The modification of thermonuclear reaction in the astrophysical plasma.

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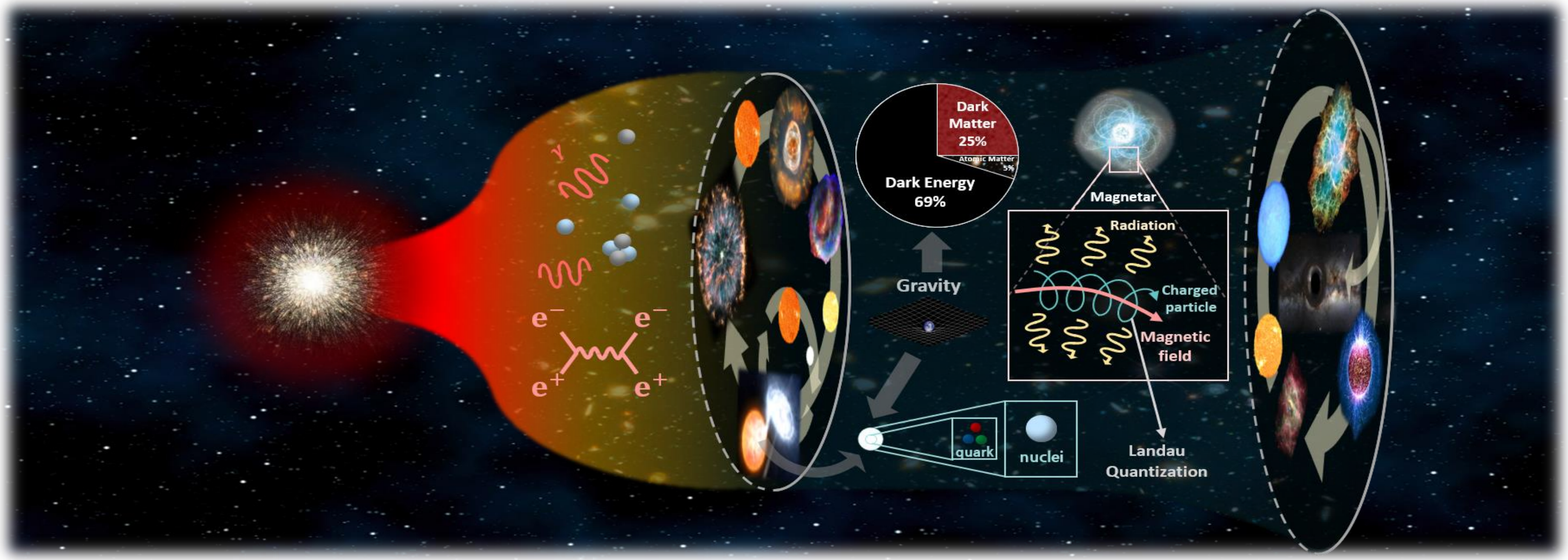
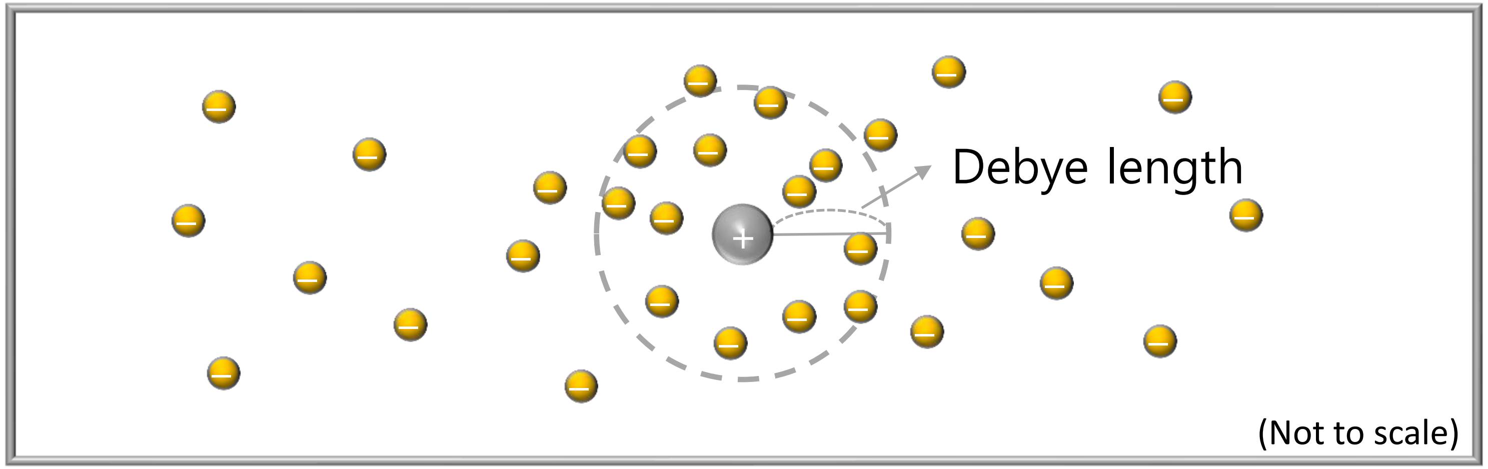
Introduction

- **Evolution of elements**
  - In the astrophysical plasma, the nuclei undergo various nuclear reactions.
  - Network calculations allow us to understand the process of nuclear abundance change.
  - Since the environmental properties of plasma, the nuclear reaction in plasma is different from what we understand in free space.



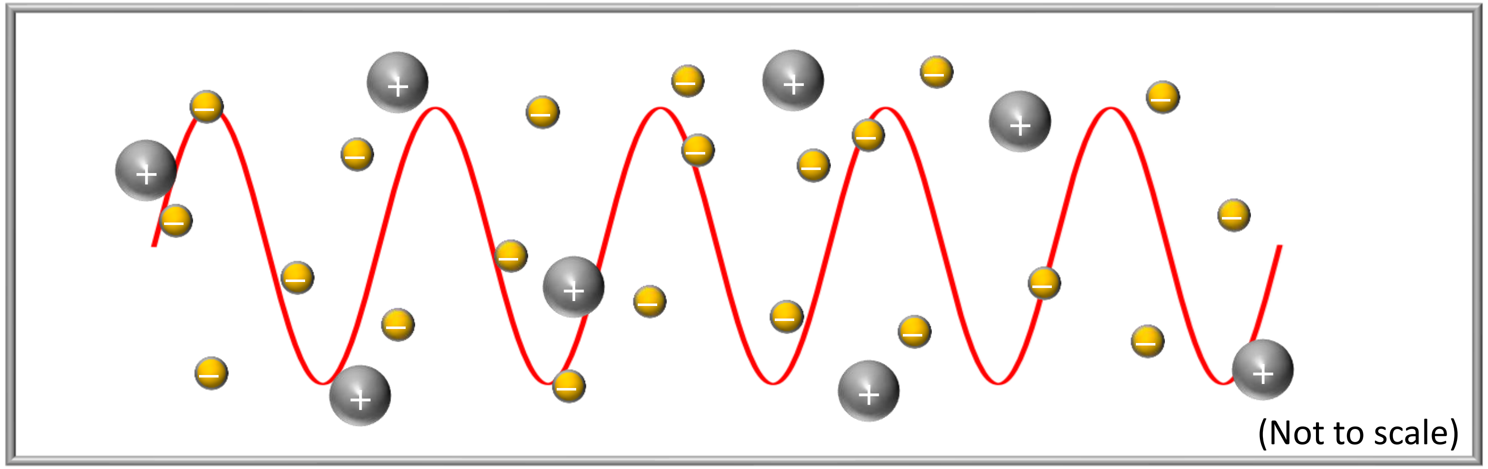
Introduction

- **Screening effect**
  - The Coulomb barrier of screened nuclei is decreased.
  - The electron screening effect gives a correction to the thermonuclear processes.
  - To thermonuclear reaction occur, nuclei moves with high energy.
  - The moving nuclei in plasma causes the distorted electric potential.



Introduction

- **Electromagnetic fluctuation**
  - ➢ The transverse permittivity can affect to the electromagnetic field.
  - ➢ The fluctuations exist even in a homogeneous plasma maintaining the thermal equilibrium.
  - ➢ Average magnetic field is zero:$\langle B \rangle = 0$, but squared average is not zero:$\langle B^2 \rangle \neq 0$.
  - ➢ The level of EM fluctuations can be evaluated by the fluctuation-dissipation theorem.



Dielectric permittivity

- **Linearize Vlasov equation**

$$\frac{\partial}{\partial t} f + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} - \frac{e}{m}\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right) \cdot \frac{\partial f}{\partial \vec{p}} = 0$$

$$f = f_0 + \delta f$$

$$\delta f \propto e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\delta f = \frac{e\vec{E}}{i(\vec{k} \cdot \vec{r} - \omega)} \cdot \frac{\partial f_0}{\partial \vec{p}}$$

- **Dielectric permittivity**

$$\vec{D} = \vec{E} + 4\pi\vec{P} = \varepsilon\vec{E}$$

$$-e\int \delta f d^3p = \rho = -\nabla \cdot \vec{P} \quad \Rightarrow \quad \varepsilon_l\left(\vec{k}, \omega\right) = 1 - \frac{4\pi e^2}{k}\int \frac{1}{k v_\parallel - \omega} \cdot \frac{\partial f}{\partial p_\parallel} d^3p$$

$$-e\int \vec{v}\delta f d^3p = \vec{j} = \frac{\partial \vec{P}}{\partial t} \quad \Rightarrow \quad \varepsilon_t\left(\vec{k}, \omega\right) = 1 - \frac{4\pi e^2}{\omega}\int \frac{\vec{v}_\perp}{k v_\parallel - \omega} \cdot \frac{\partial f}{\partial p_\perp} d^3p$$

Dielectric permittivity

- **Thermonuclear reaction rate**

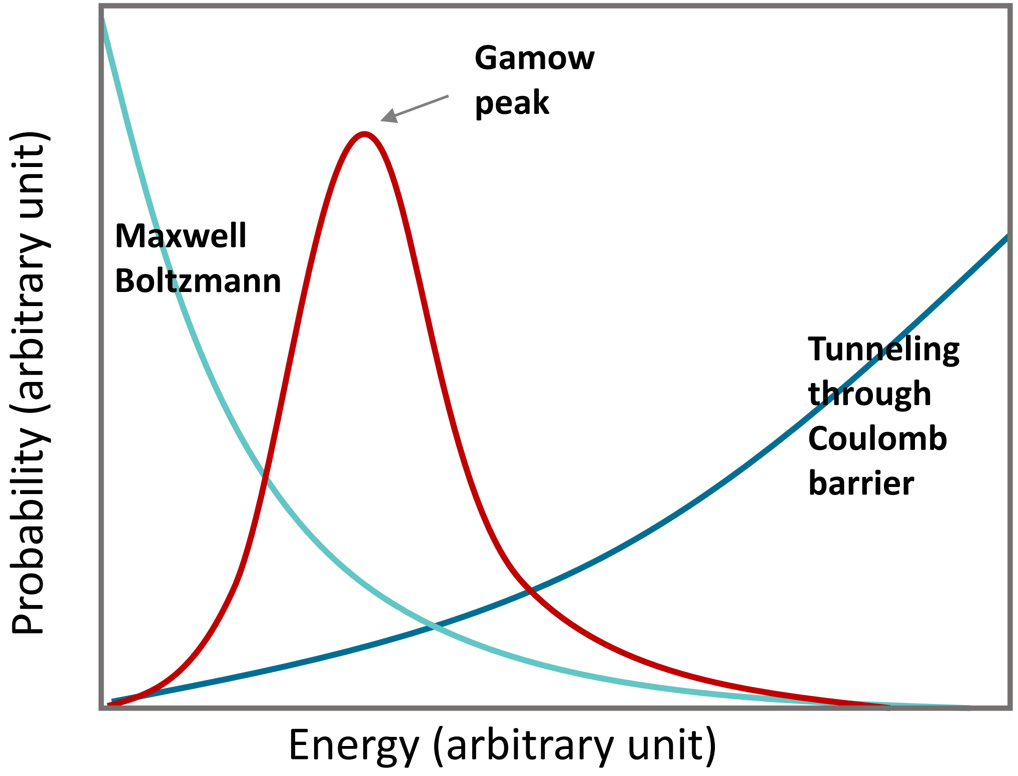
$$\langle \sigma v \rangle \propto \int E e^{-\frac{E}{T}} \sigma(E) dE$$
$$= \int e^{-\frac{E}{T}} P(E) S(E) dE$$

σ: cross section P: penetration factor S: astrophysical S-factor

- **Barrier penetration factor**

$$P(E) = \exp\left(-2\sqrt{2\mu} \int_{R_0}^{R_c} \sqrt{\frac{Z_1 Z_2 e^2}{r} - E dr}\right) \equiv e^{-2\pi\eta}$$

R_c: turning point R_0: nuclear radius μ: reduced mass



Dynamical screening

- **Longitudinal permittivity**

$$f_i(p_x) = \frac{1}{n_i}\int f_i(p)dp_y dp_z$$

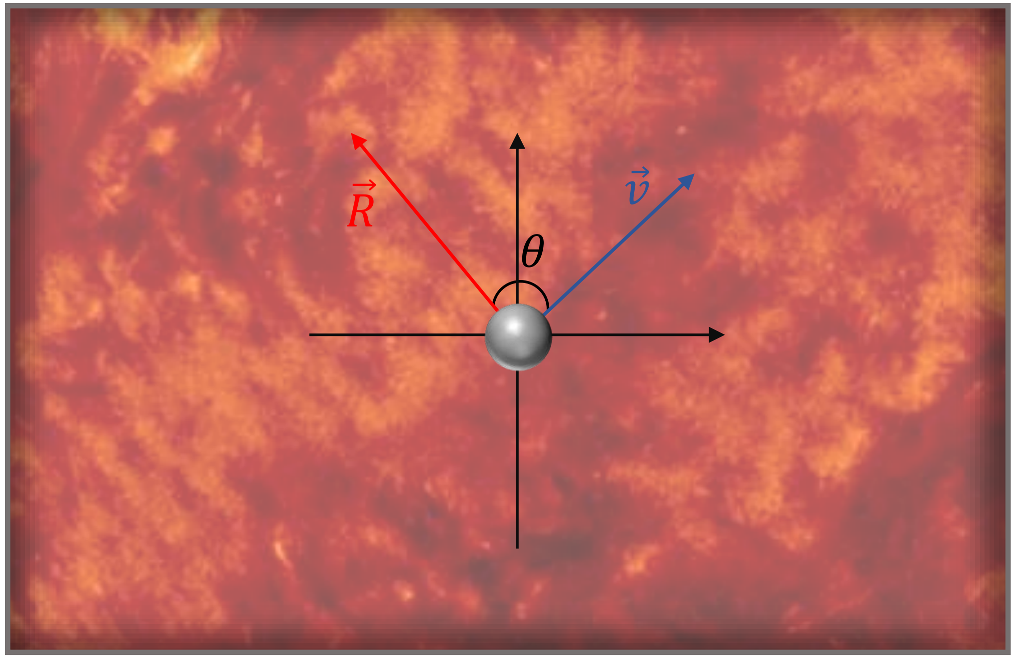
f_i: Distribution of particle i

[1]
$$\varepsilon_l\left(\vec{k},\omega\right) = 1 - \frac{T}{(k\lambda_{d,e})^2}\left[\int_{-\infty}^{\infty}\frac{df_e(p_x)}{dp_x}\frac{dp_x}{v_x - \frac{\omega}{k}}\right]_{electron} - \frac{T}{(k\lambda_{d,i})^2}\left[\int_{-\infty}^{\infty}\frac{df_i(p_x)}{dp_x}\frac{dp_x}{v_x - \frac{\omega}{k}}\right]_{ion} - \cdots$$

- **Moving charge in the plasma**

$$\nabla\cdot\varepsilon_l\vec{\mathrm{E}}(\vec{\mathrm{r}},t) = 4\pi Z_0 e\delta(\vec{r}-\vec{v}t) \qquad \vec{R}\equiv\vec{r}-\vec{v}t$$

[1,2]
$$\Rightarrow \quad \phi(R,v,\theta) = \frac{1}{2\pi^2}\int\frac{d\vec{k}}{k^2}\frac{\exp\left(i\vec{k}\cdot\vec{R}\right)}{\varepsilon_l(\vec{k},\vec{k}\cdot\vec{v})}$$



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- **Enhancement factor**

$$P(E) = \exp\left(-2\sqrt{2\mu}\int_{R_0}^{R_c}\sqrt{\frac{Z_1Z_2e^2}{r}+W-E}\,dr\right) \approx e^{-2\pi\eta}e^{\frac{W(v_1,v_2,\theta_1,\theta_2)}{T}}$$

$$\langle\sigma v\rangle_{screen} = \int_0^{\infty}\sqrt{\frac{8}{\pi\mu T^3}}\sigma(E)Ee^{-\frac{E}{T}}f_s(E)dE \approx f_s(E_G)\langle\sigma v\rangle_{bare}$$

E_G: Gamow energy

[1] M. Lifshitz and L. P. PitaevskiT, Physical Kinetics, Pergamon Press, Oxford (1981)

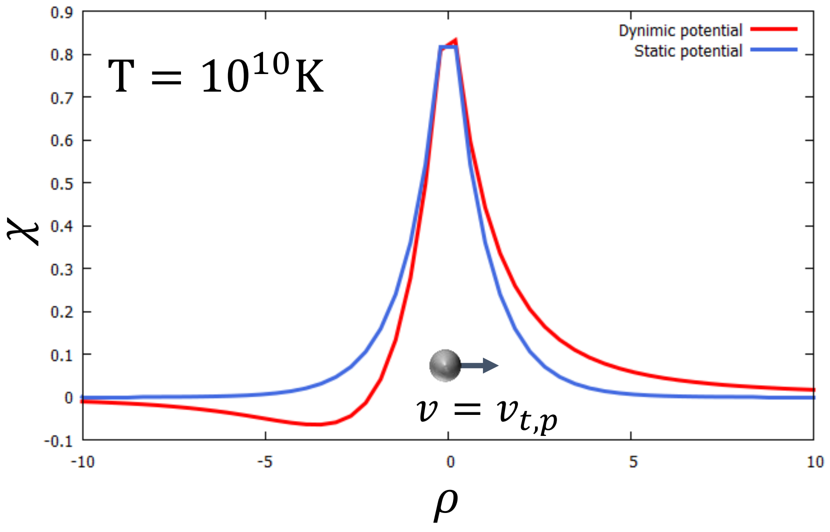
[2] E. E. Trofimovich and V. P. Krainov , JETP 77 (6). December (1993)

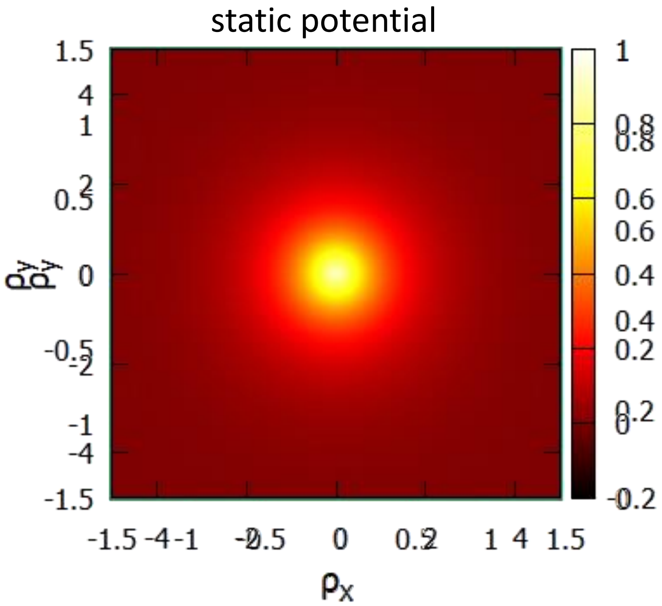
Result : Dynamical screening effect

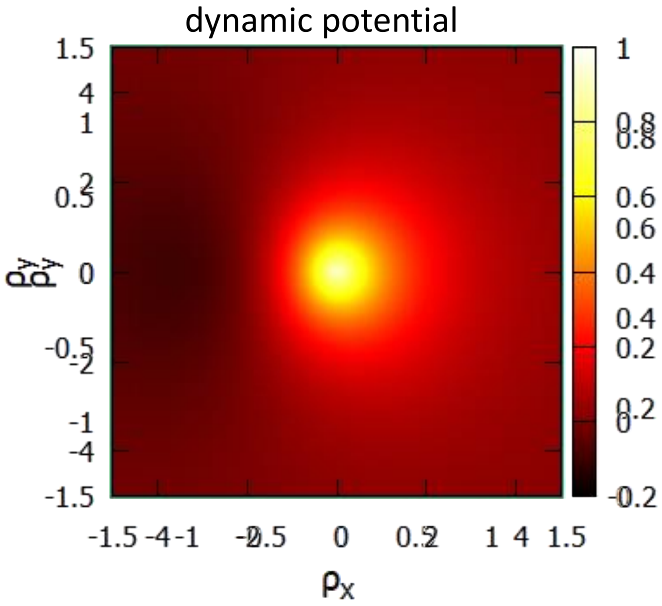
- **Effective potential**

$$\chi \equiv \frac{\phi_{screen}}{\phi_{Coulomb}} \qquad \rho \equiv \frac{r}{\lambda_D}$$

λ_D: Debye length

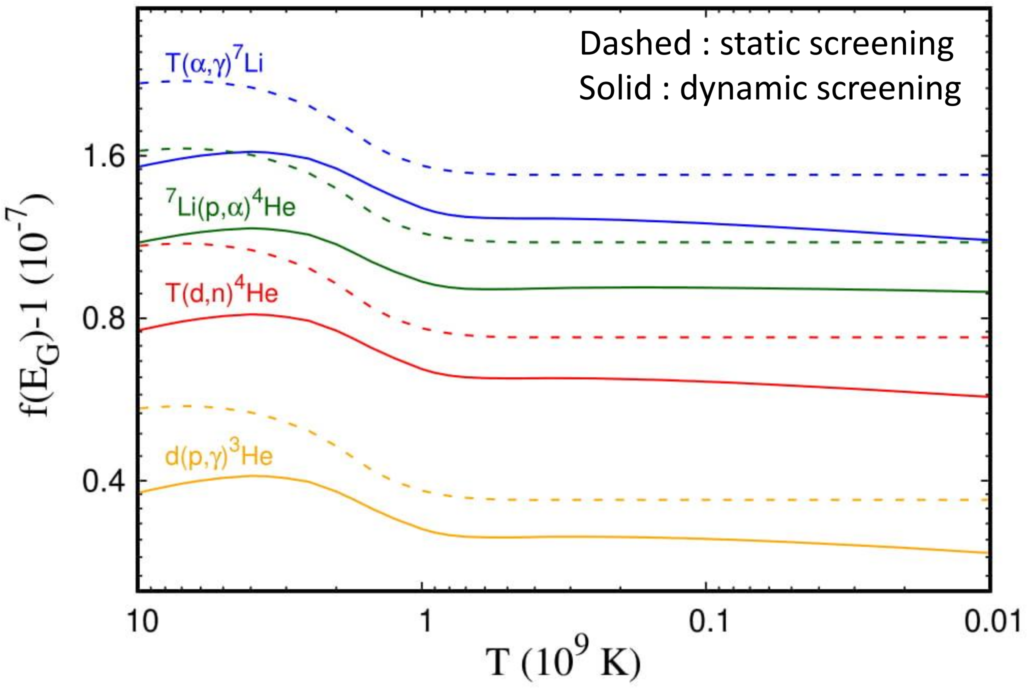


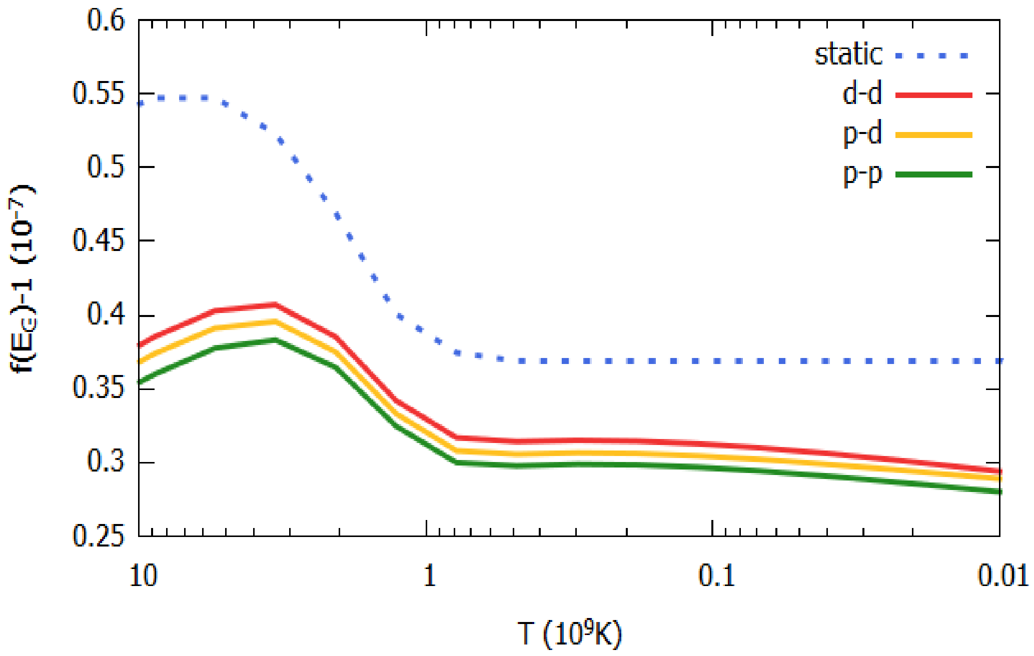




Result : Dynamical screening effect

- **Enhancement factor**

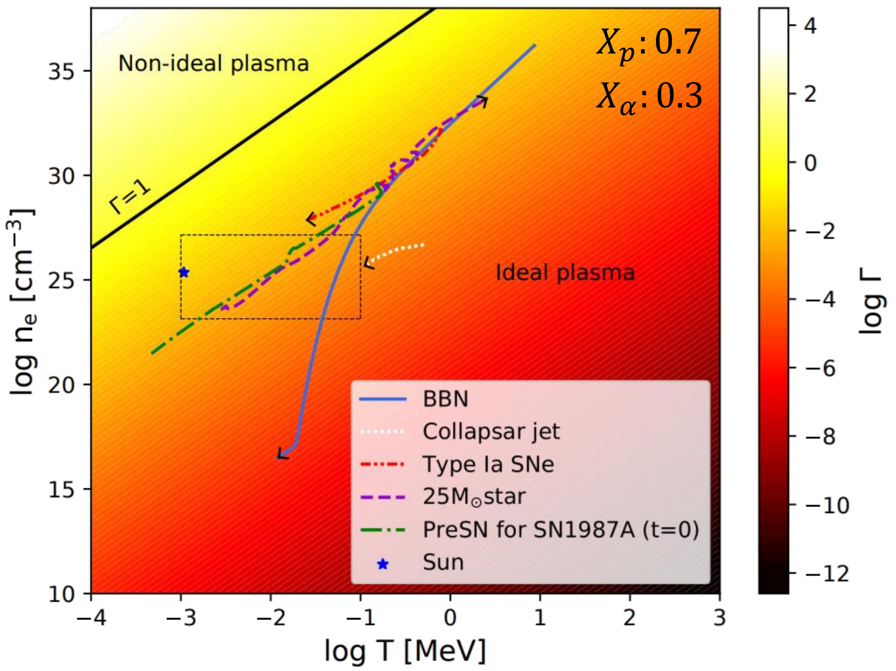


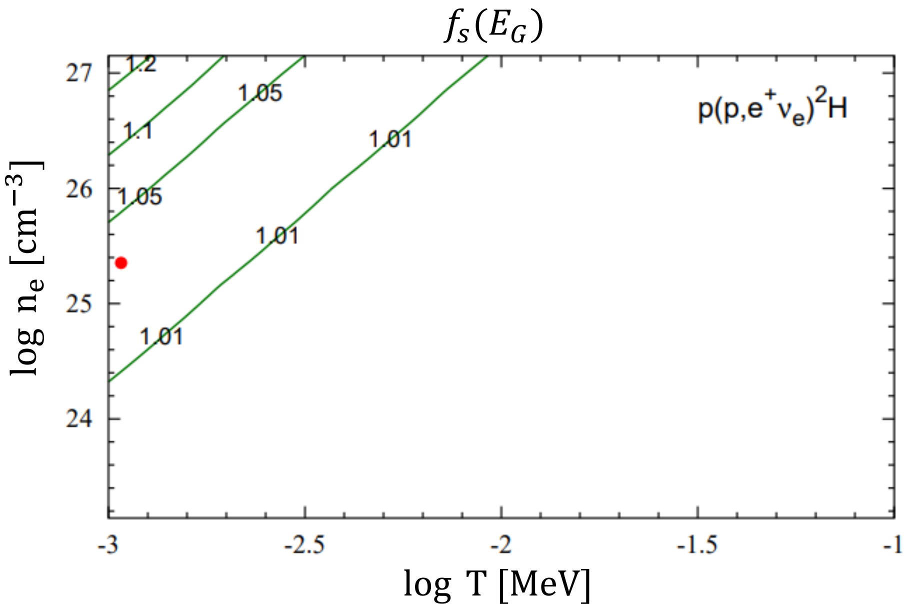


Result : Dynamical screening effect

- **Various conditions**

plasma parameter





Reaction	E_g/T	$f_s(E_g)$
$p-p$	4.5689	1.0262
^{3}He $-$ ^{3}He	16.5938	1.1078
^{3}He $-$ ^{4}He	17.3375	1.1087
$p-$ ^{7}Be	13.8710	1.1187
$p-$ ^{14}N	20.5473	1.2242

$T = 1.25\times10^7$(K) $\rho = 1.6\times10^2$(g/cm^3)

$$\Gamma = \frac{n_e^{1/3}(Ze)^2}{k_B T}$$

$\Gamma \gtrsim 1$: non-ideal plasma

$\Gamma \ll 1$: ideal plasma

Electromagnetic fluctuation

- **Transverse permittivity**

$$\varepsilon_t\left(\vec{k},\omega\right)=1+\frac{\omega_{p,e}^2}{\omega}\left(\frac{\omega}{\sqrt{2}kv_{T,e}}\right)Z\left(\frac{\omega+i\eta_e}{\sqrt{2}kv_{T,e}}\right)+\frac{\omega_{p,p}^2}{\omega}\left(\frac{\omega}{\sqrt{2}kv_{T,p}}\right)Z\left(\frac{\omega+i\eta_p}{\sqrt{2}kv_{T,p}}\right)+\frac{\omega_{p,{}^4\mathrm{He}}^2}{\omega}\left(\frac{\omega}{\sqrt{2}kv_{T,{}^4\mathrm{He}}}\right)Z\left(\frac{\omega+i\eta_{{}^4\mathrm{He}}}{\sqrt{2}kv_{T,{}^4\mathrm{He}}}\right)$$

- **Electromagnetic fluctuation**

$$\frac{\langle E^2\rangle_{\vec{k}\omega}}{8\pi}=\frac{2}{e^{\frac{\omega}{T}}-1}\frac{Im\varepsilon_t}{\left|\varepsilon_t-\left(\frac{k}{\omega}\right)^2\right|^2}$$

$$\frac{\langle B^2\rangle_{\vec{k}\omega}}{8\pi}=\frac{2}{e^{\frac{\omega}{T}}-1}\left(\frac{k}{\omega}\right)^2\frac{Im\varepsilon_t}{\left|\varepsilon_t-\left(\frac{k}{\omega}\right)^2\right|^2}$$

- **Electromagnetic spectrum**

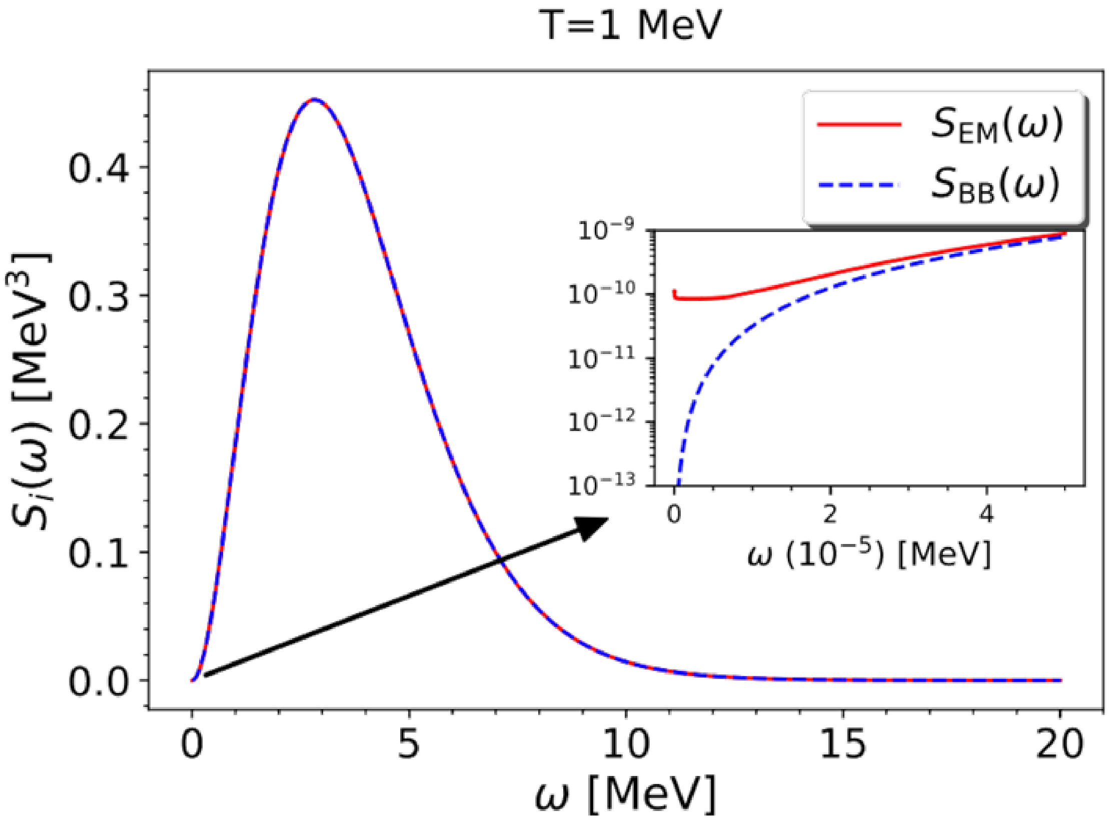
[1]

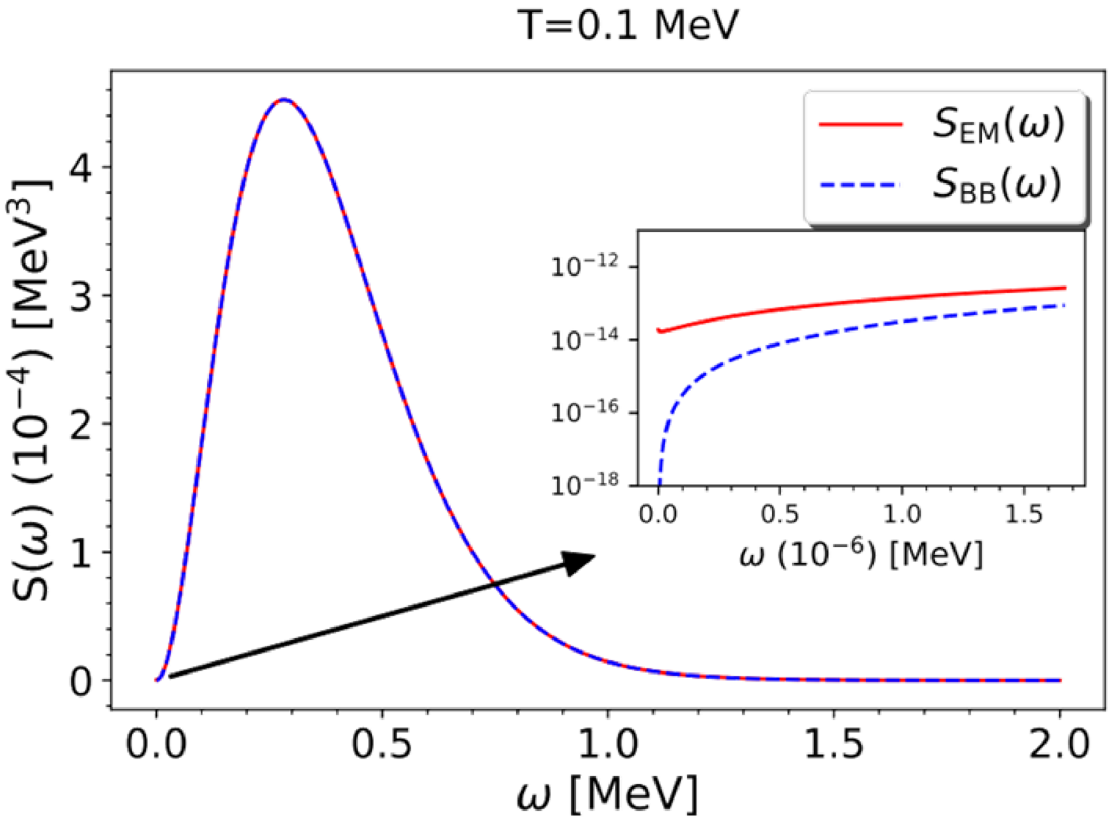
$$S(\omega)=\int\frac{\langle B^2\rangle_{\vec{k}\omega}}{8\pi}+\frac{\langle E^2\rangle_{\vec{k}\omega}}{8\pi}d\vec{k}=\frac{\langle B^2\rangle_{\omega}}{8\pi}+\frac{\langle E^2\rangle_{\omega}}{8\pi}$$

[1] Merav Opher and Reuven Opher , PRL 79 2628. October (1997)

Result : Electromagnetic fluctuation

- **Electromagnetic spectrum**



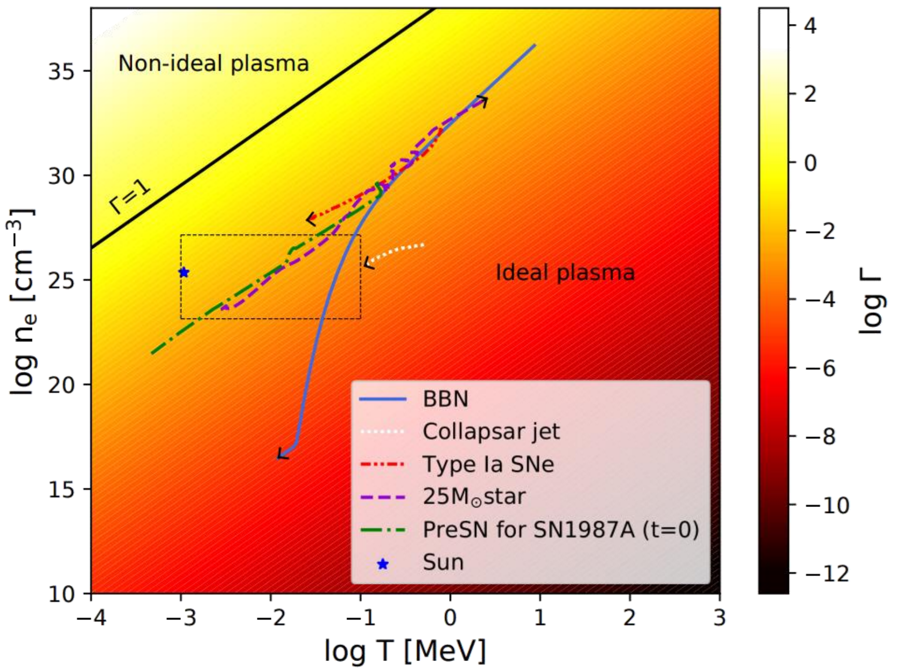


Result : Electromagnetic fluctuation

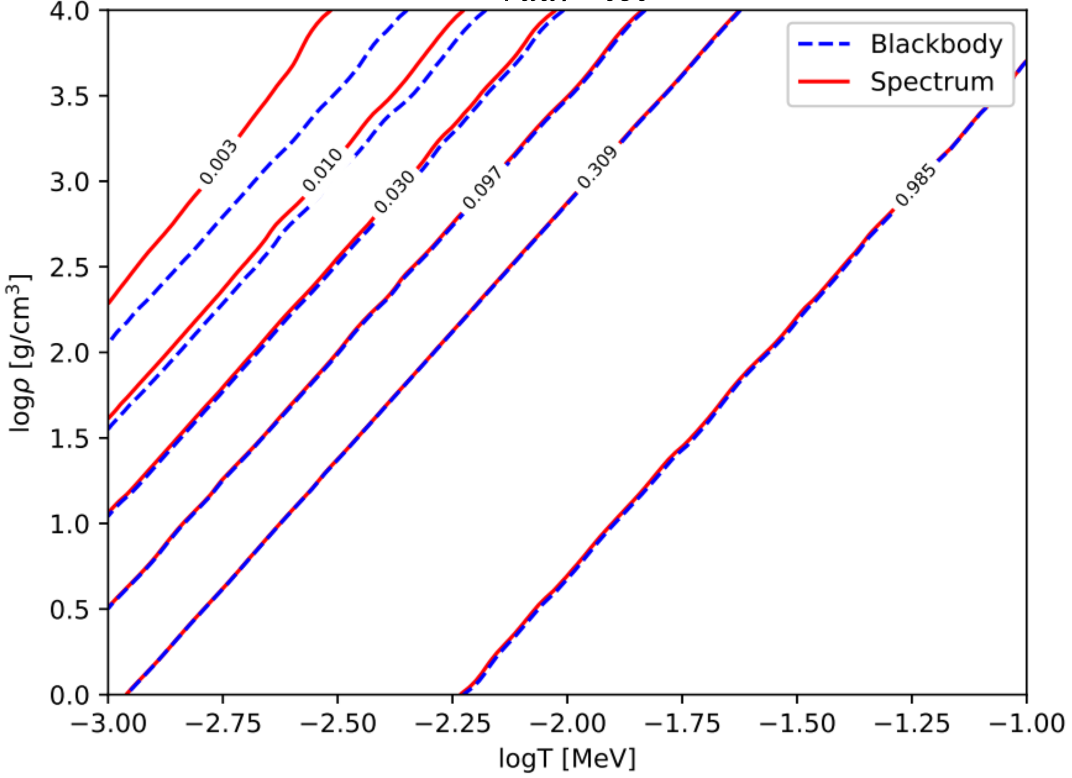
- **Various conditions**

$X_p: 0.7$ $X_\alpha: 0.3$

plasma parameter



P_{rad}/P_{tot}



$$\Gamma = \frac{n_e^{1/3}(Ze)^2}{k_B T}$$

$\Gamma \gtrsim 1$: non-ideal plasma

$\Gamma \ll 1$: ideal plasma

Pressure $P_{rad} = \frac{1}{3}\int S(\omega)d\omega$ $P_{tot} = P_{gas} + P_{rad}$

Result : Electromagnetic fluctuation

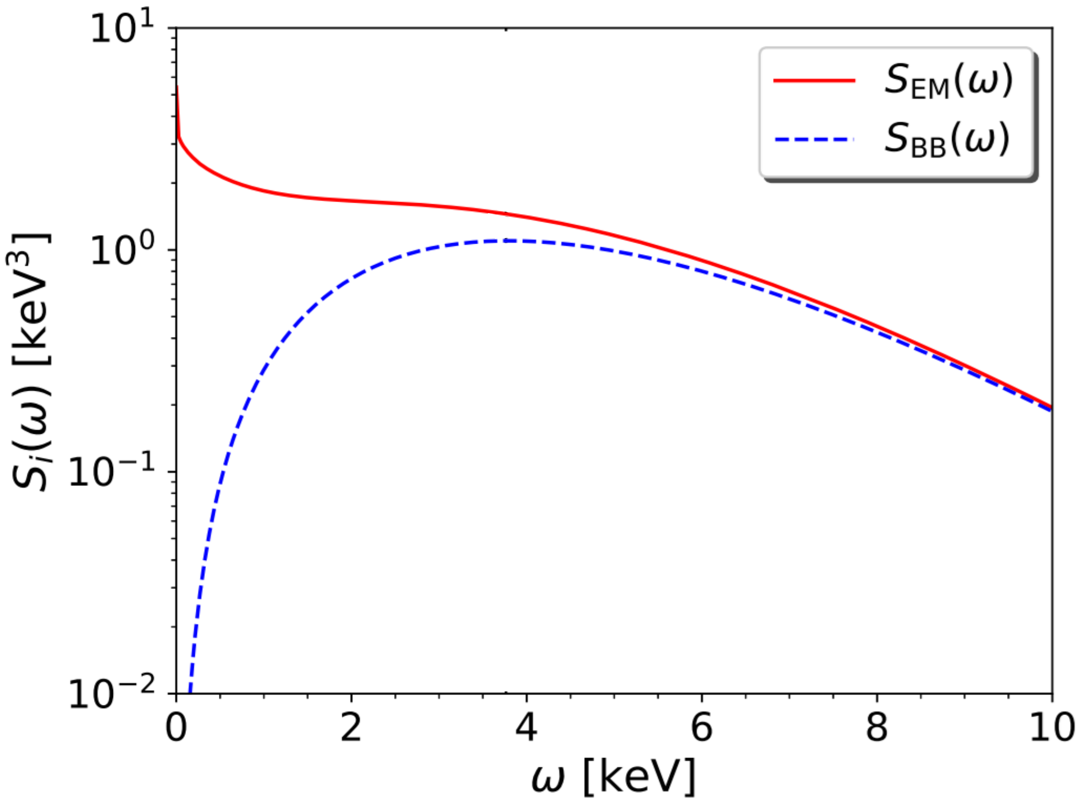
- **Solar core condition**

  - ➢ Temperature

    $$T = 1.56 \times 10^7 \mathrm{K}$$

  - ➢ Baryon density

    $$\rho = 1.48 \times 10^2 \mathrm{g/cm^3}$$



Result : Electromagnetic fluctuation

- **Radiation pressure**

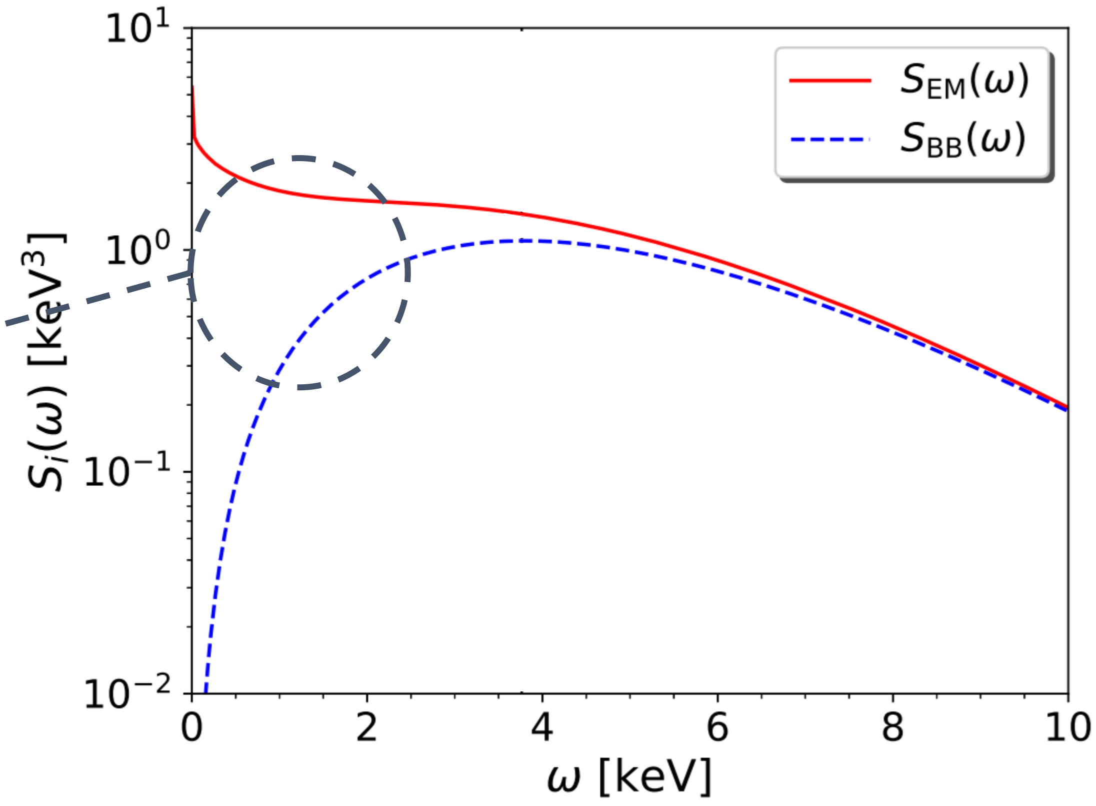
$$P_{rad} = \frac{1}{3}\int S(\omega)d\omega$$

Additional pressure
by fluctuation

$$P_{tot} = P_{gas} + P_{rad} = \sum_i n_i T_i + \frac{aT^4}{3} + \delta P$$

Ideal gas

Radiation pressure
by blackbody spectrum



- **Temperature**

$$P_{core} \rightarrow P_{core} + \delta P$$

$$T_{core} \rightarrow T_{core} + \delta T$$

$$\frac{\delta T}{T_{core}} = \frac{1}{4}\frac{\delta P}{P_{core}} = 3.922 \times 10^{-4}$$

0.039% increase of the central temperature.

Result : Electromagnetic fluctuation

- **Core temperature and solar neutrino fluxes**

Empirically, it is known that the neutrino fluxes obey the scaling law for small deviation of the central temperature. (Bahcall & Ulrich 1988)

$$\phi_i \propto T_{core}^{m_i} \qquad m_i = \frac{d\, ln\phi}{d\, lnT}$$

$$\frac{\Delta\phi_i}{\phi_i} = m_i \frac{\Delta T_{core}}{T_{core}} = m_i \left(\frac{\delta P}{4P_{SSM}} \bigg|_{core} \right) \phi_i$$

ϕ_i : neutrino flux produced by a source i

T_{core} : core temperature of sun

m_i : corresponding exponent

PRELIMINARY

Neutrino fluxes	Exponents m_i (Bahcall & Ulmer 1996)	$\Delta\phi_i/\phi_i(\%)$
ϕ_{pp}	-1.1	-4.31×10^{-2}
ϕ_{pep}	-2.4	-9.41×10^{-2}
$\phi_{^7Be}$	10	3.92×10^{-1}
$\phi_{^8B}$	24	9.41×10^{-1}
$\phi_{^{13}N}$	24.4	9.56×10^{-1}
$\phi_{^{15}O}$	27.1	1.06
$\phi_{^{17}F}$	27.8	1.09

Summary

- **Summary**

  - We found the dielectric permittivity through the Vlasov equation.
  - Using the longitudinal permittivity, we can determine the dynamical screening effect.
  - We applied the screening effect to BBN conditions and solar core condition.
  - Considering EM fluctuations, we can confirm the change of blackbody radiation.
  - For the condition of solar core in the SSM, we obtain 0.039% increase of the central temperature.
  - As a result, we have obtained a 1% change in CNO neutrino fluxes

Thank you for your attention!