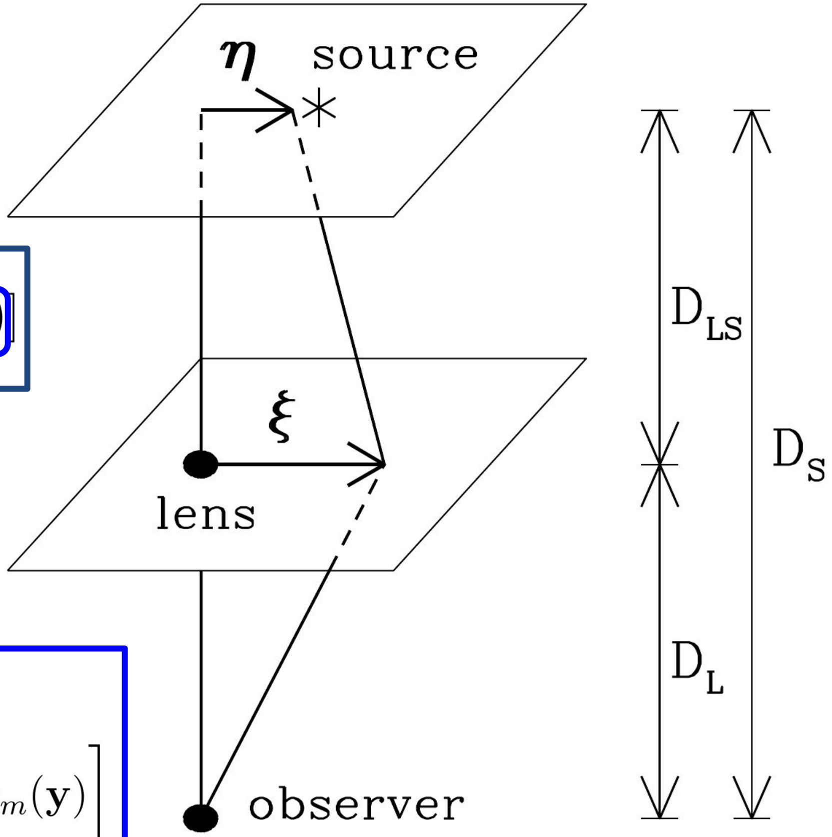
Angular diameter distances

Lens redshift

Lensing time delay

Lens potential

$$t_d(\mathbf{x}, \mathbf{y}) = \frac{D_S \xi_0^2}{D_L D_{LS}} (1+z_L) \left[\frac{1}{2} |\mathbf{x} - \mathbf{y}|^2 - \psi(\mathbf{x}) + \phi_m(\mathbf{y}) \right]$$



Basics of gravitational-wave lensing

Simplest example: **Point Mass Lens**

$$F(w,y) = \exp\left[\frac{\pi w}{4} + i\frac{w}{2}\left\{\ln\left(\frac{w}{2}\right) - \frac{(\sqrt{y^2+4}-y)^2}{4} + \ln\left(\frac{y+\sqrt{y^2+4}}{2}\right)\right\}\right]\Gamma\left(1-\frac{i}{2}w\right) \times {}_1F_1\left(\frac{i}{2}w, 1; \frac{i}{2}wy^2\right),$$

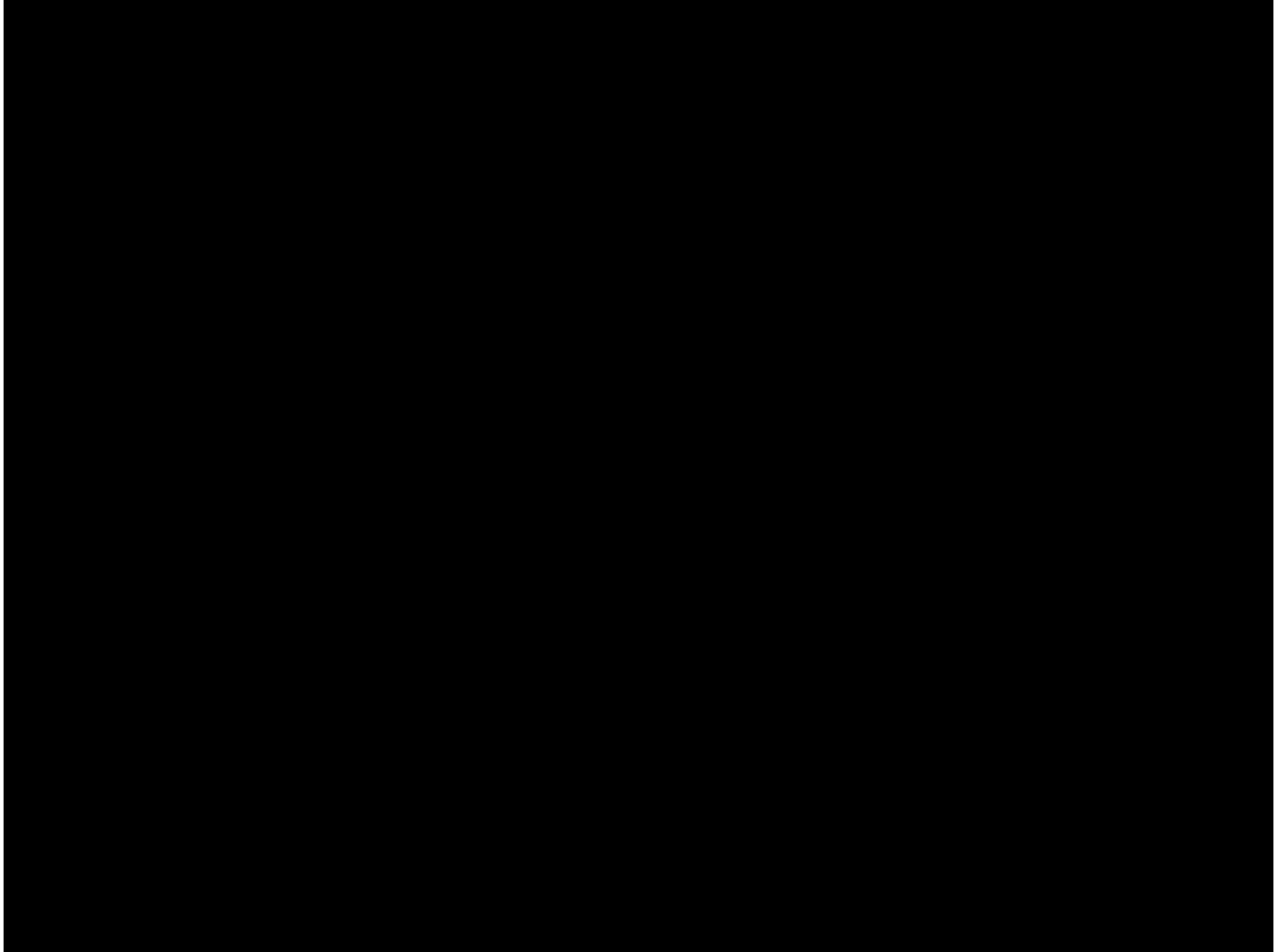
Described by:

- Lens mass
- Lens position
- Angular diameter distances

Simplifications:

1) Analytical solution to the amplification factor
2) Spherically symmetric model

$$h_L(f) = F(f)h(f)$$



Basics of gravitational-wave lensing

Geometrical optics limit (λ<<M□)

$$F(f) = \sum_j |\mu_j|^{1/2} \exp\left[\, 2\pi i f t_{d,j} - i\pi n_j \right]$$

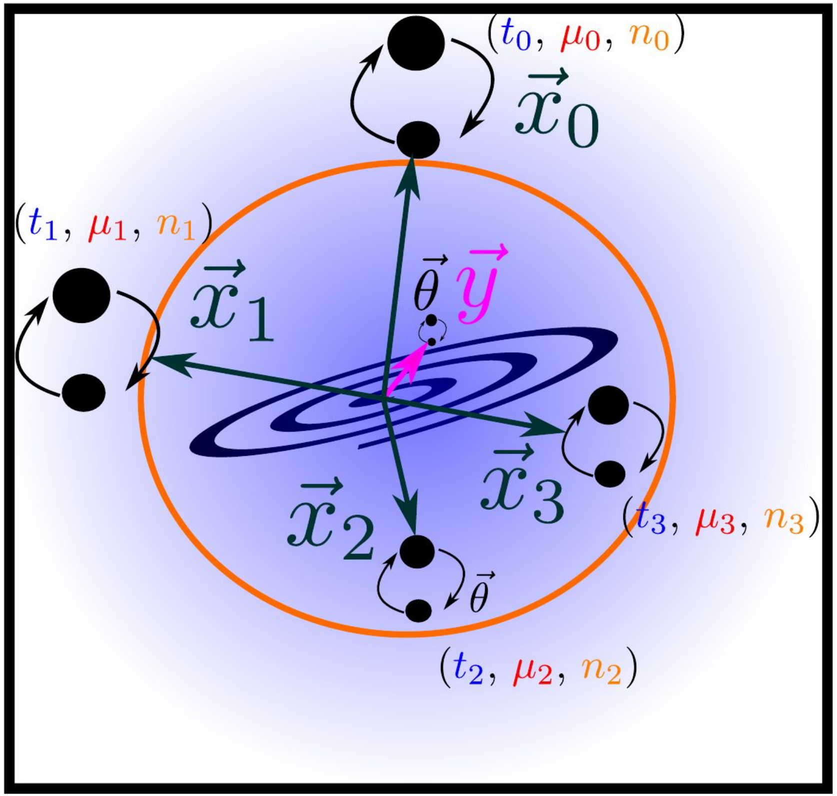
Time delay

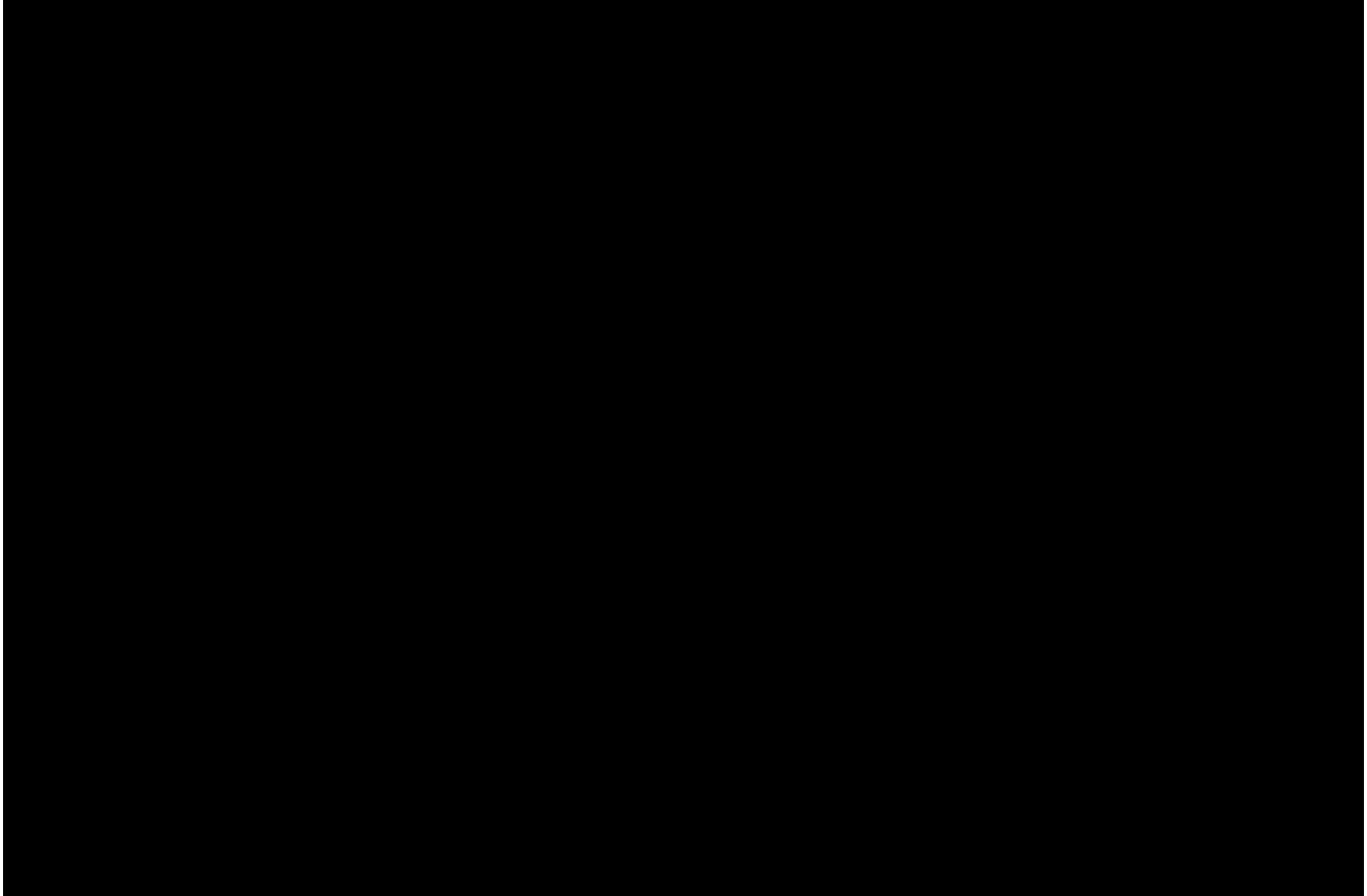
Image magnification

Morse phase

$$h_L(f) = F(f)h(f)$$

Summed over number of images





Basics of gravitational-wave lensing

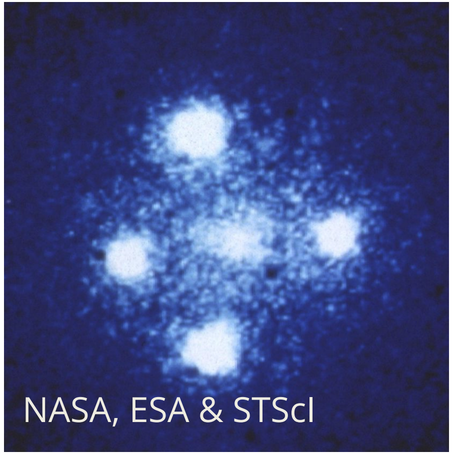
Wave optics lensing (GW lensing)	**Geometrical optics (typical light lensing scenario)**
Valid for all lenses	Valid for large lenses
Suppresses small-scale structure	Does not suppress small-scale structure
Solved using amplification factor (expensive) $F(f) = \frac{D_S\ \xi_0^2(1+z_L)}{D_L D_{LS}} \frac{f}{i} \int d^2\mathbf{x}\ \exp\left[\ 2\pi i f t_d(\mathbf{x},\mathbf{y})\right]$	Solved using the lens equation (cheap) $F(f) = \sum_j \|\mu_j\|^{1/2} \exp\left[\ 2\pi i f t_{d,j} - i\pi n_j\right]$

How do we detect gravitational-wave lensing?

Detection: GW vs EM observatories

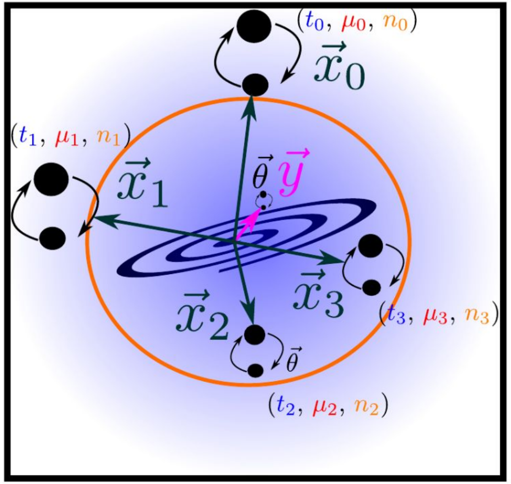
Electromagnetic observatories	**Gravitational-wave observatories**
Main observable: Electromagnetic sources	Main observable: Gravitational sources
Good angular resolution	Poor angular resolution
Fair time resolution	Extremely precise time resolution
Detection method: Measure flux intensities	Detection method: Waveform templates
Wealth of gravitational lensing detections	No confirmed gravitationally lensed detections

Strong lensing challenge



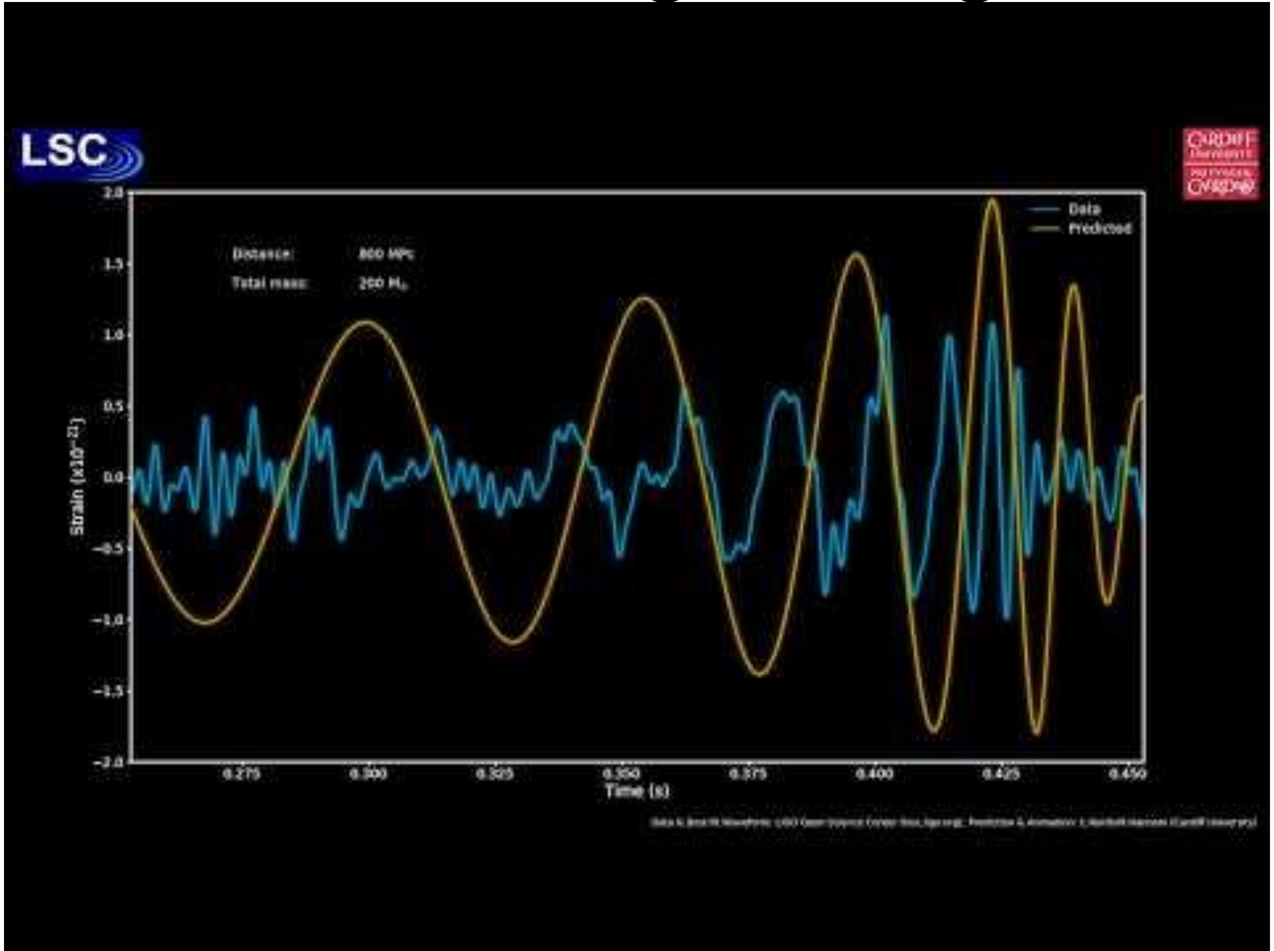
Question: How can we detect lensing if we can not see the multiple images in the sky?

arcsecond~10^(-4) deg



- Targets: Galaxies, galaxy clusters

Strong lensing challenge



Parameter estimation follows Bayesian analysis

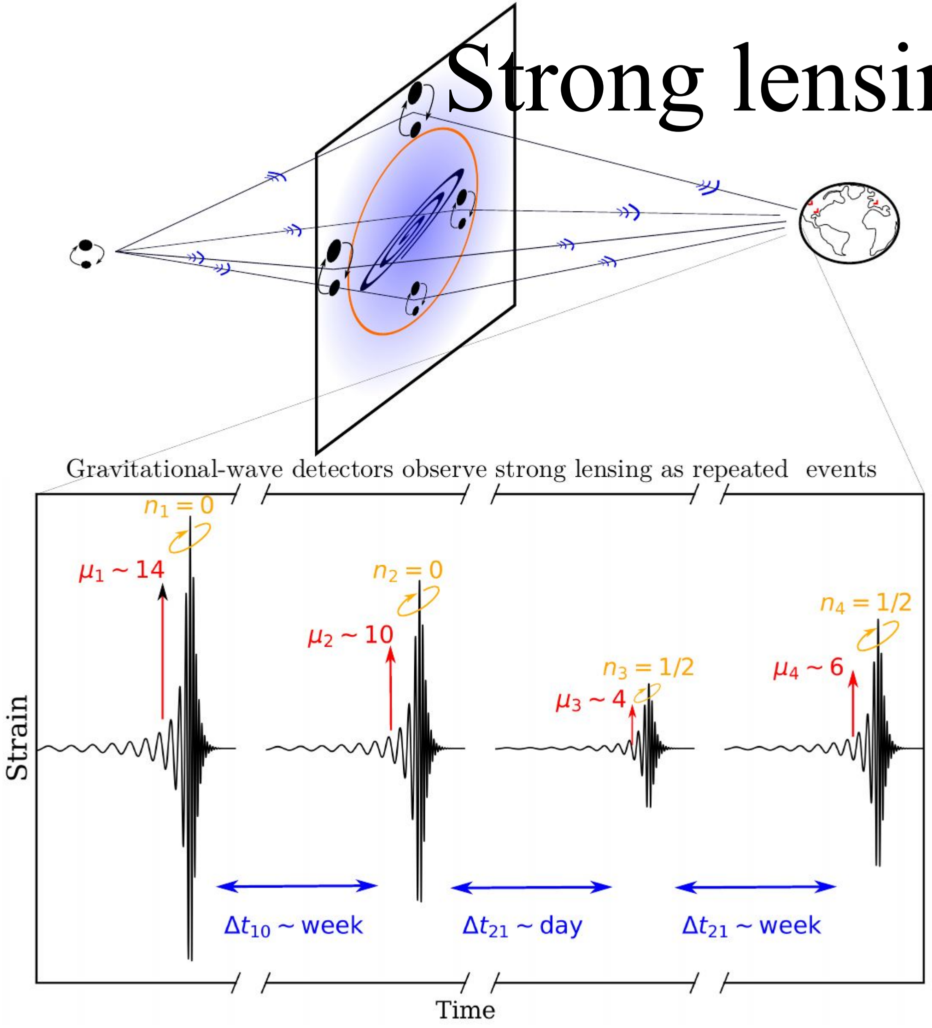
Key components:

- Waveform model known
- Noise model known

Gives us for extremely precise measurement of the binary parameters and the signal

Credits: Cardiff University

Strong lensing challenge

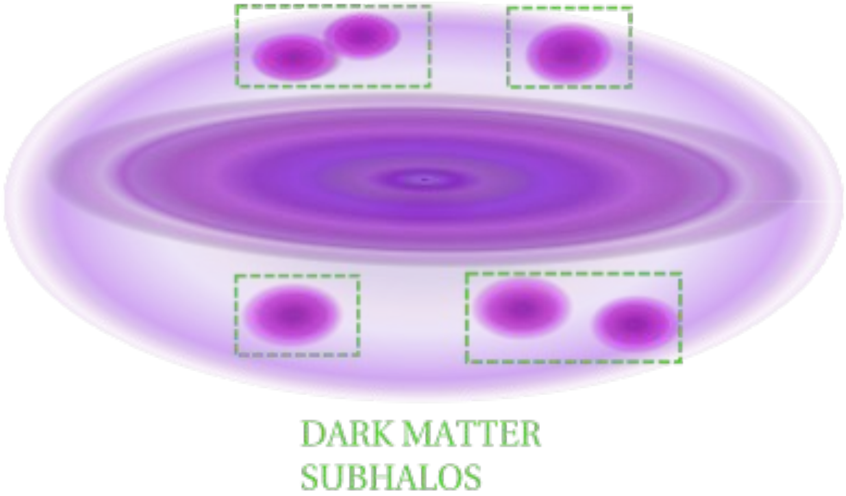


Strong lensing:

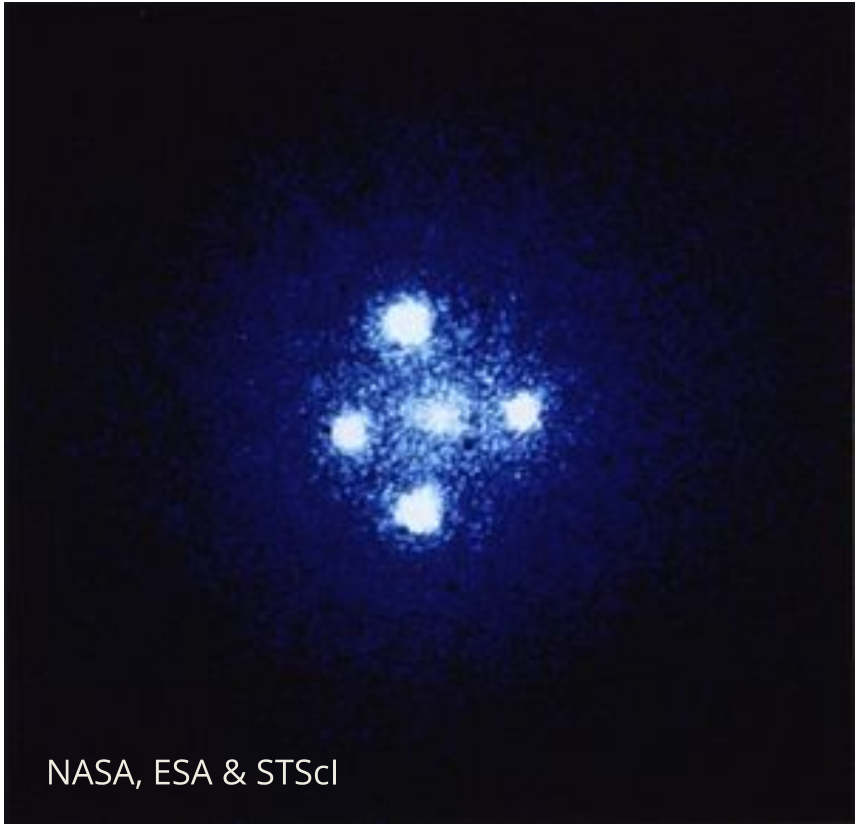
- Identify "repeated events" in the data
- Targets: Galaxies, galaxy clusters

Precise measurement of gravitational-wave image properties

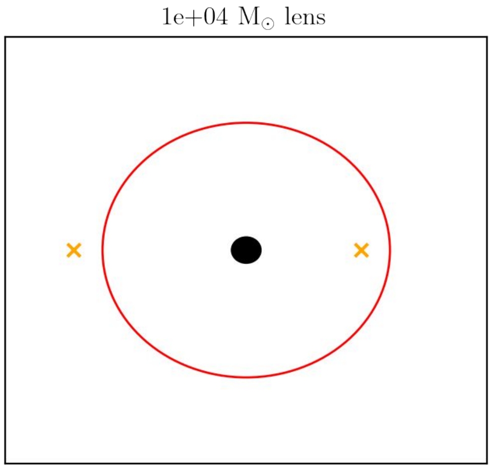
Millilensing challenge

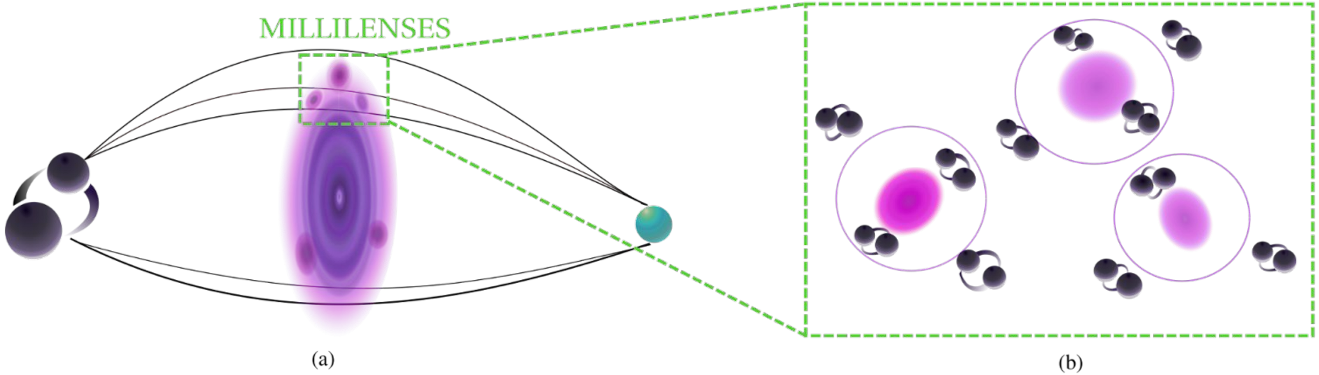


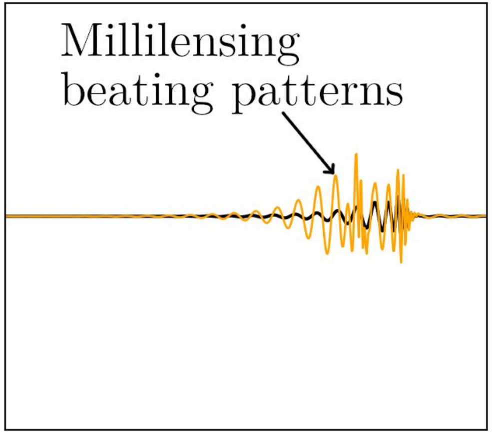
Millilensing by subhalos or more massive black holes can produce milli-images or changes in the intensity of light



Millilensing challenge





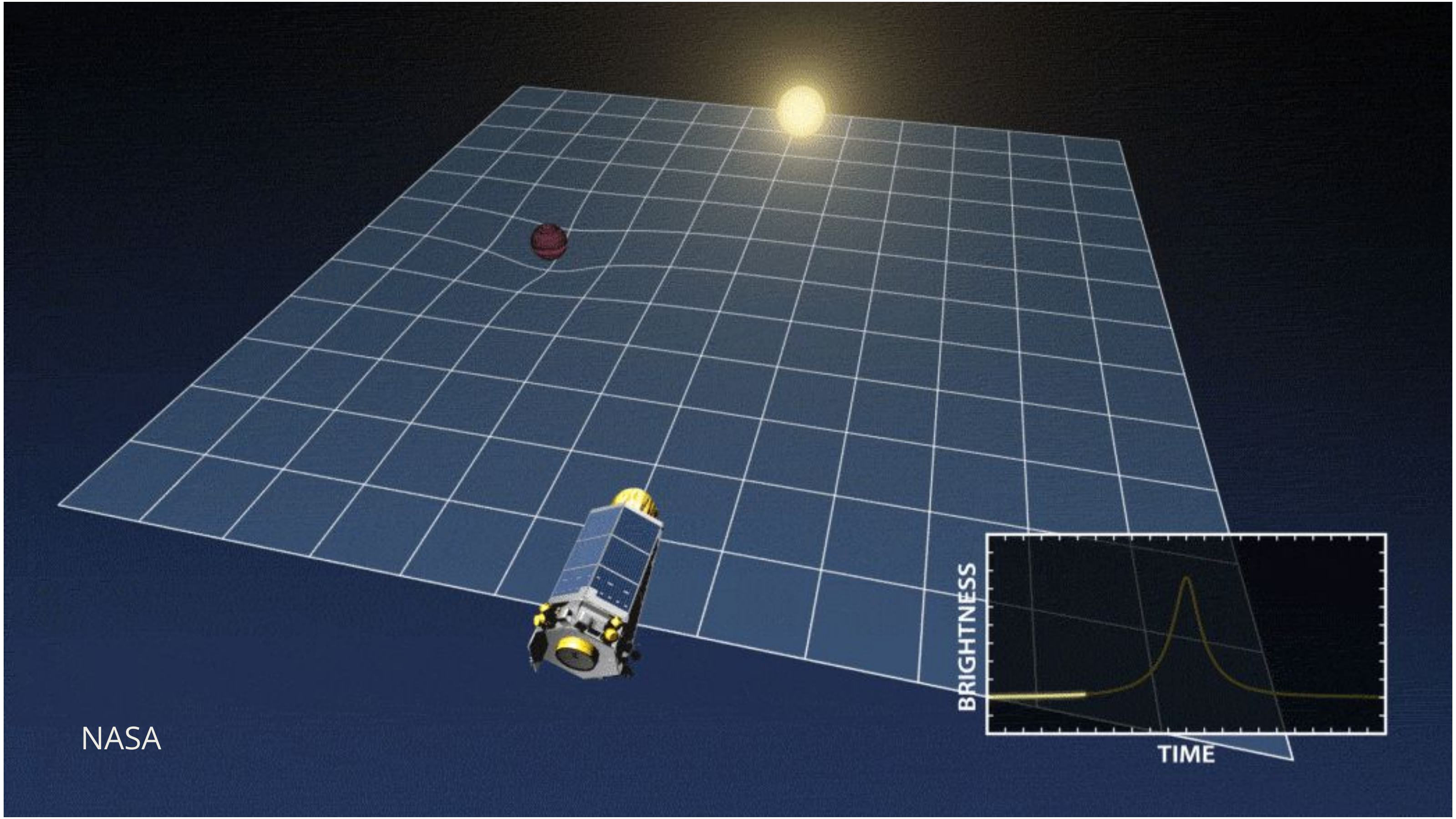


Millilensing:

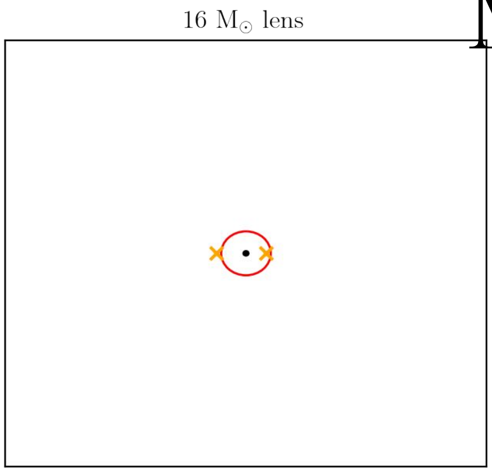
- Modify gravitational-wave templates to identify millilensing “beating patterns”
- Targets: Subhalos, massive black holes

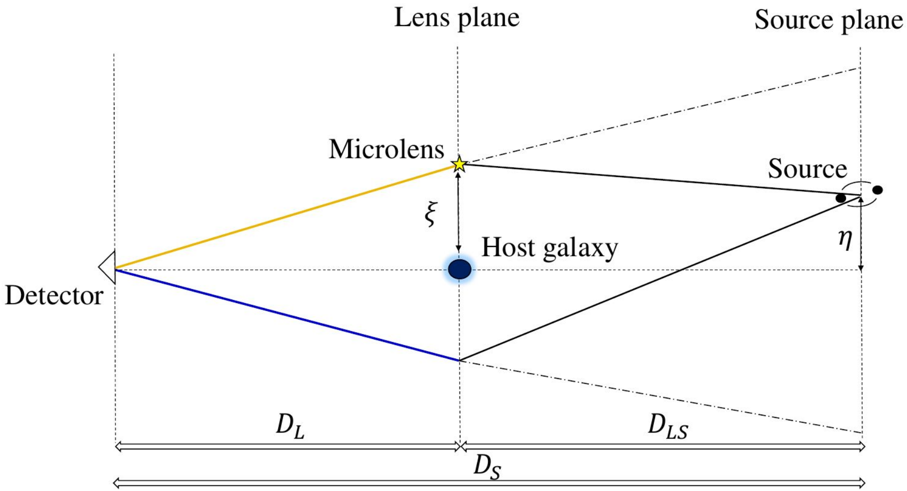
Measurement of the millilens **mass** and source position

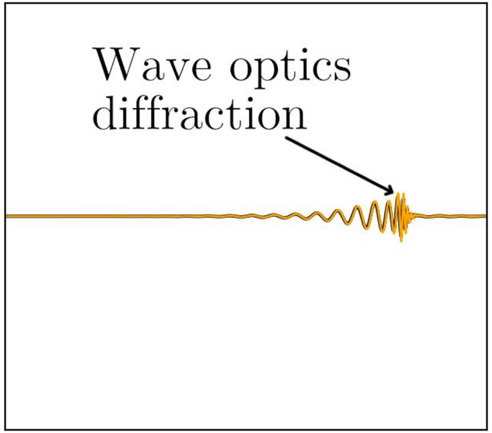
Microlensing challenge



Microlensing challenge







Microlensing:

- Modify gravitational-wave templates to identify microlensing wave optics effects
- Targets: Stars, stellar-mass black holes and primordial black holes

Measurement of the microlensing **mass** and source position

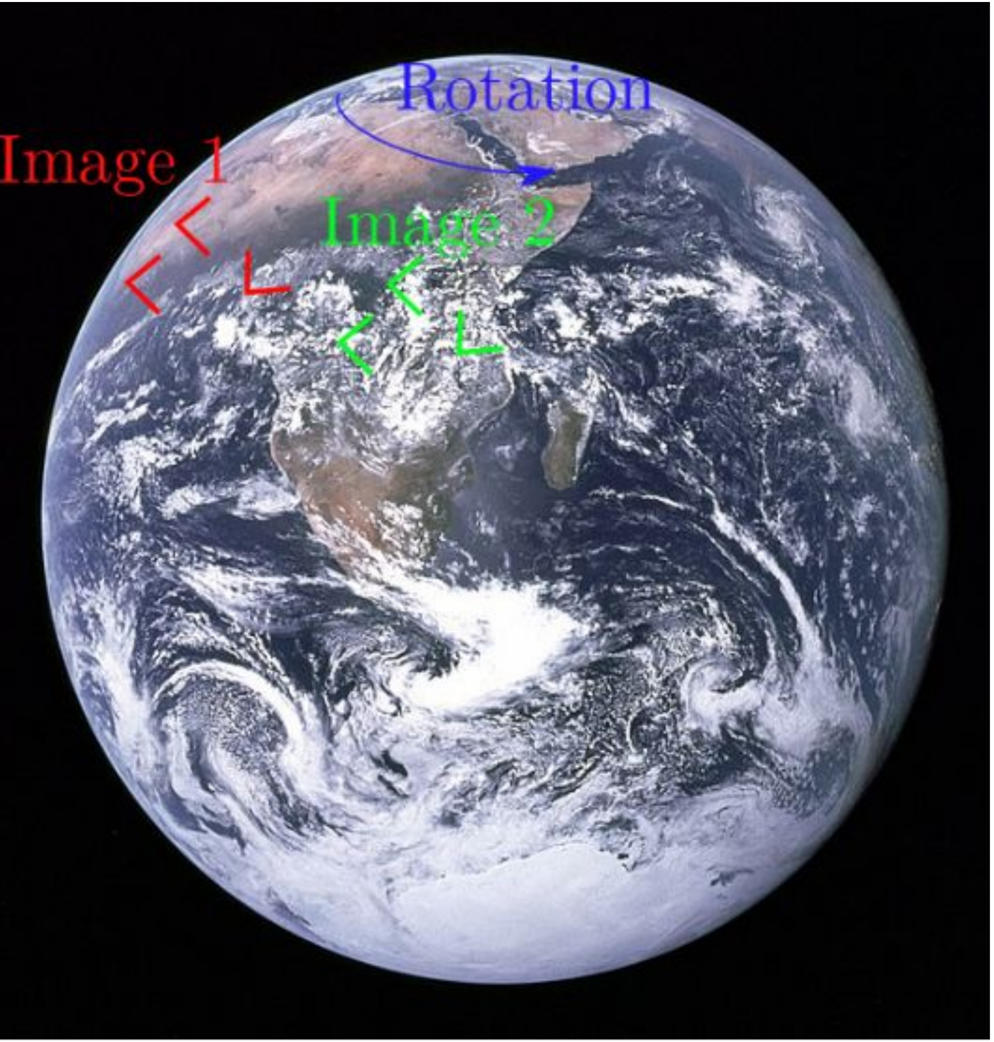
Gravitational-wave detection

Electromagnetic observatories	Gravitational-wave observatories
Strong lensing: Multiple images	...epeated events
Millilensing: Strong lensing flux anomalies, lens system imaging	...eform beating patterns
Microlensing: Time-dependent li... change	...ve optics diffraction in

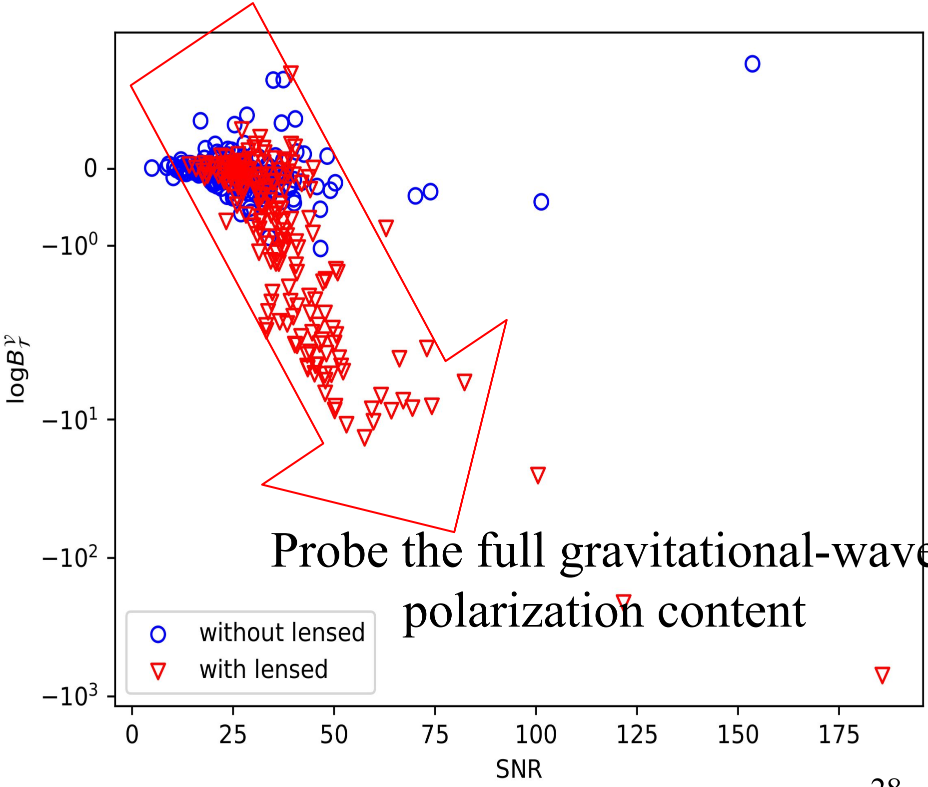
Searches for gravitational-wave lensing have started

- Hannuksela et al. (2019)
- Li et al. (2019)
- McIsaac et al. (2019)
- Pang (2020)
- Dai et al. (2020)
- Liu et al. (2020)
- LVK (2021)
- ….

Why detect gravitational-wave lensing?



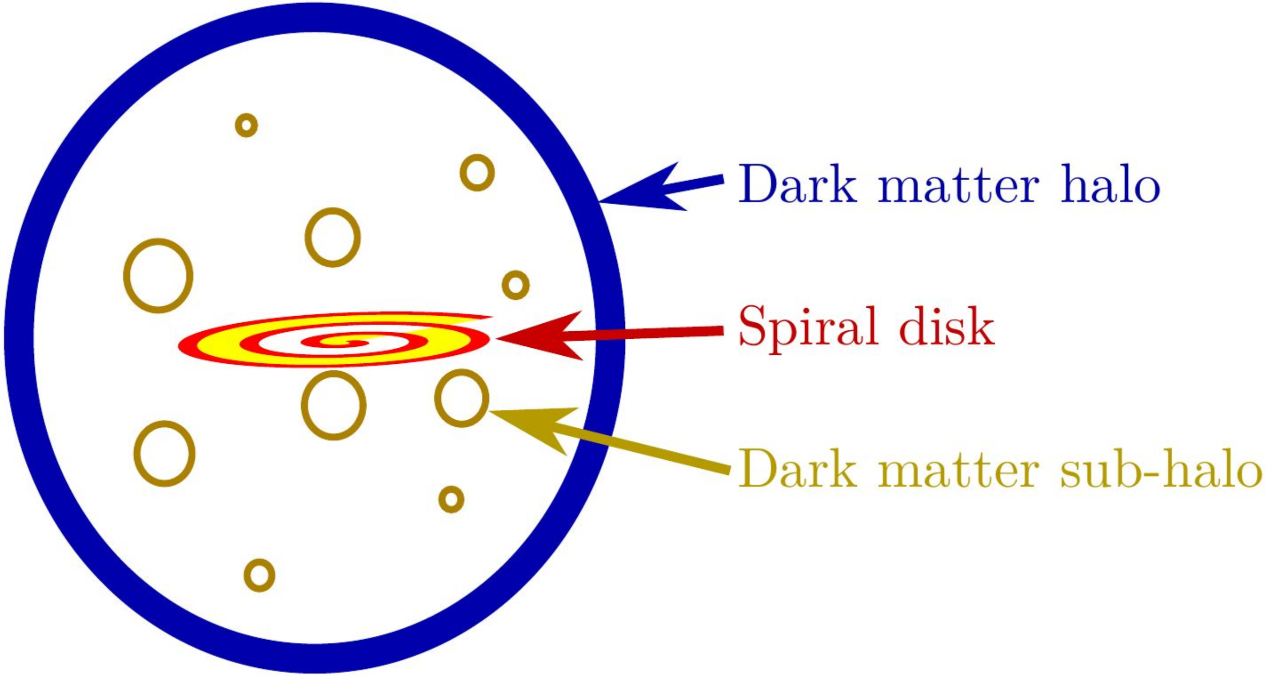
Test gravity



Probe the full gravitational-wave polarization content

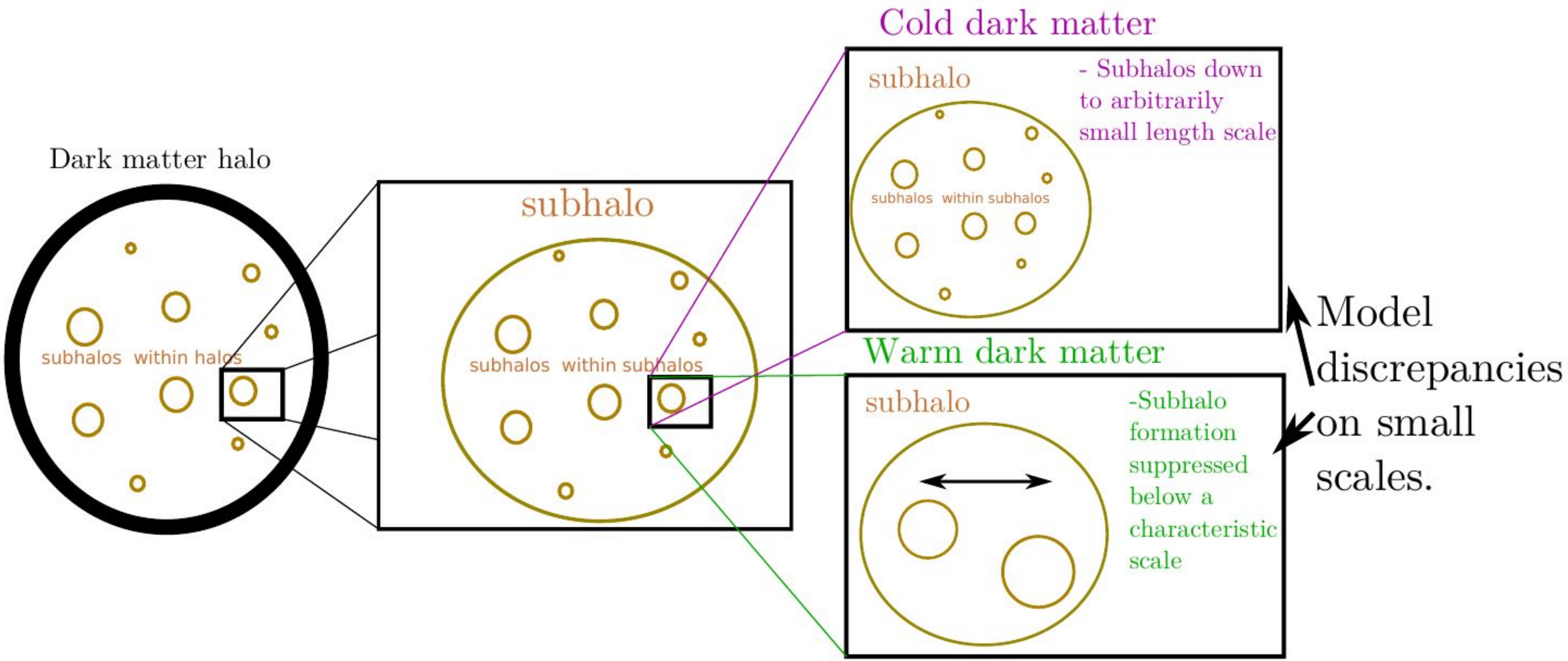
Wong et al. (in prep); see also Goyal et al. (2020)

Probe dark matter



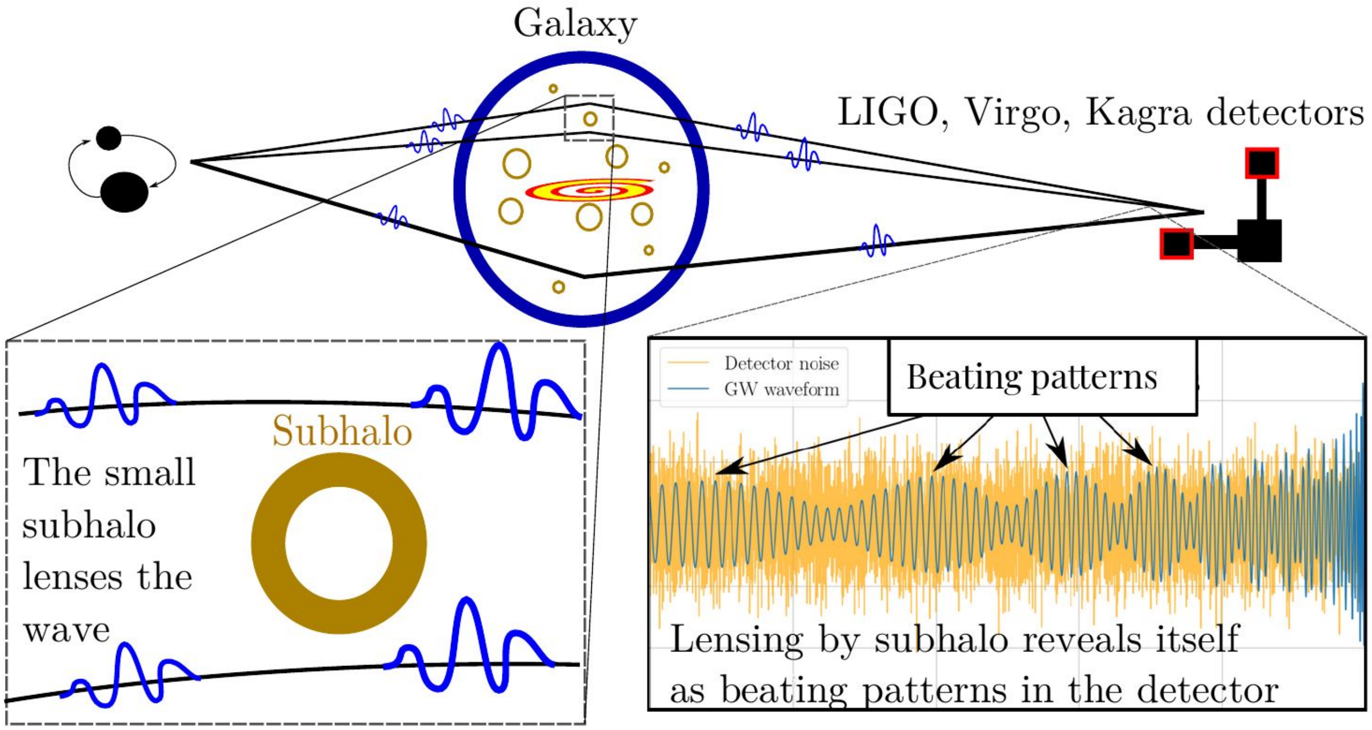
Liu et al., in preparation

Probe dark matter



Liu et al., in preparation

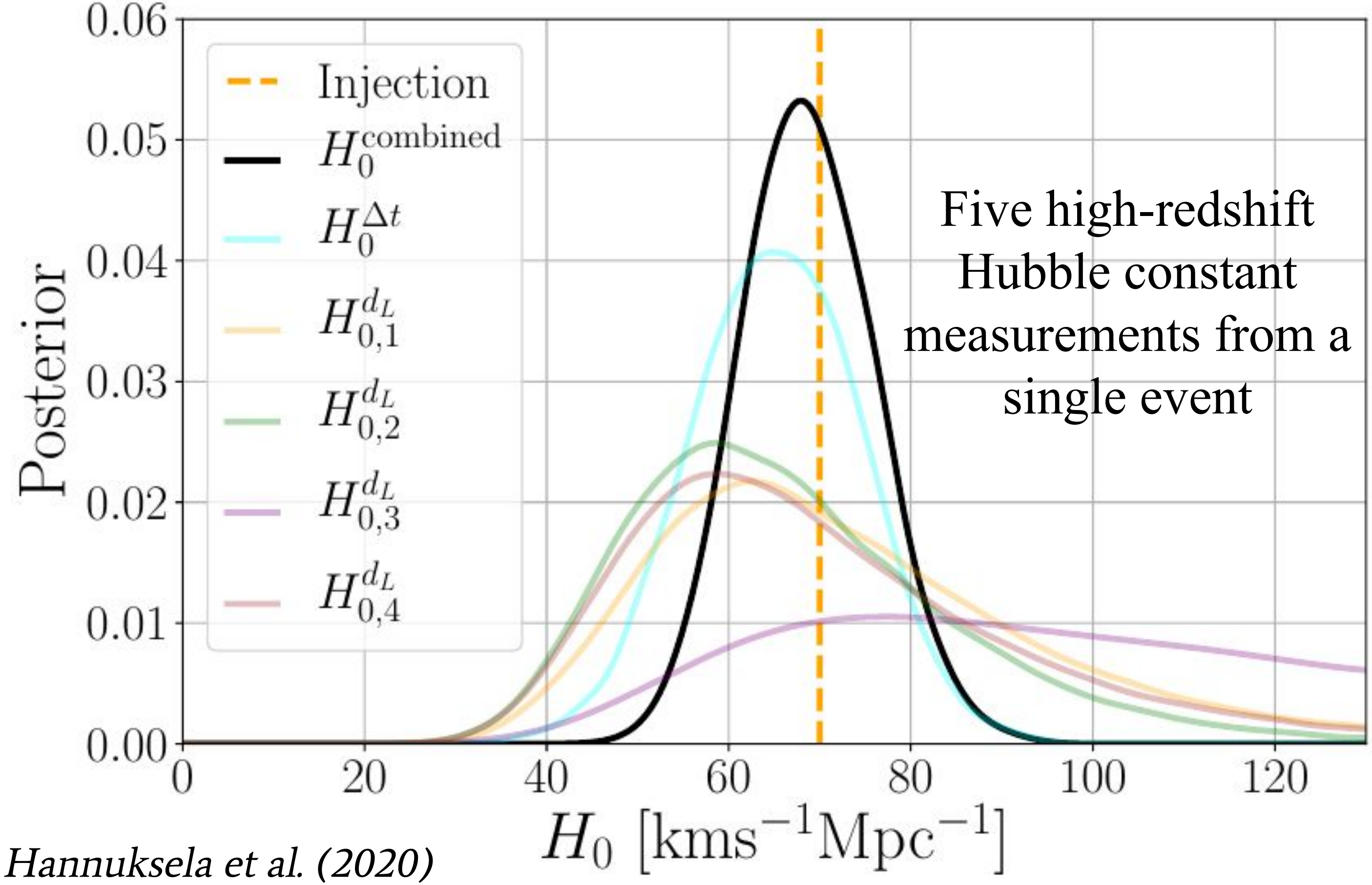
Probe dark matter



See also applications to compact dark matter *(Basak et al., 2022)* and low-mass subhalos using wave optics effects *(Oguri & Takahashi, 2022)*, among others

Liu et al., in preparation

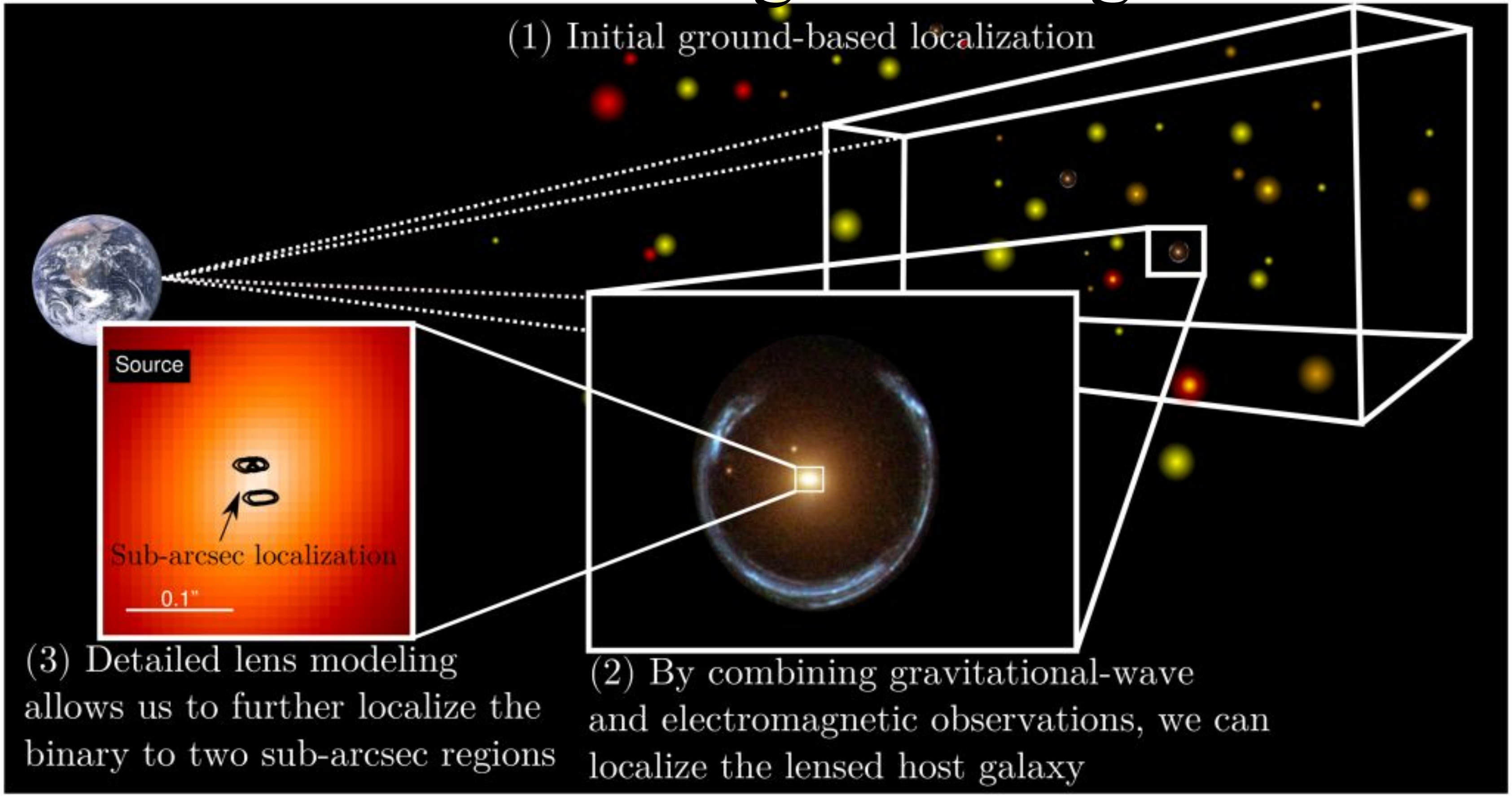
Study the expansion of the Universe



See also studies using electromagnetic counterparts *(Liao et al., 2017, 2019, and others)*

Hannuksela et al. (2020)

Do multi-messenger lensing science



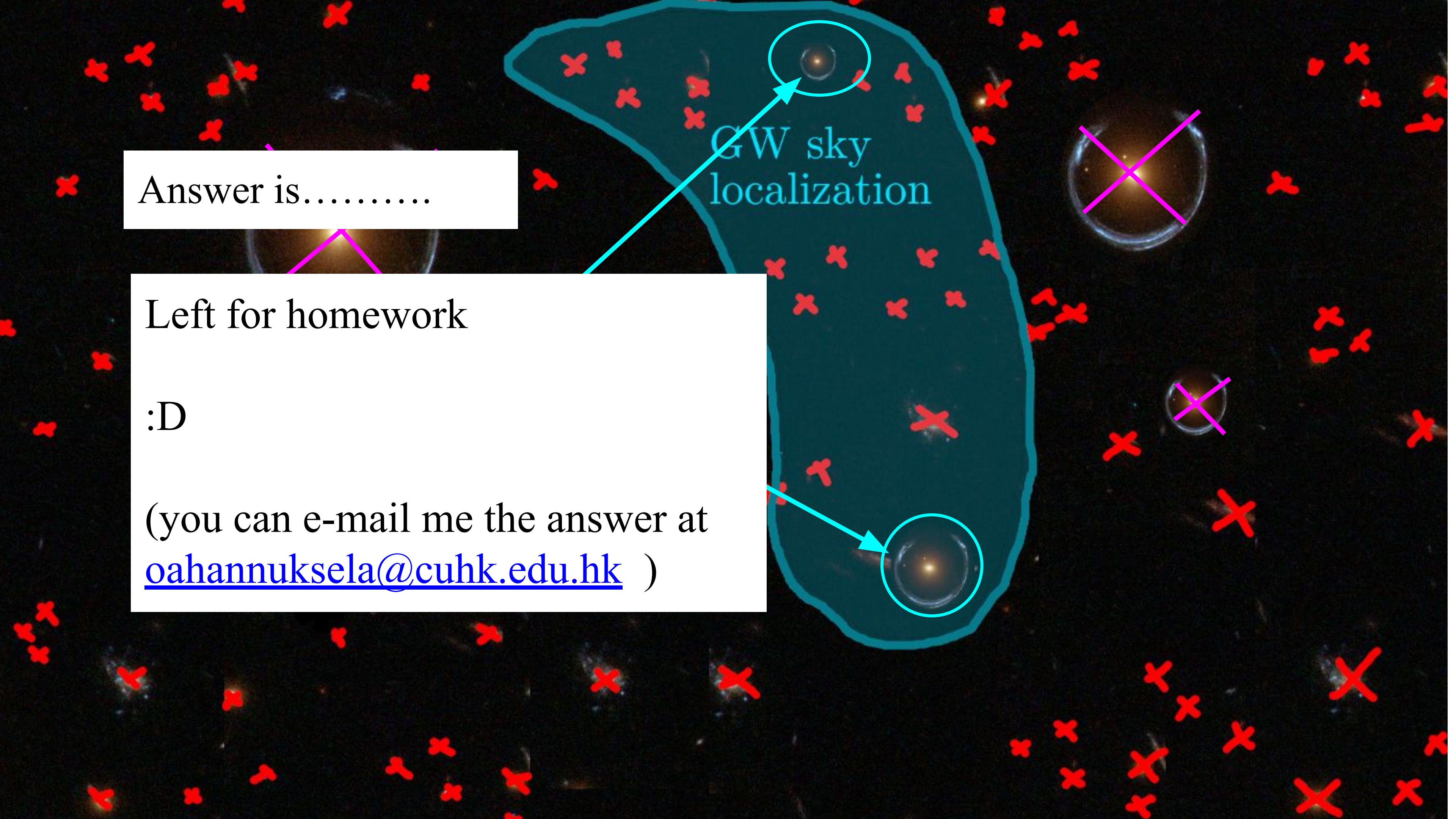
Hannuksela et al. (2020); see also Smith et al., (2018, 2019, 2020) for cluster lensing

Answer is……….

Left for homework

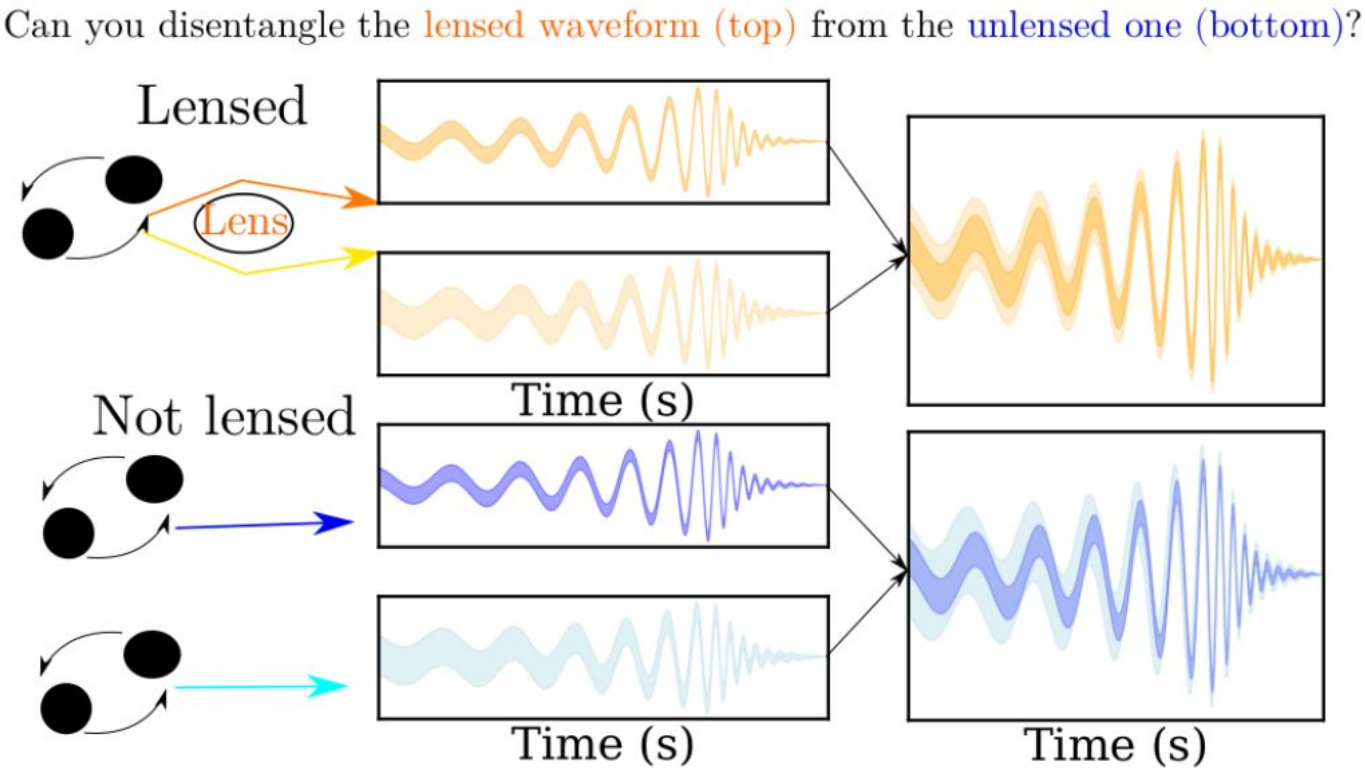
:D

(you can e-mail me the answer at oahannuksela@cuhk.edu.hk)

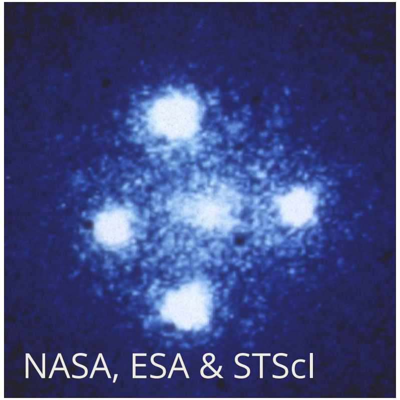


What are some of the challenges?

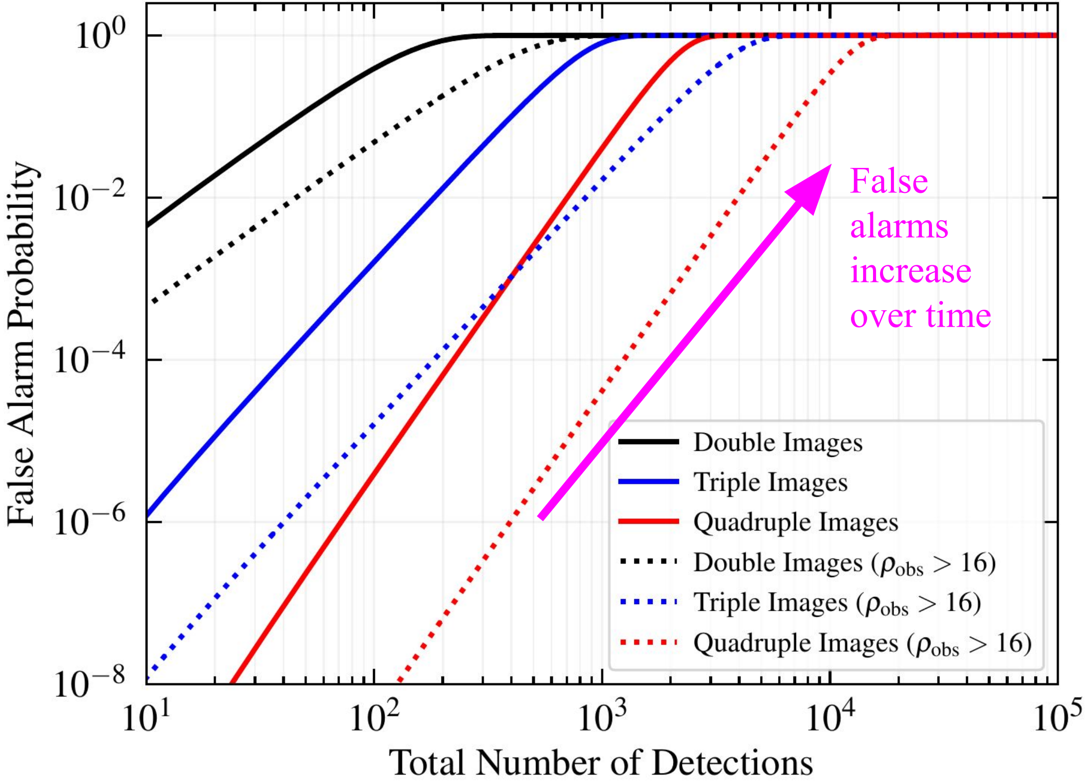
Why is it so difficult to detect strong lensing?



Caliskan et al. (2022)



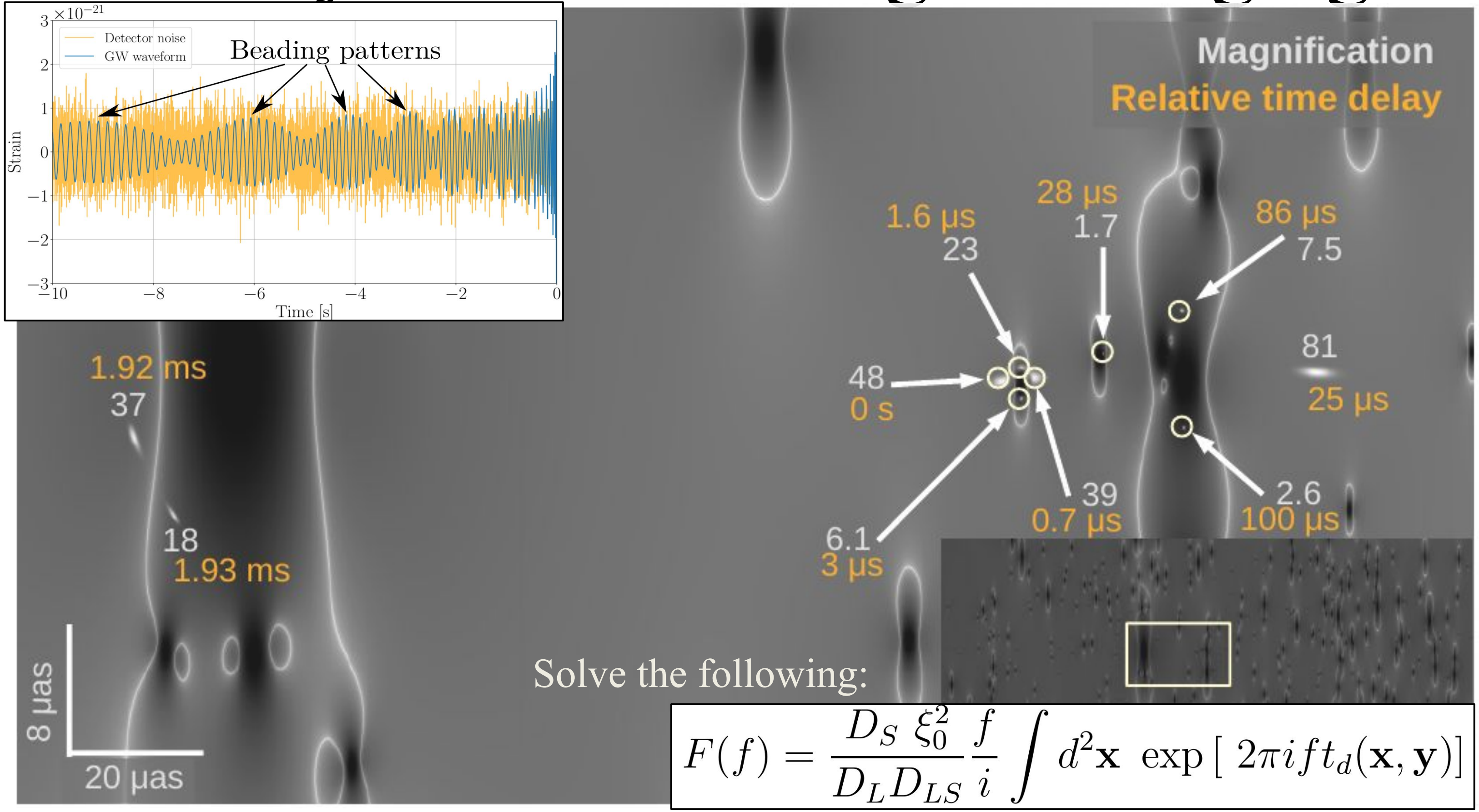
Why is it so difficult to detect strong lensing?



- Probability of false alarm increases rapidly over time
- Solutions perhaps in statistical lens modelling

Caliskan et al. (2022)

Why is microlensing challenging?

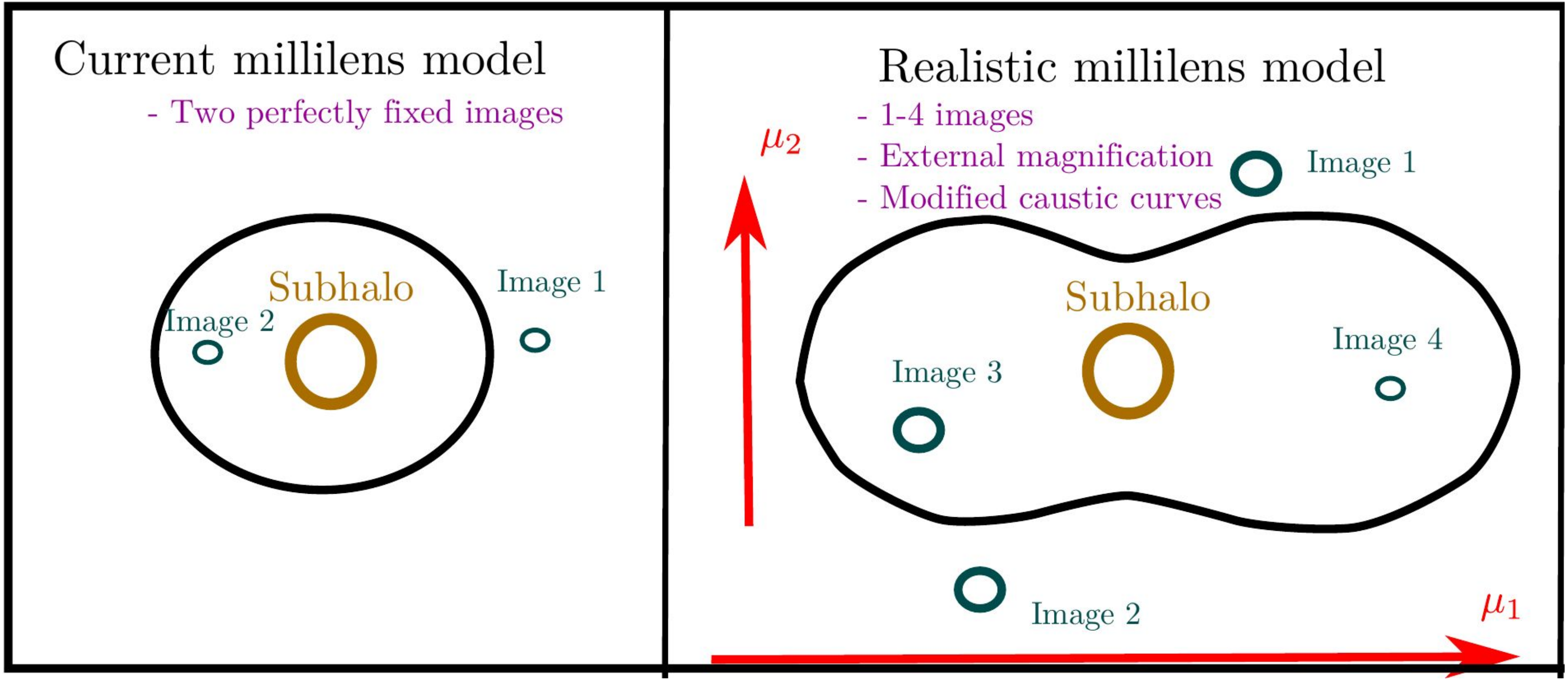


Solve the following:

$$F(f) = \frac{D_S \, \xi_0^2}{D_L D_{LS}} \frac{f}{i} \int d^2\mathbf{x} \; \exp\left[\, 2\pi i f t_d(\mathbf{x}, \mathbf{y})\right]$$

Diego et al. (2019)

Why is millilensing challenging?



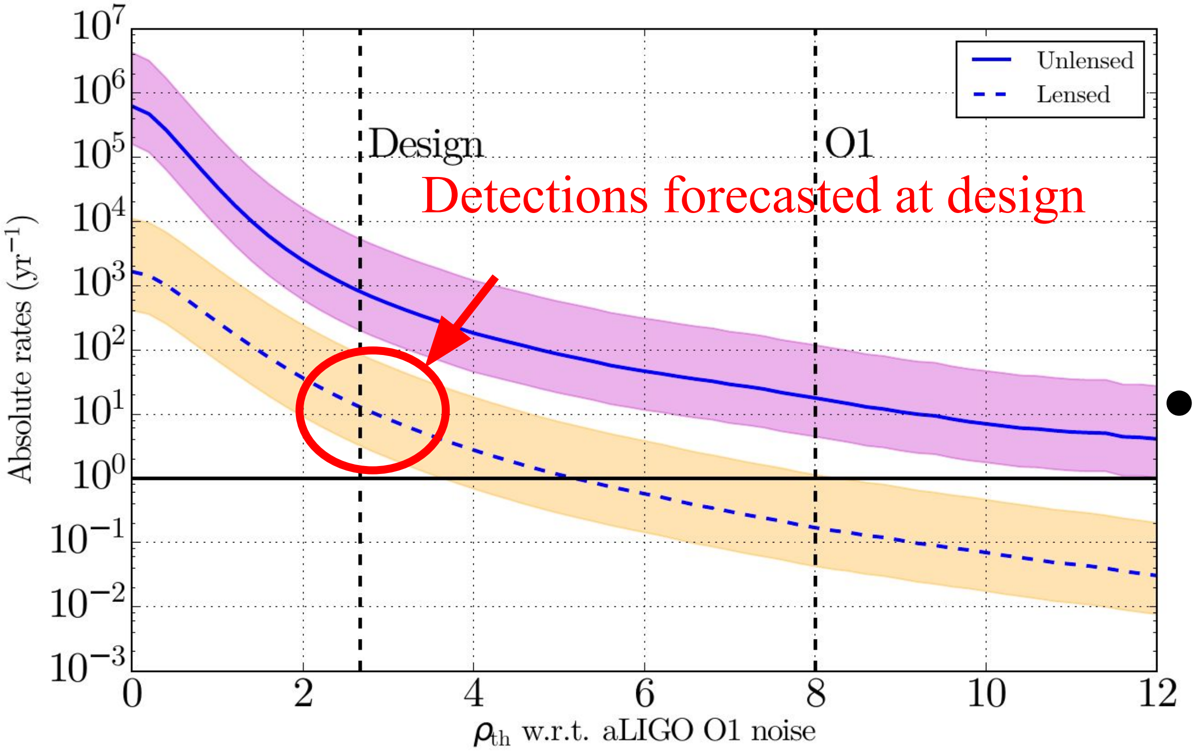
Rapidly working on implementing better models and using phenomenological searches

Why is multimessenger lensing challenging?

- The current lens models are behind in progress
- Instrument capabilities largely unknown
- Science cases not fully explored

What to look forward to in the future?

Searches on-going



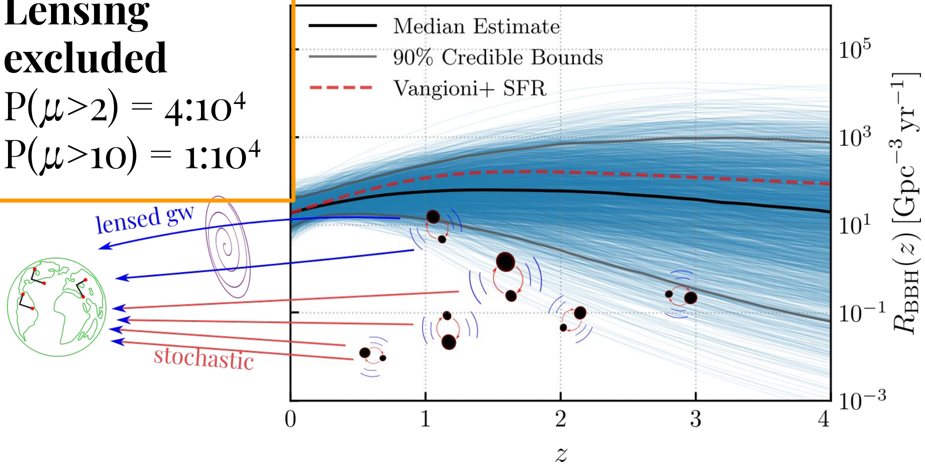
- Forecasts predict strong lensing at a reasonable rate

Ng et al. (2017); see also *Li et al. (2018)*, *Oguri (2018); Wierda et al. (2021); Xu et al. (2021)*

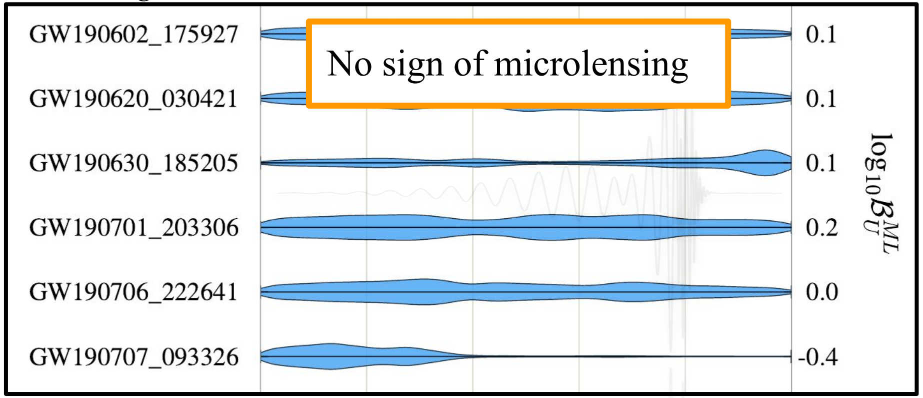
GWTC-2 lensing study

LIGO-Virgo Collaboration

Lensing excluded
$P(\mu>2) = 4{:}10^4$
$P(\mu>10) = 1{:}10^4$



No sign of microlensing



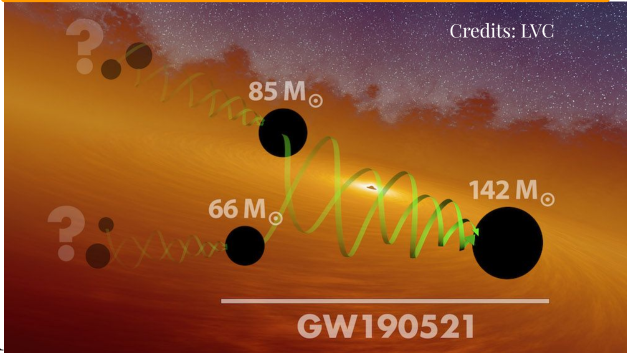
Events analysed:

BNS:
GW190425

NSBH:
GW190426_152155

BBHs:
GW190521
GW190602_175927
GW190706_222641

No sign of lensing magnification



No sign of multiple images or sub-threshold lensed detections

Bayes factor

Event 1	Event 2	$\log_{10}(\mathcal{B}^L_U)$ HANABI
GW190412	GW190708_232457	−9.7
GW190421_213856	**GW190910_112807**	**−3.8**
GW190424_180648	**GW190727_060333**	**−3.9**
GW190424_180648	**GW190910_112807**	**−3.9**
GW190513_205428	**GW190630_185205**	**−5.5**

Google "**LVK lensing webinar**" for more information

Thank you

Summary

- Introduction
- Basics of gravitational-wave lensing
- How to detect lensing
- Science applications
- Challenges
- Future outlook

Gravitational-wave lensing is a relatively new observational field promising an entirely different way to study gravity and observe lensing, with applications ranging from modified gravity to dark matter and cosmology.

Detections have been forecasted, and searches for lensing have started recently.