THE NOVEL SIGNAL DETECTION FOR GRAVITATIONAL WAVES VIA AUTO-REGRESSIVE APPROACH

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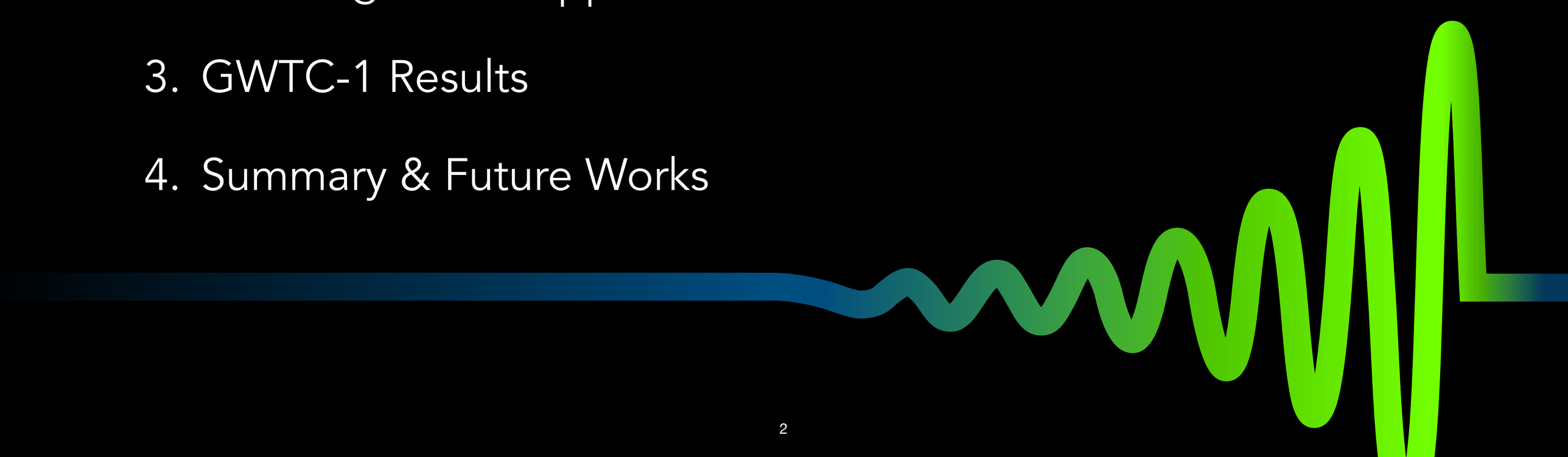
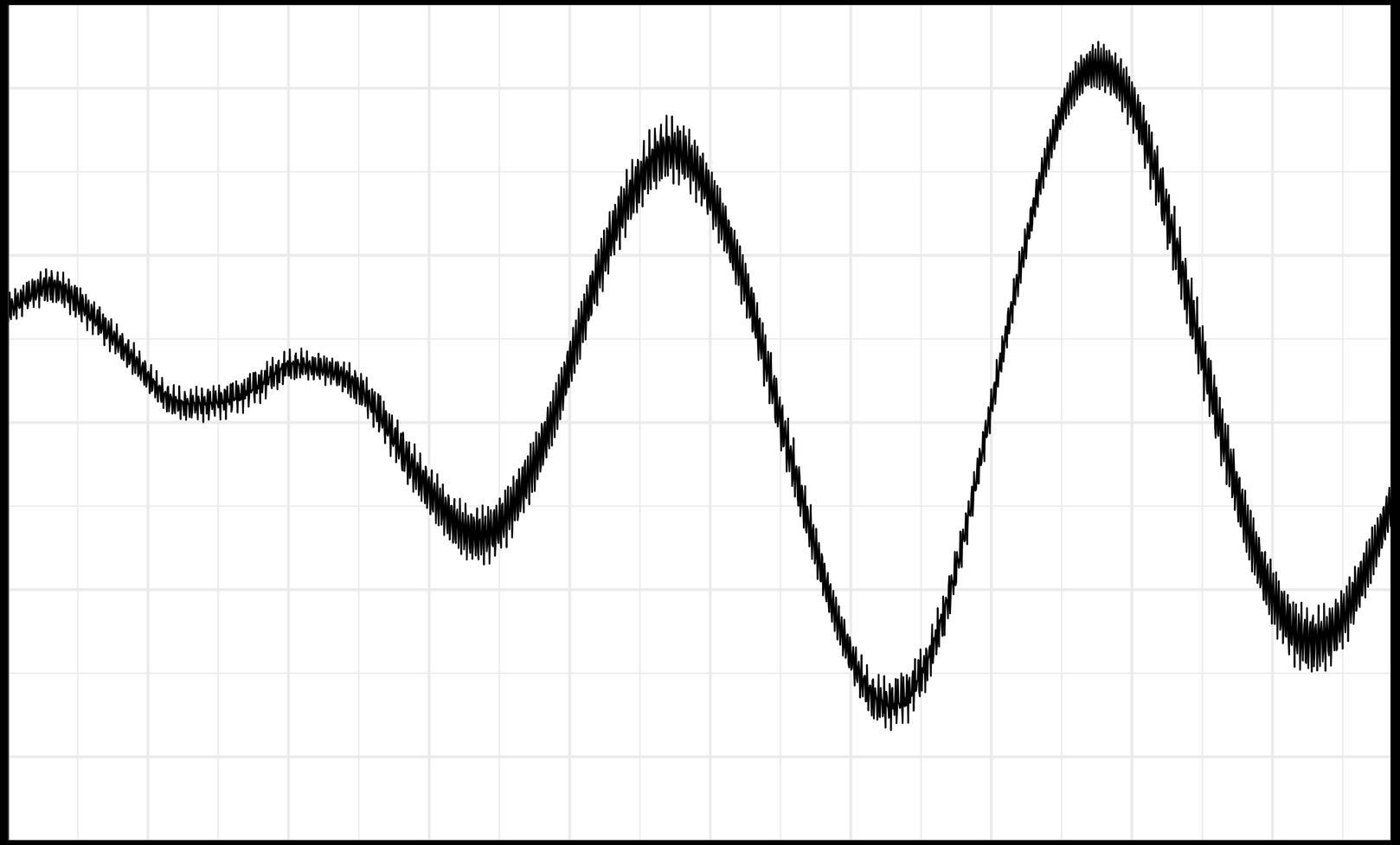
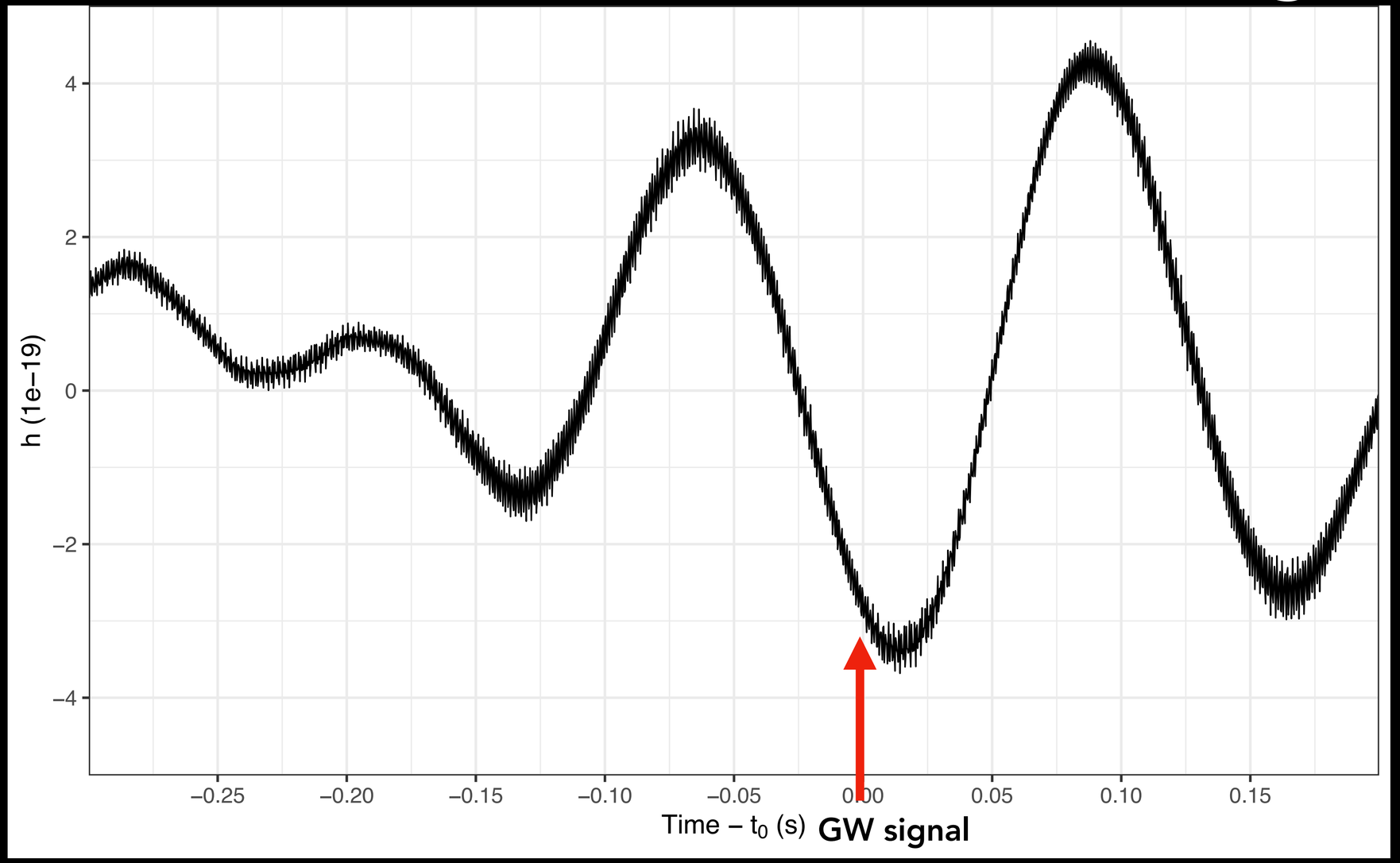


Signal Processing

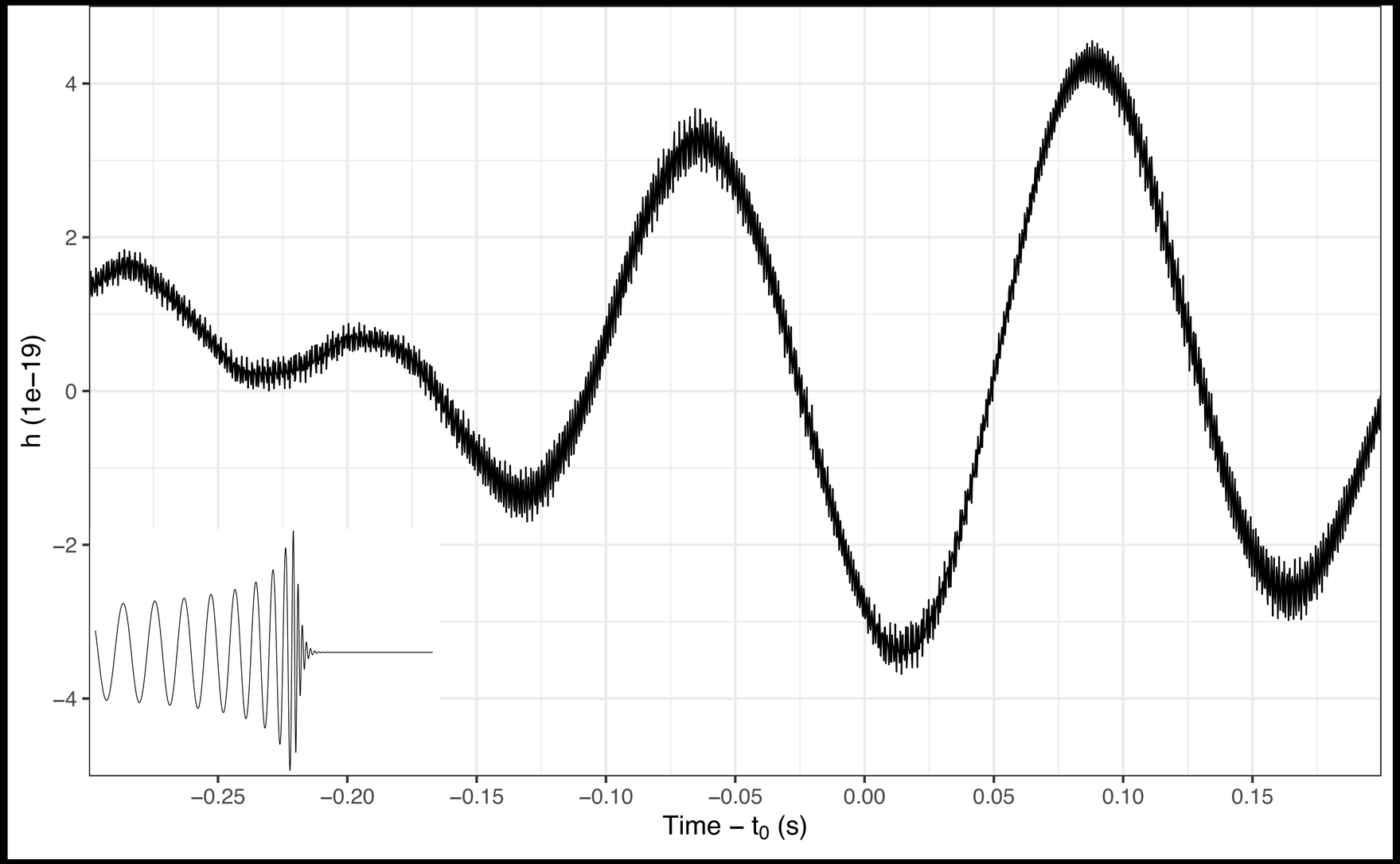
Where is the Gravitational Wave (GW) signal?



Where is the Gravitational Wave (GW) signal?



Matched filter



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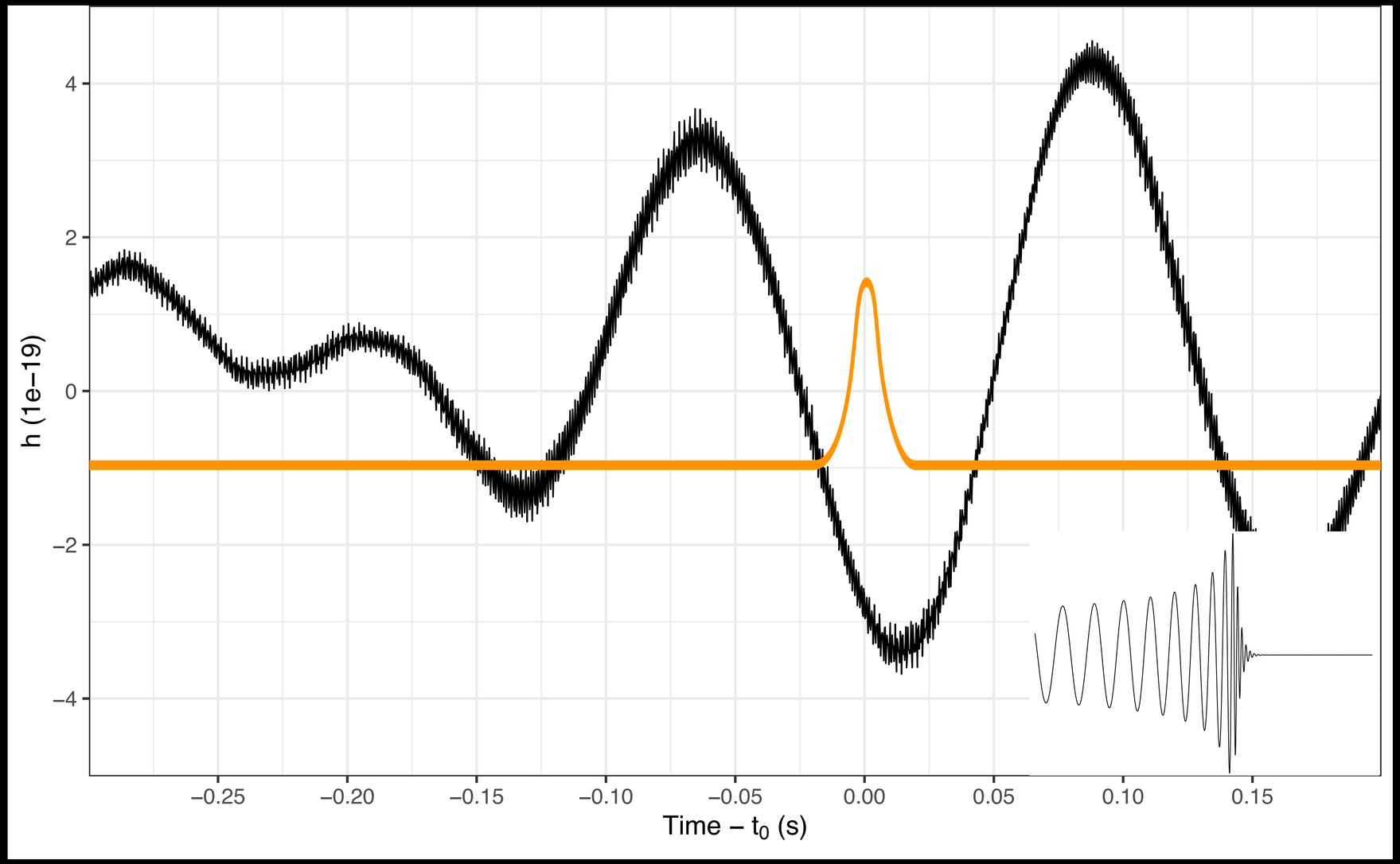
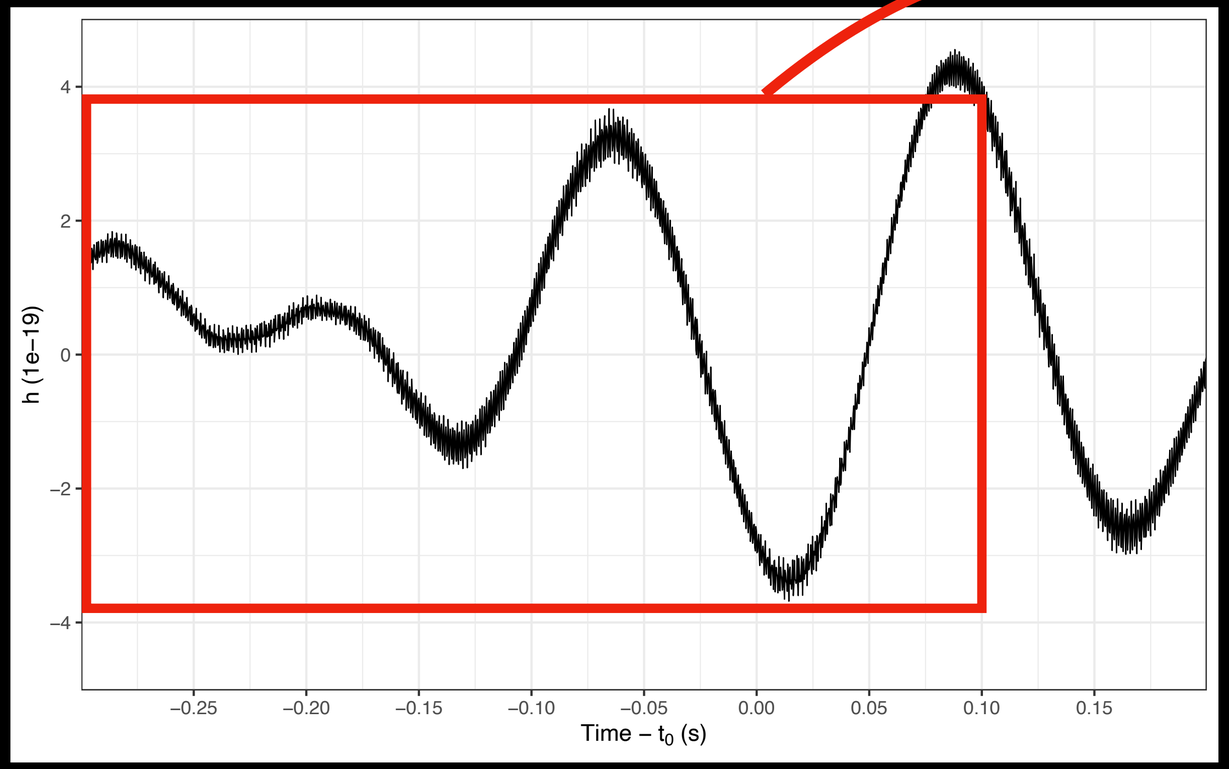
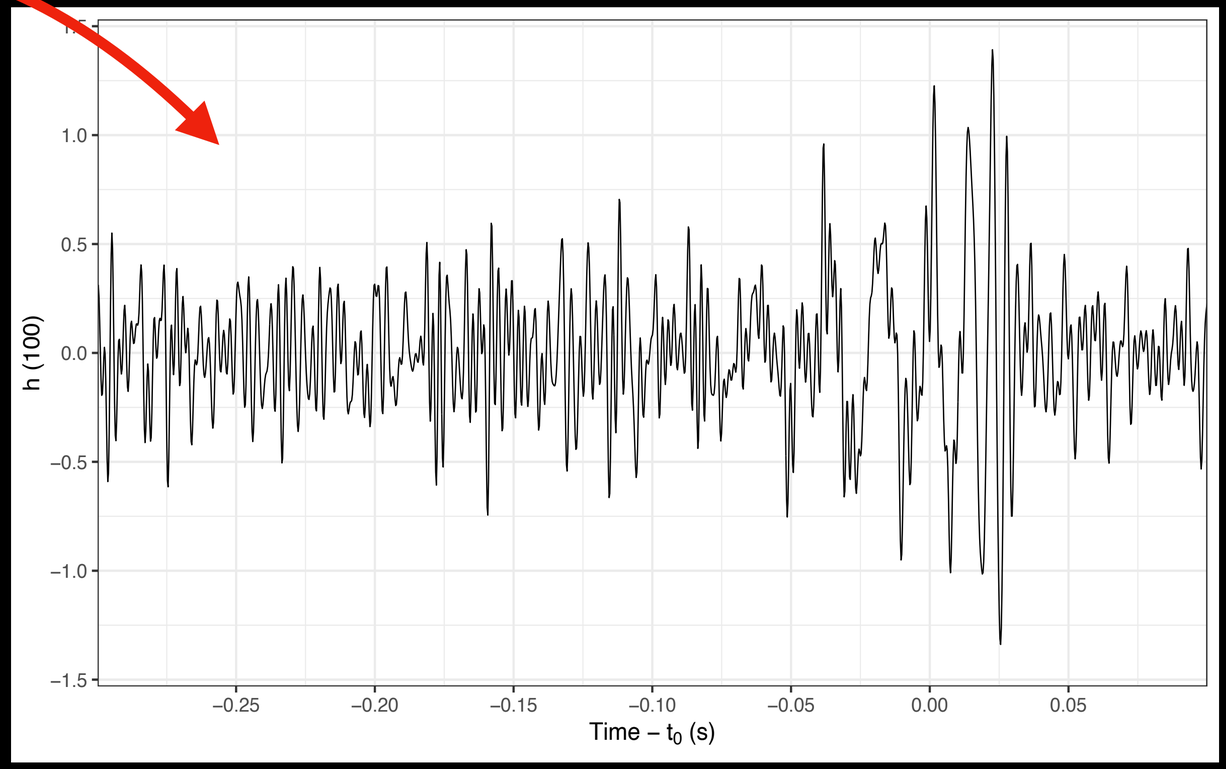


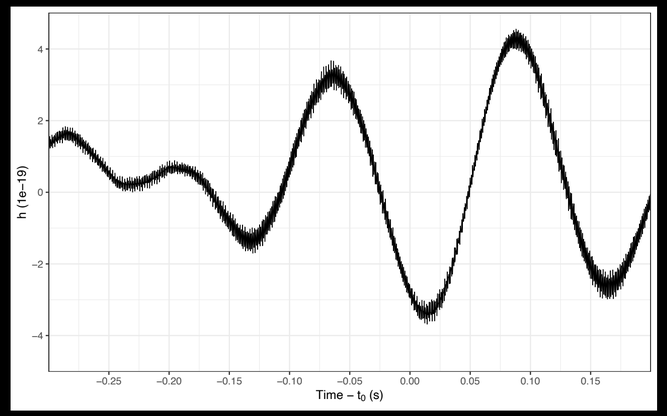
Matched filter & Spectral whitening



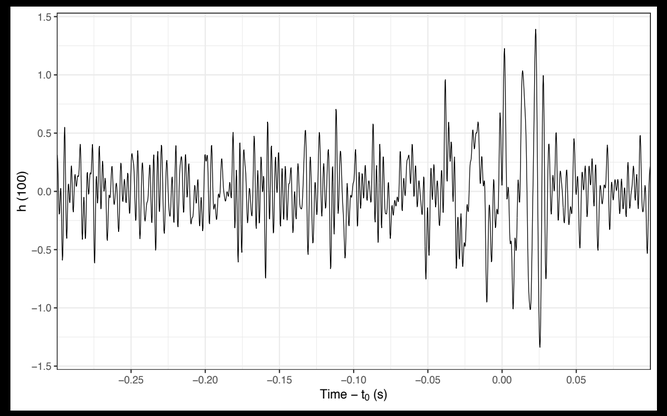


Spectral whitening

Time series

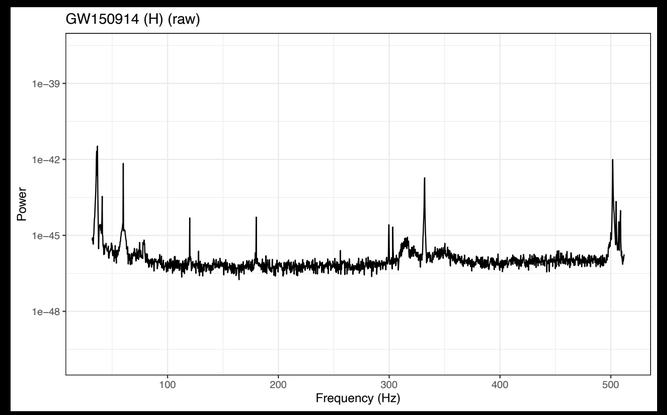


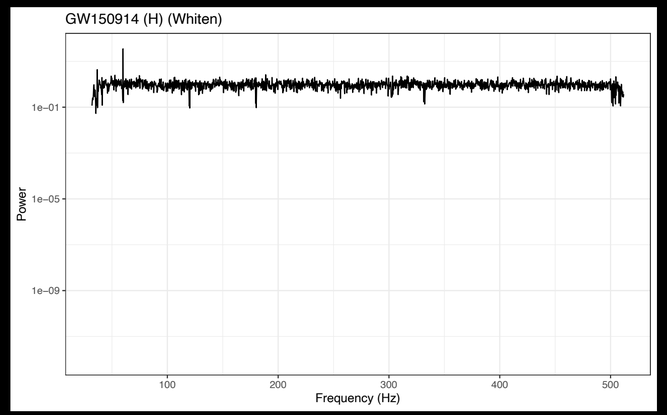
Flatten PSD



estimate

PSD





Matched filter & Spectral whitening

Shortcomings:

- Computational efforts to generate waveforms
- Even those waveforms can not cover all possible kinds of GW signal
- The order of magnitude in amplitude is lost during spectral whitening
- Less flexibility to optimize whitening the data

Auto-regressive Approach

Auto-regressive (AR)

Auto-regressive (AR)

*Auto*regressive = *self*-regressive

"Predict a future value based on past values"

Linear regression

Stochastic process

Auto-regressive (AR)

*Auto*regressive = *self*-regressive

"Predict a future value based on past values"

Linear regression

Stochastic process

$$x_t = \sum_{i=1}^{p} a_i x_{t-i} + \epsilon_t + c$$

< AR model with the order of p >

ARIMA

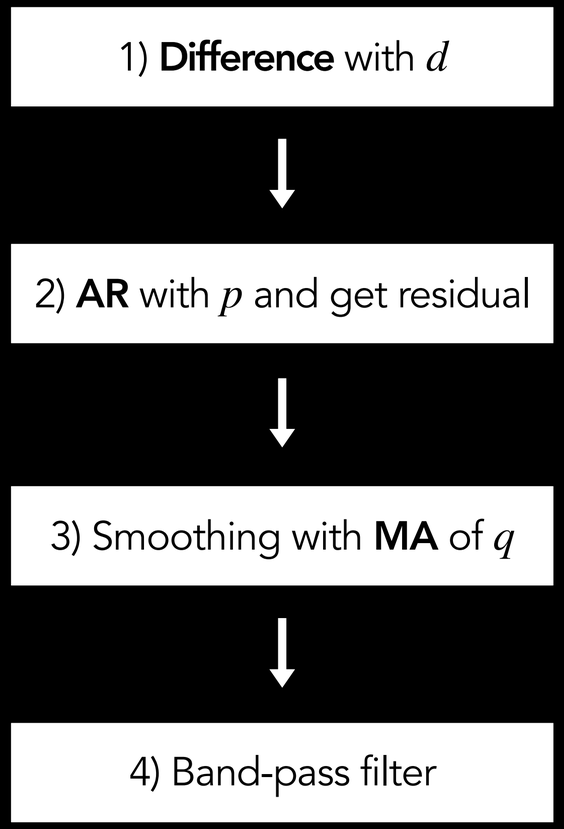
Autoregressive Integrated Moving Average (ARIMA) model

ARIMA

Autoregressive Integrated Moving Average (ARIMA) model

$$\underbrace{(1-B)^d x_t}_{\text{Difference}} = \underbrace{\sum_{i=1}^{p} a_i x_{t-i}}_{\text{Auto-regression}} + \underbrace{\sum_{j=1}^{q} b_i \epsilon_{t-j}}_{\text{Moving Average}} + \epsilon_t$$

Customized Sequential ARIMA



$$x_t = (1 - B)^d x_t$$

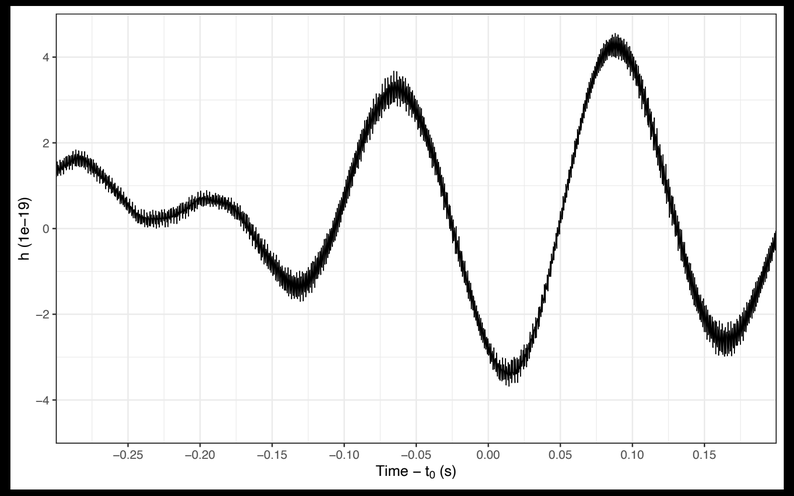
$$x_t = \sum_{i=1}^{p} a_i x_{t-i}$$

$$x_t = \frac{1}{q} \sum_{j=-k}^{k} x_{t+j}$$

(Butterworth filter)

Customized Sequential ARIMA

0) **Raw**



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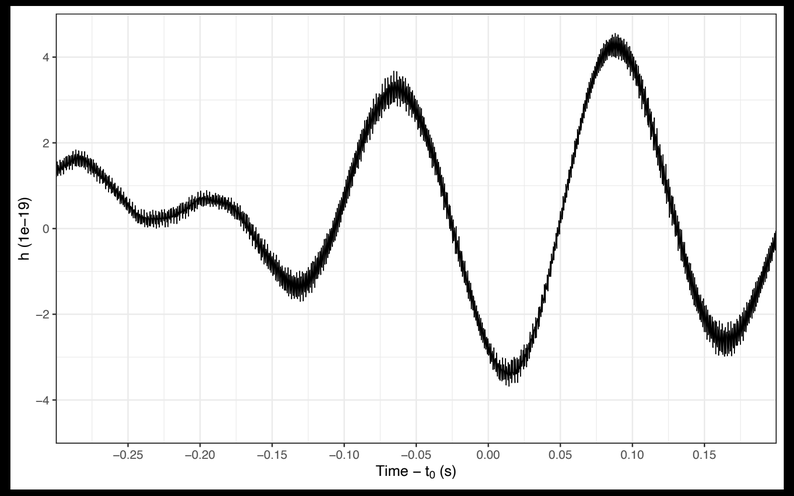
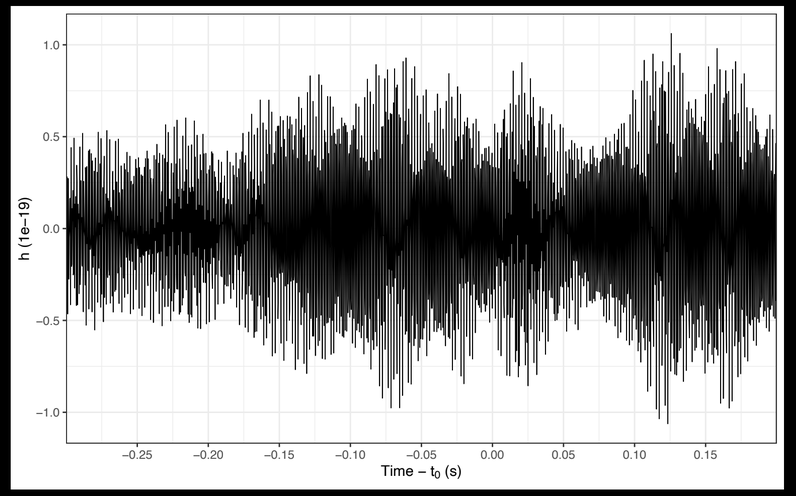
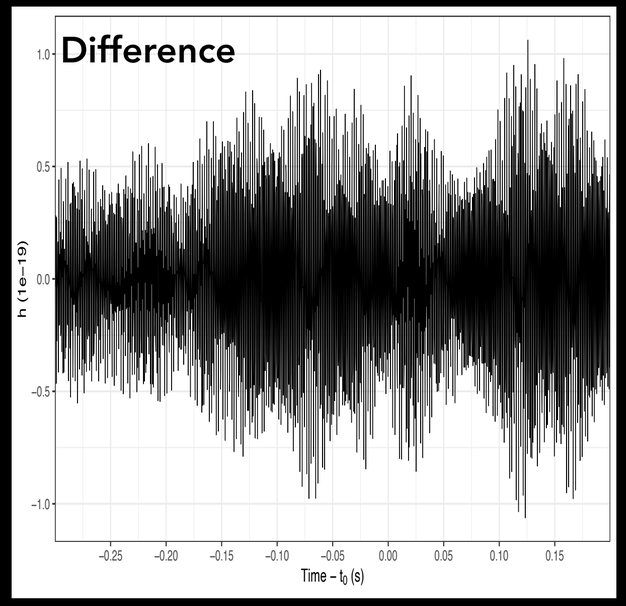


1) **Difference** with d



Customized Sequential ARIMA

2) **AR** with p



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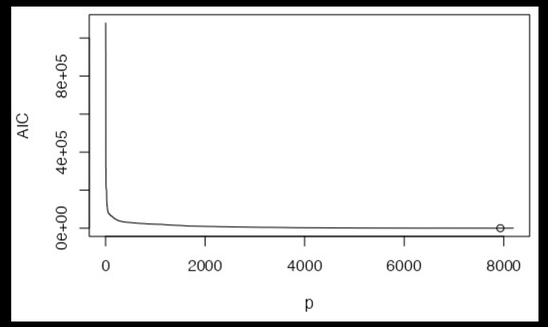

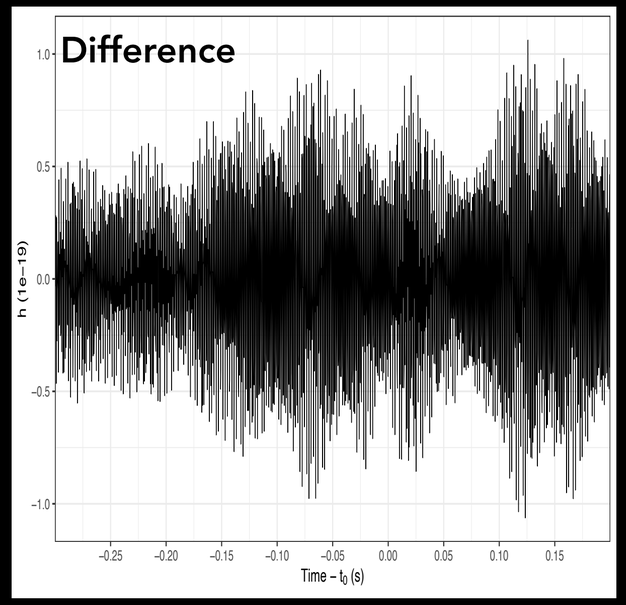

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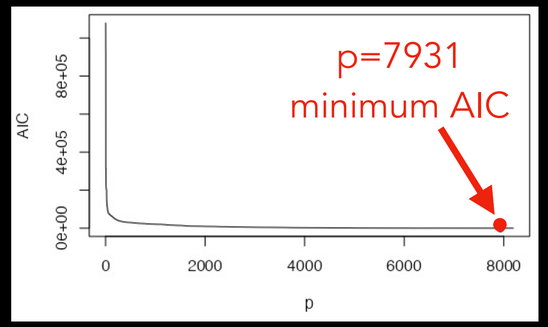
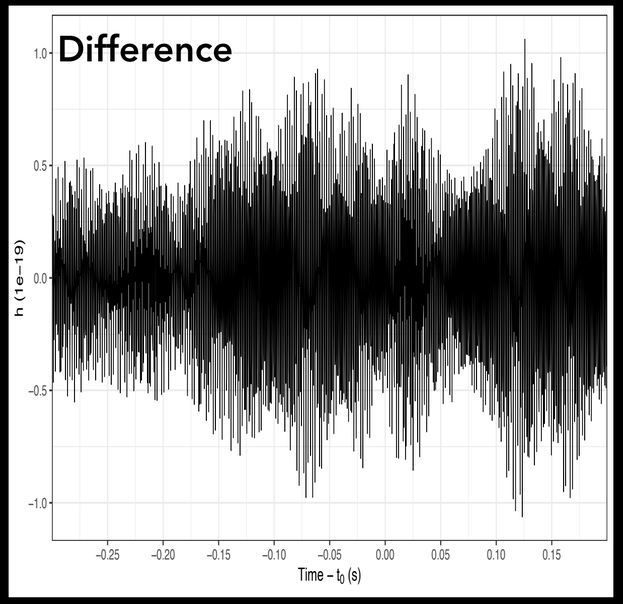
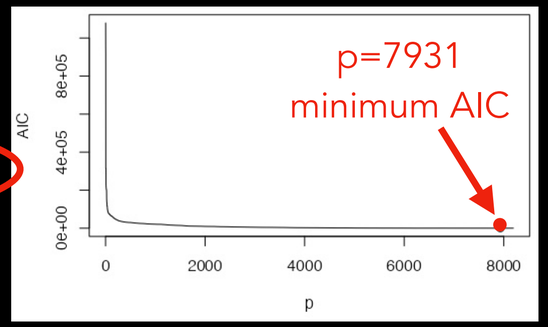


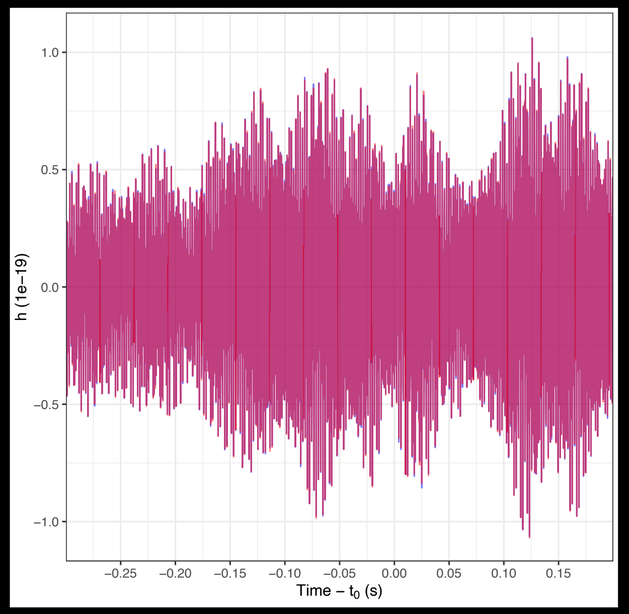


Customized Sequential ARIMA

2) **AR** with p

2-1) **Fitted model**





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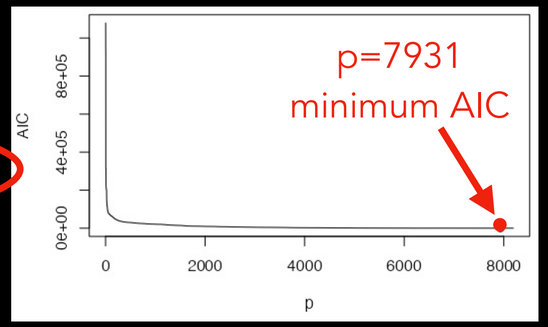

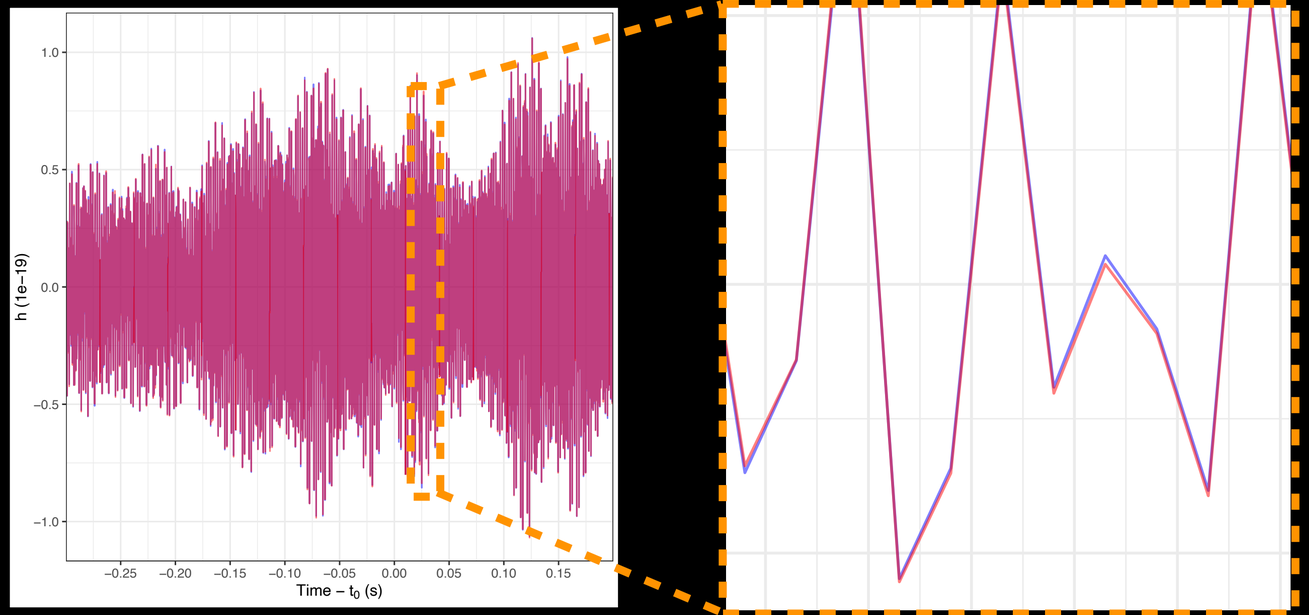

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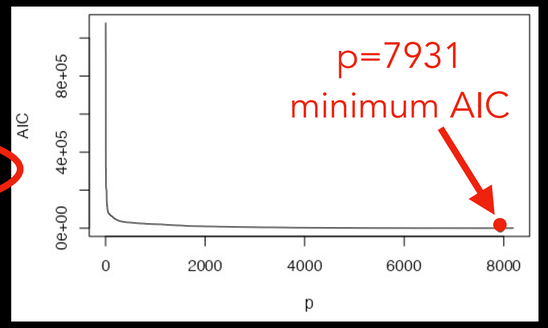
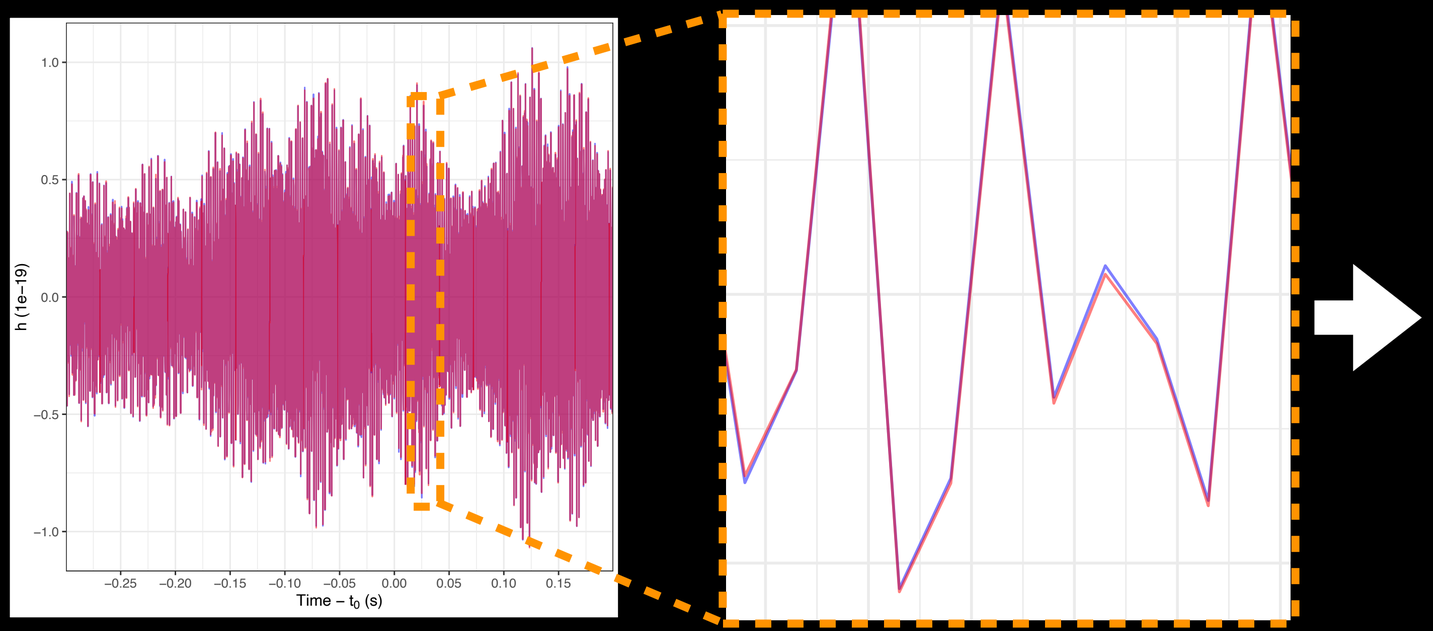


2-2) **Residuals**

$$y_{\text{resid}} = y - \hat{y}$$

(Residuals) = (Data) - (Fitted model)



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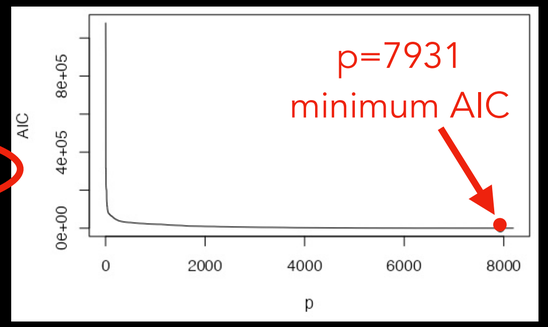

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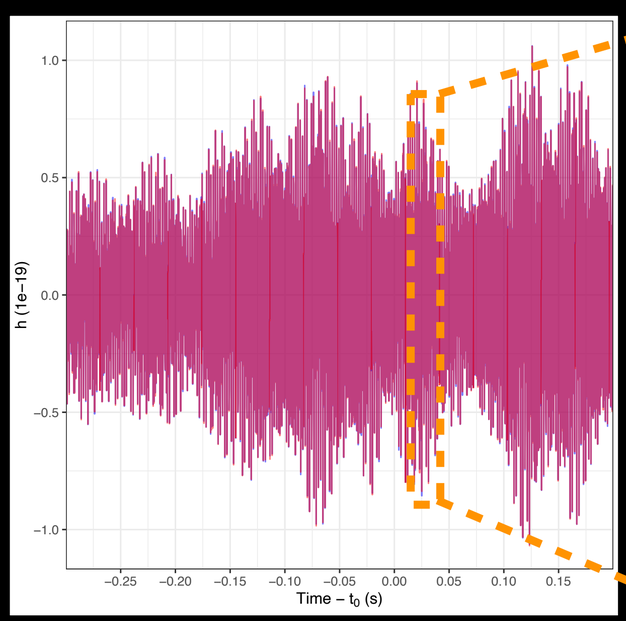
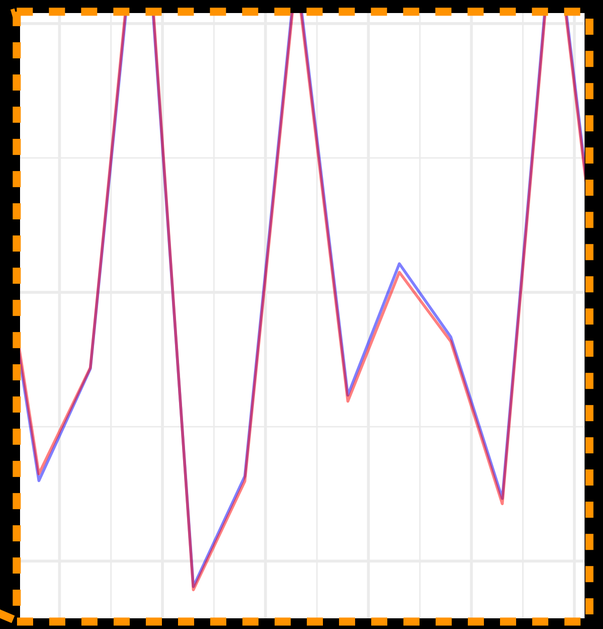
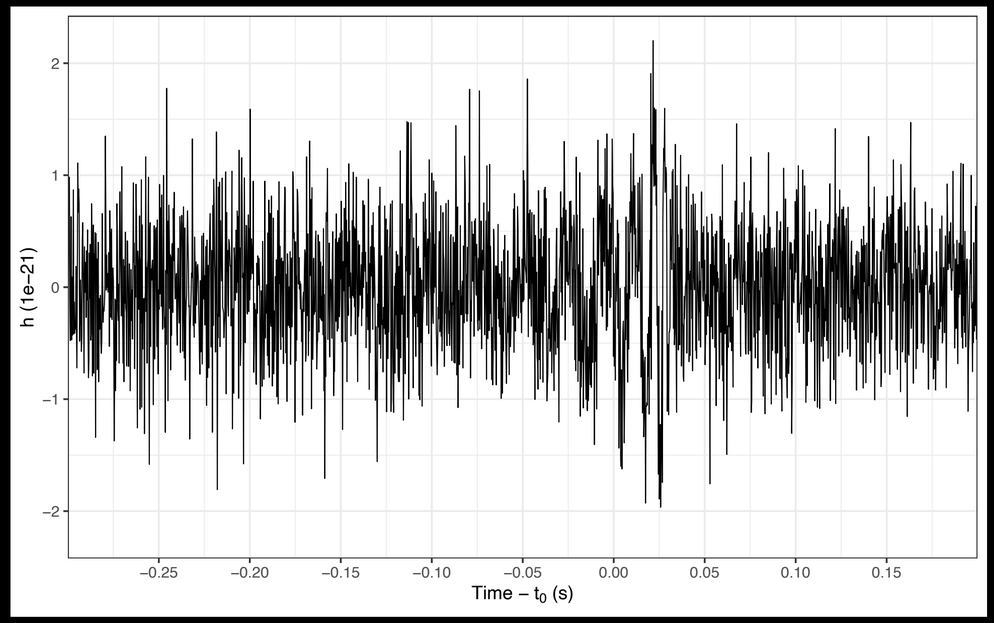
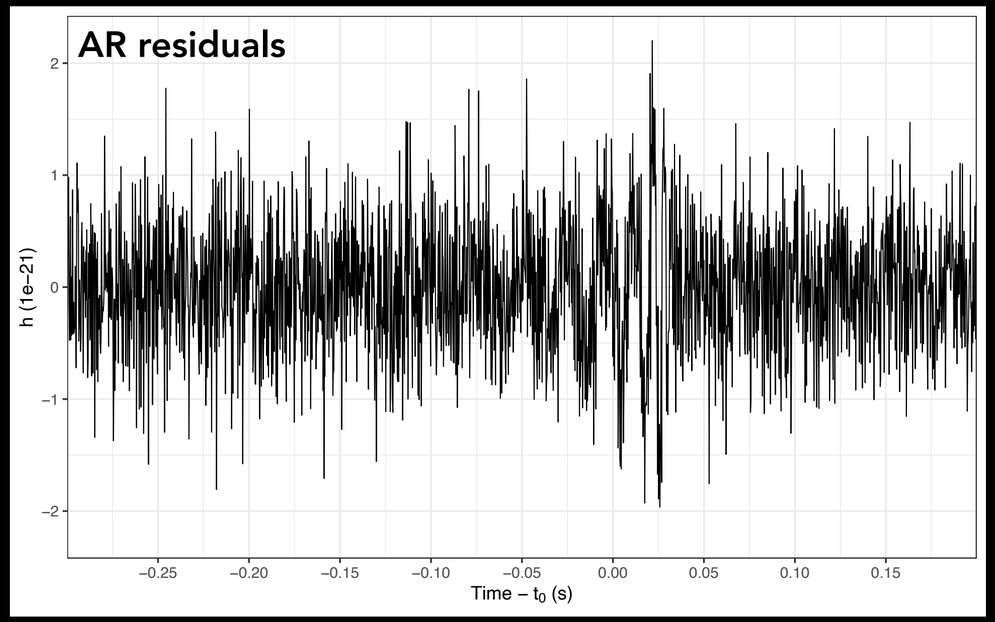






Customized Sequential ARIMA

3) **MA** with q



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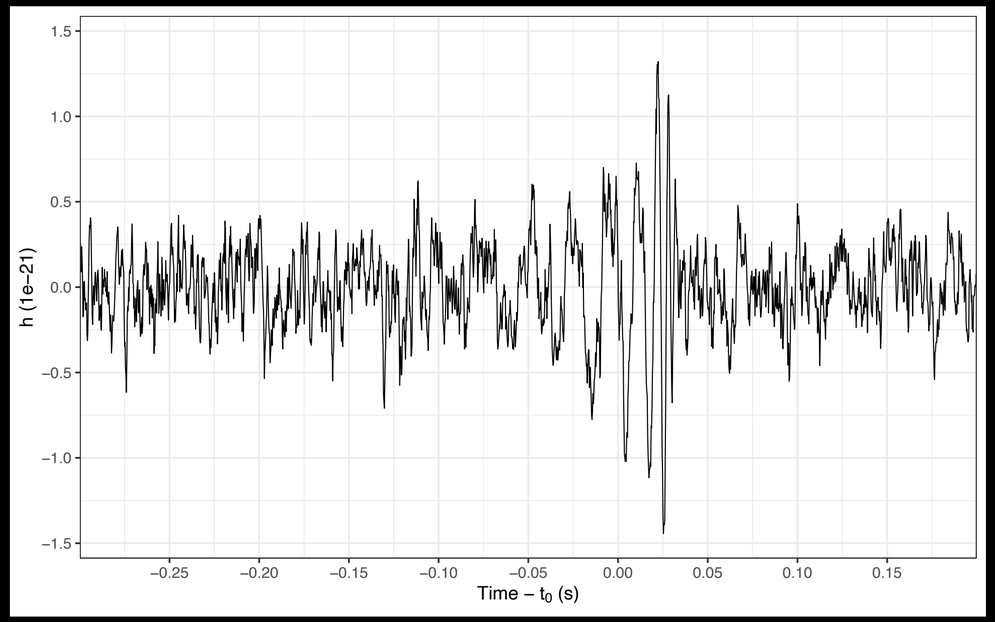

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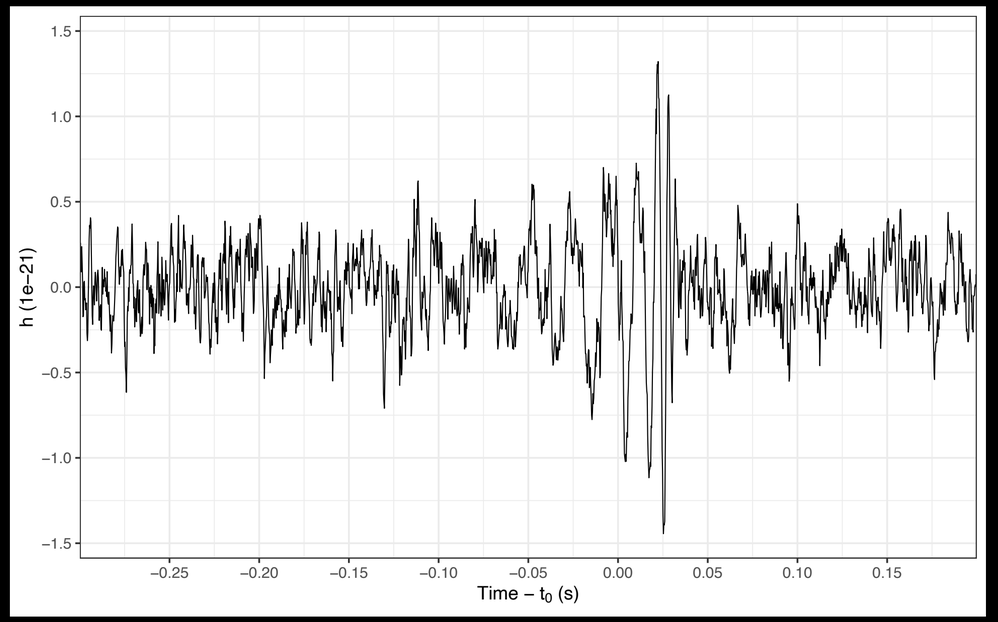
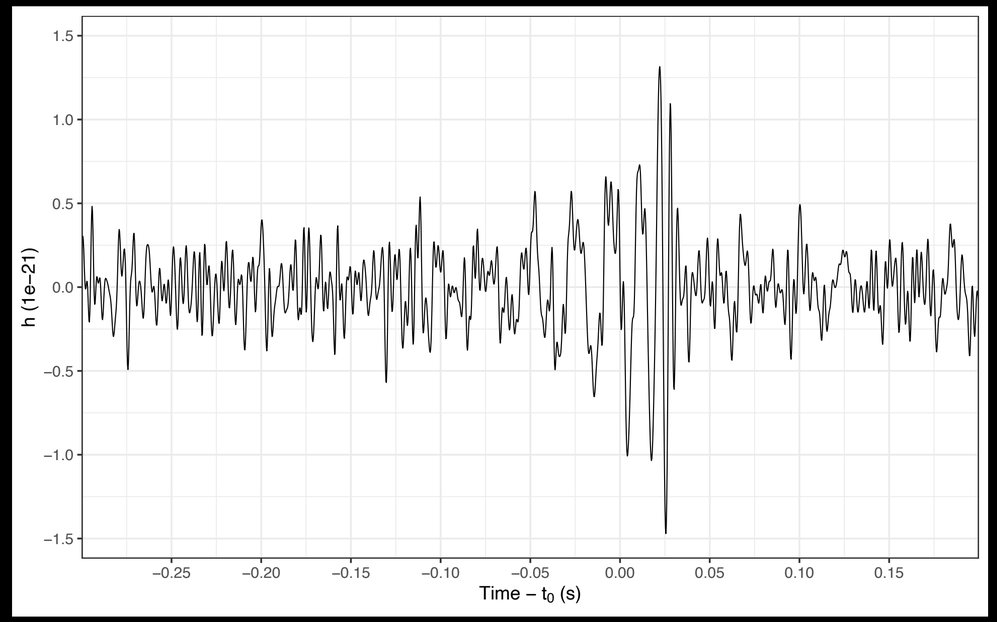
3) **MA** with q



4) Band-pass filter



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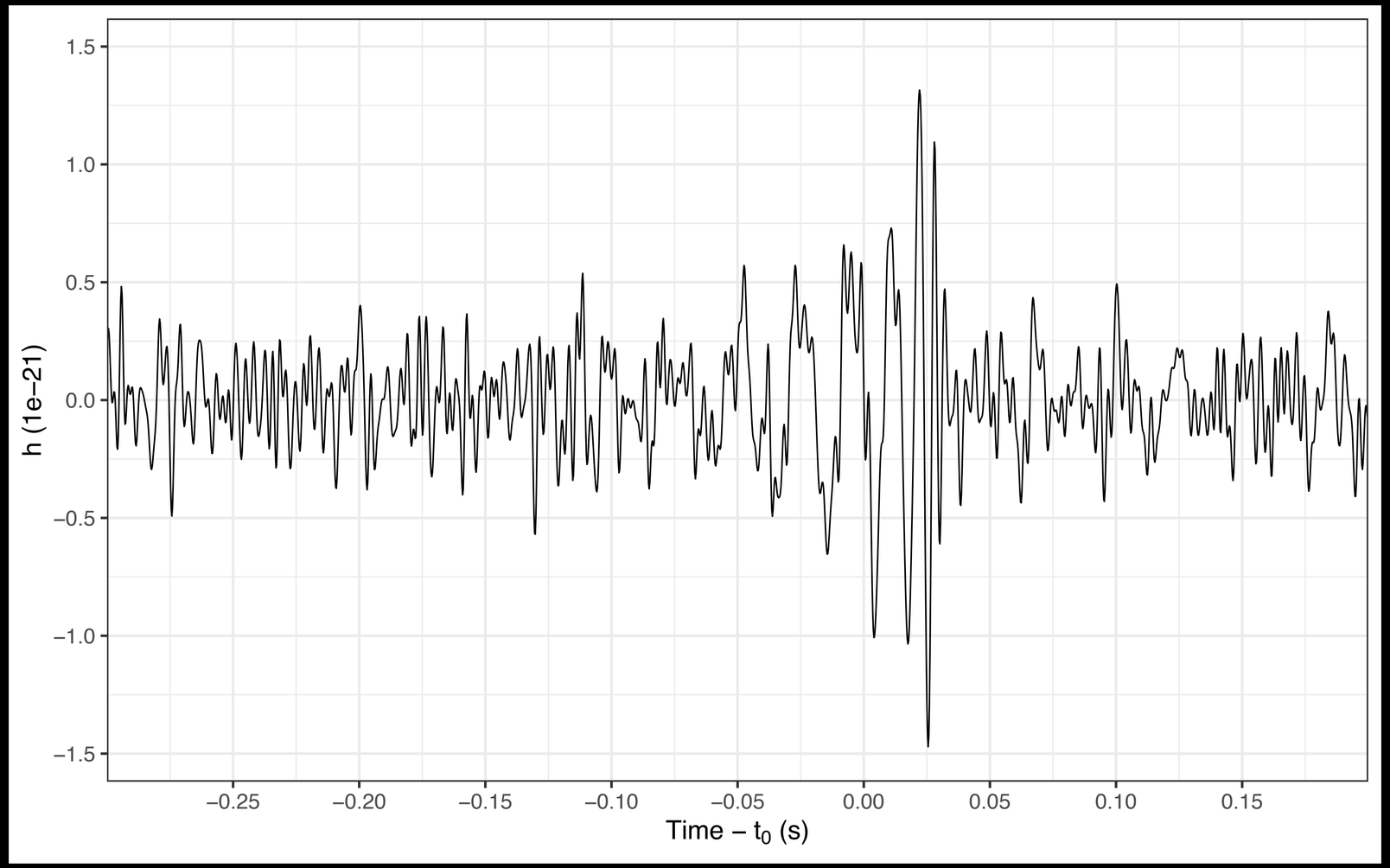

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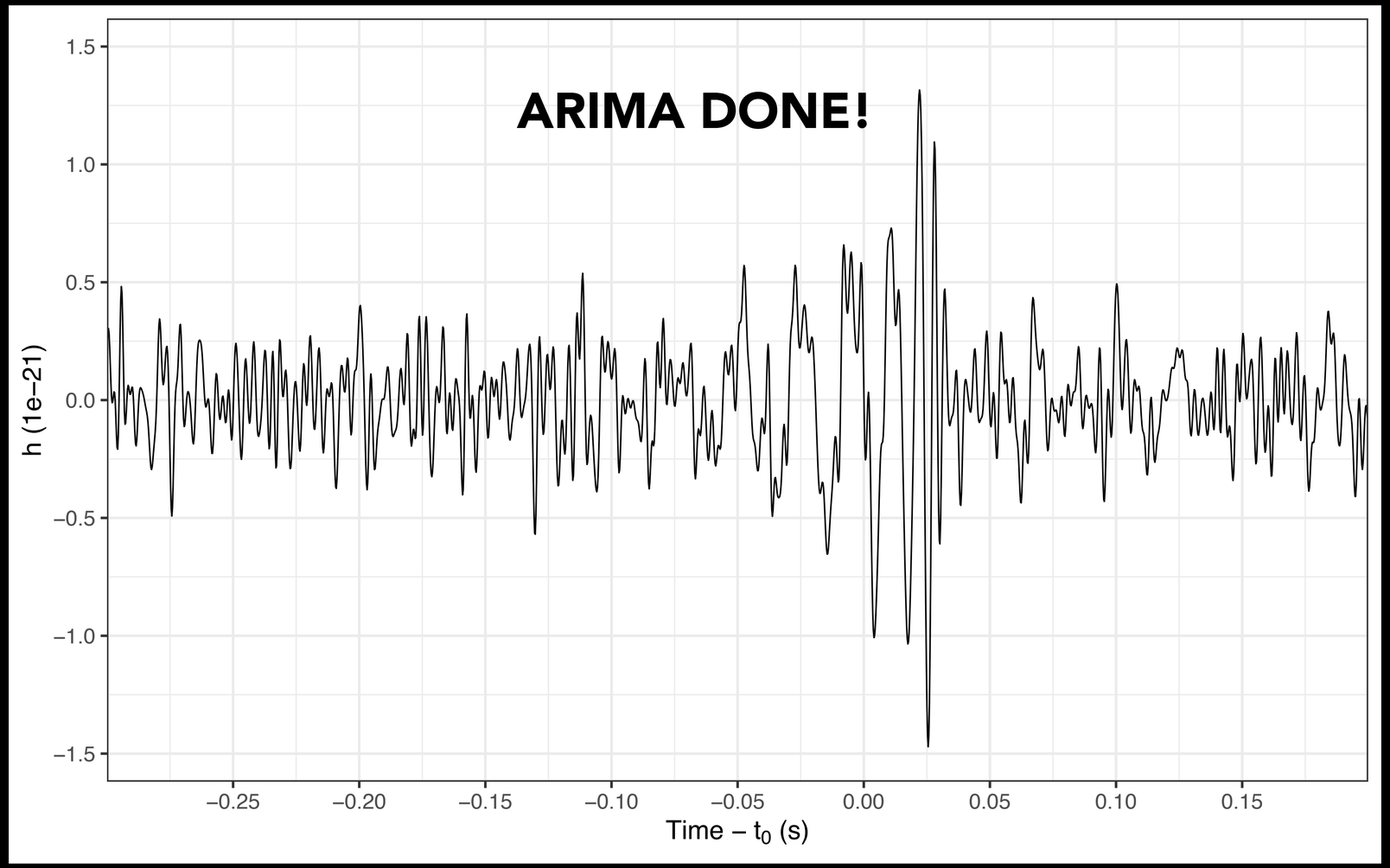

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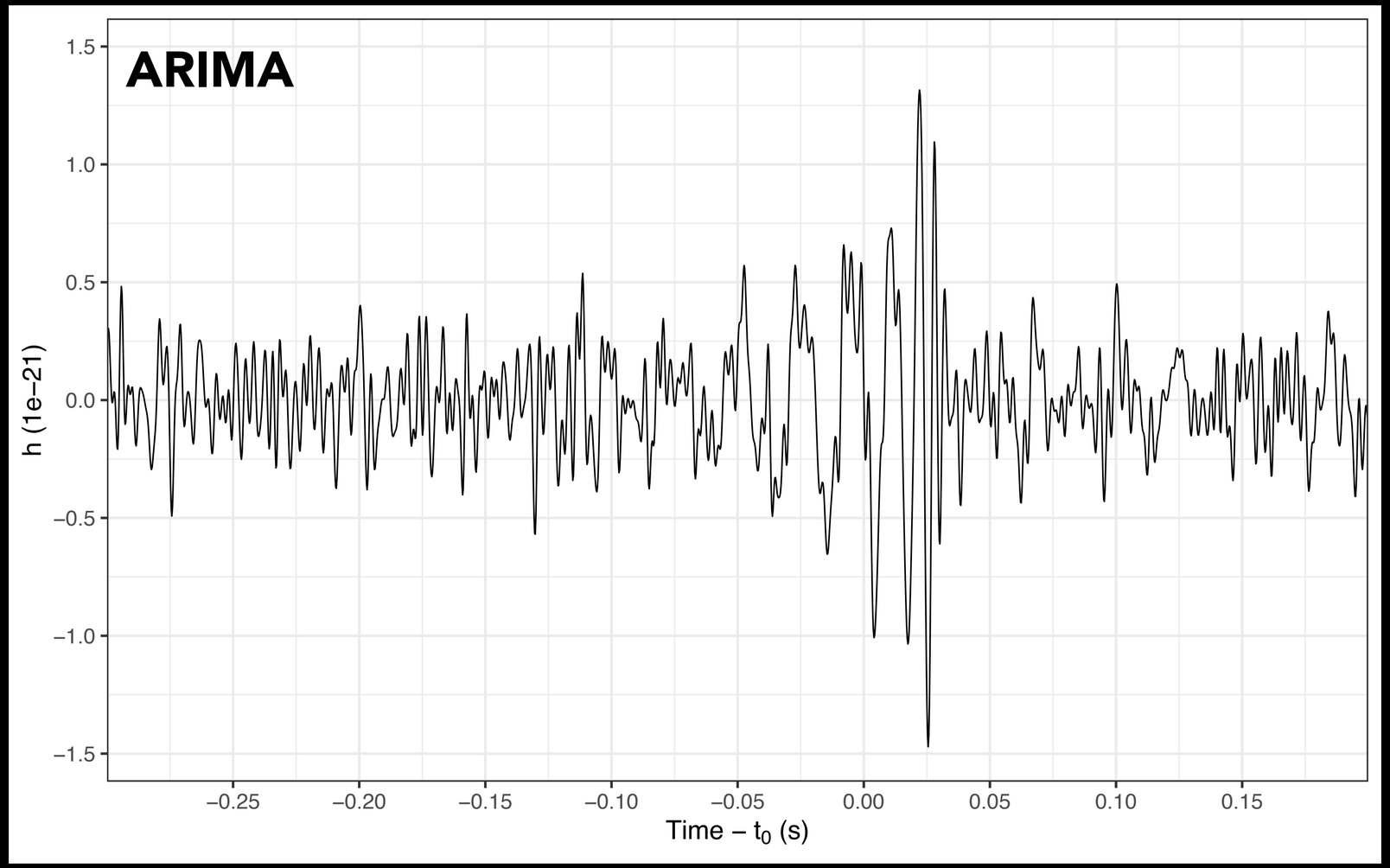

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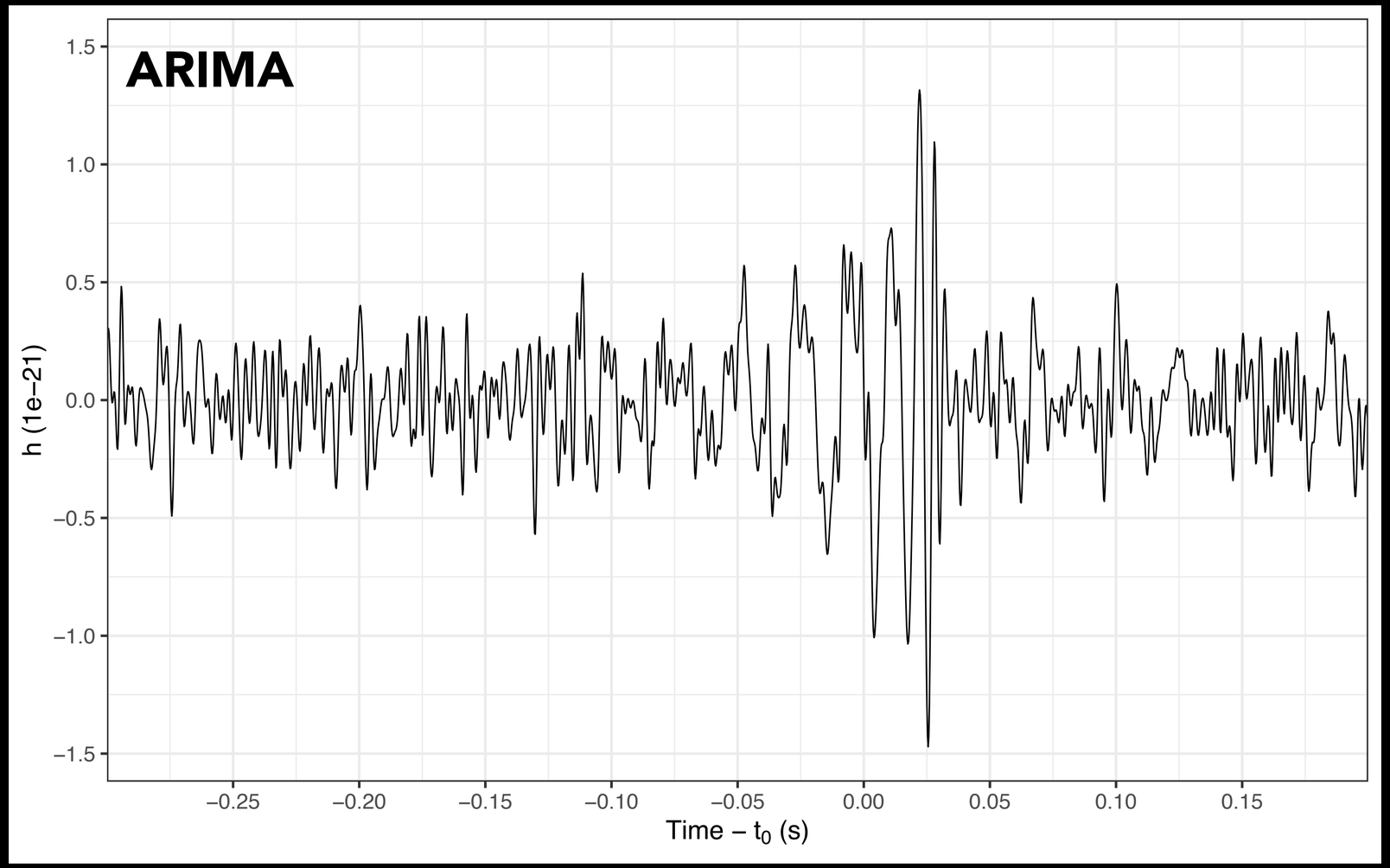


Advantages of ARIMA

1) More degree of freedom in modeling noise
2) Preserving scale of signals
3) Flexibility to use in low-latency search (lower p)
4) Flat PSD as well as noise reduction
5) Higher cross-correlation between two detectors

GWTC-1 Results

1) Power Spectral Density
2) Spectro- & Oscillogram
3) Cross-correlation & Data Combination
4) Cross-correlation comparison

Experiment control

p: AR order (auto search)
q: MA order (fixed = 7)
d: Difference (fixed = 2)
fl: Lower frequency bound (fixed = 32 Hz)
fu: Upper frequency bound (fixed = 512 Hz)

1) Power Spectral Density

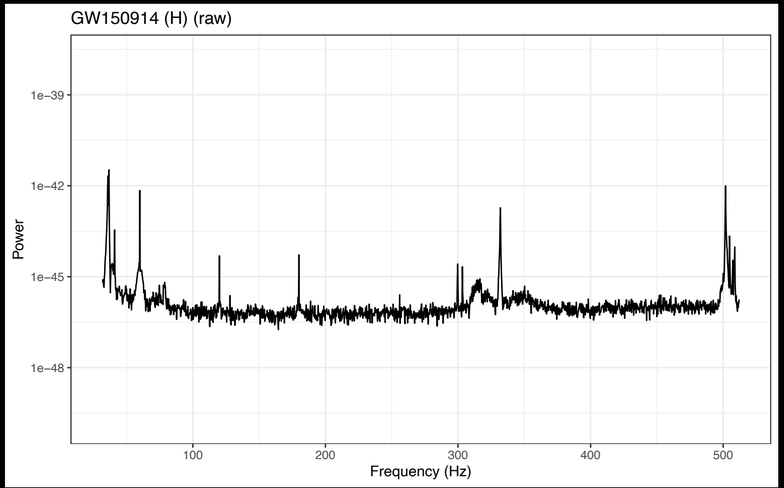
GW150914
(spectral whitening)

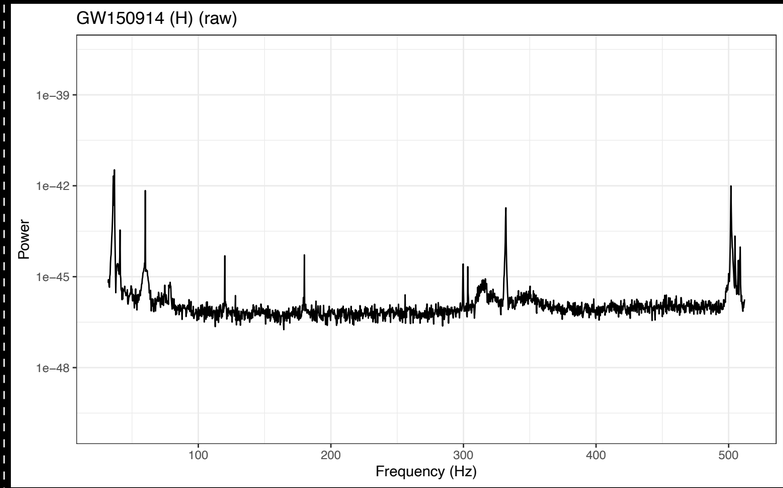
GW150914

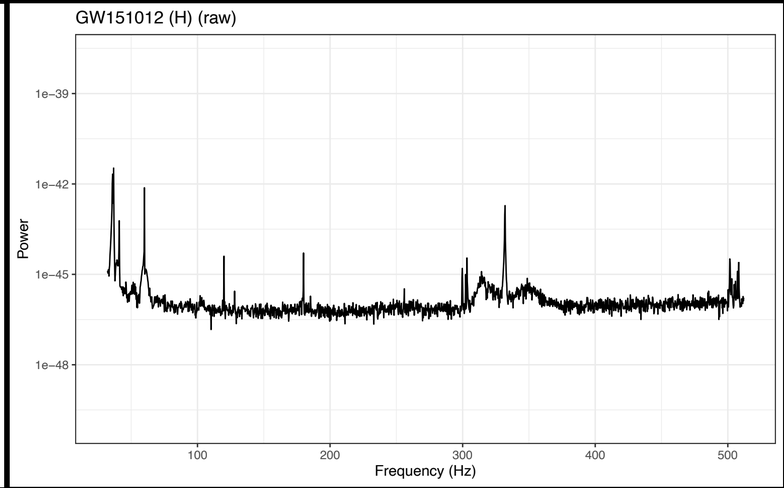
GW151012

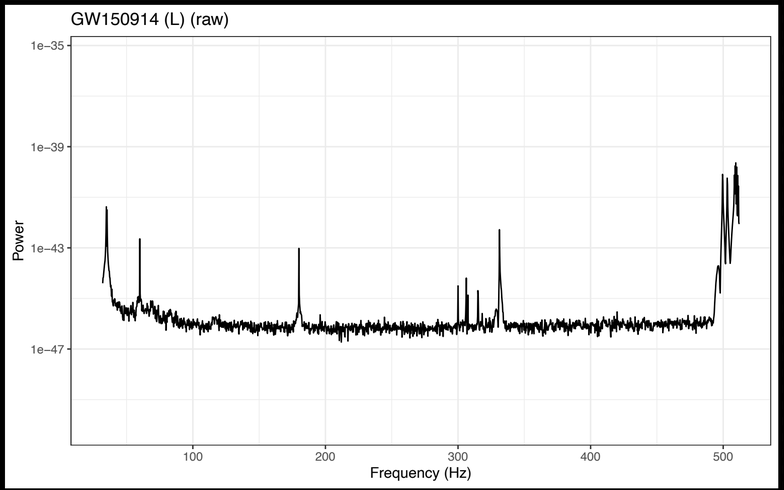
Hanford

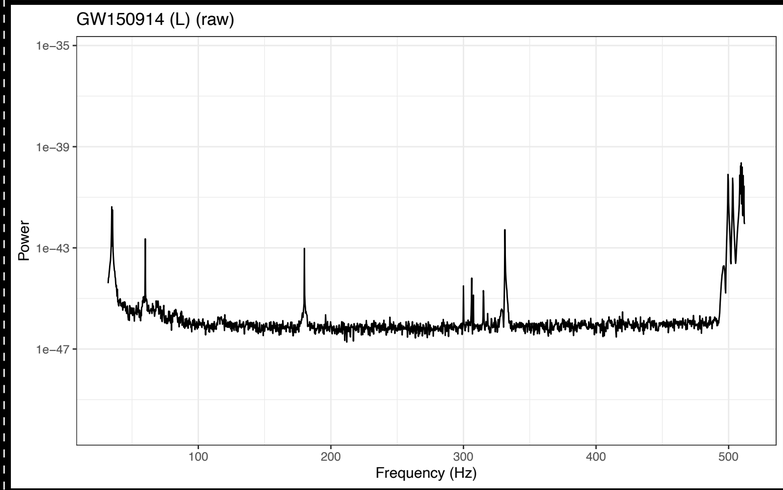
Livingston

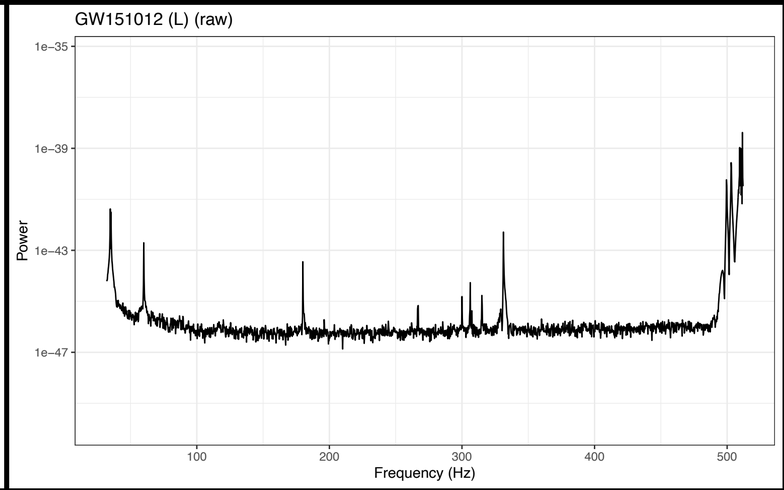












ARIMA

GW150914
(spectral whitening)

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GW151012

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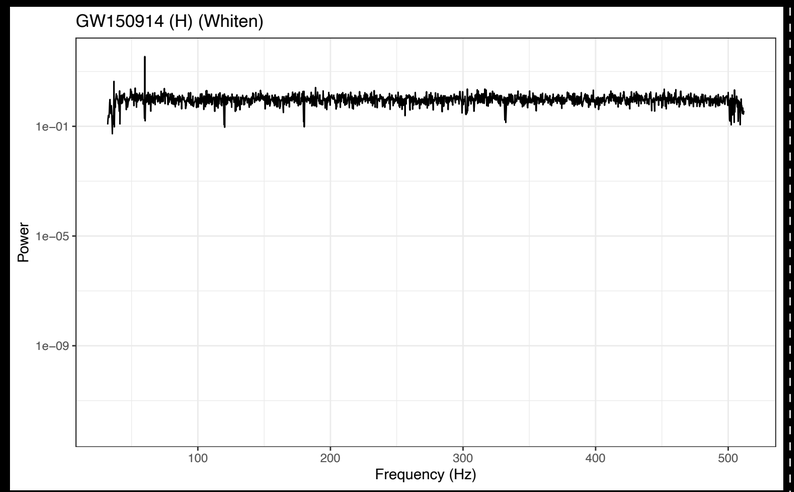

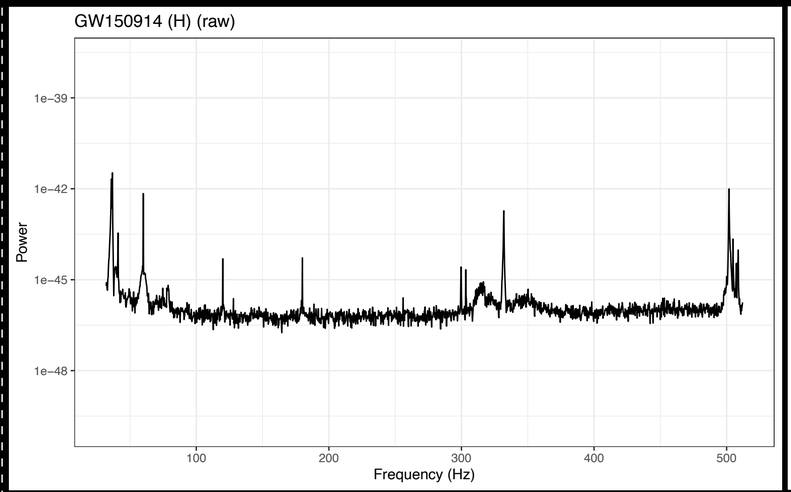

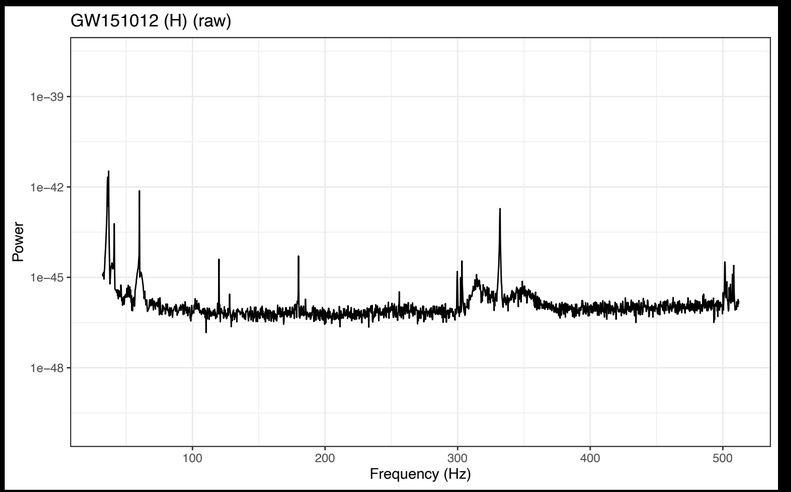

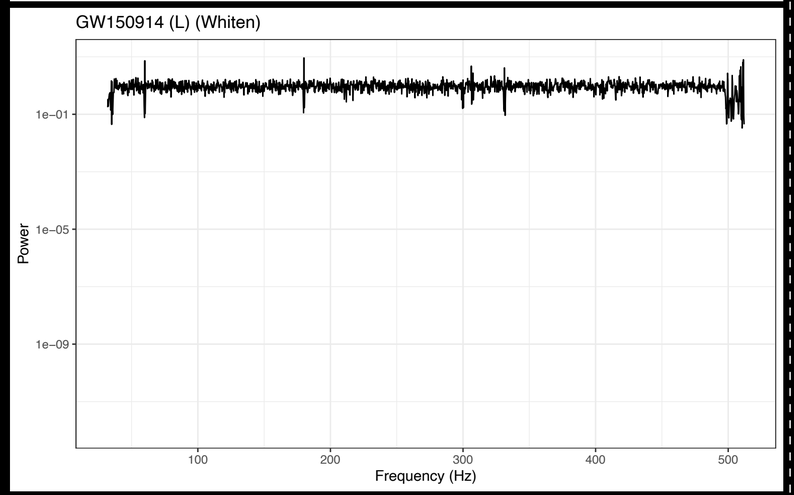

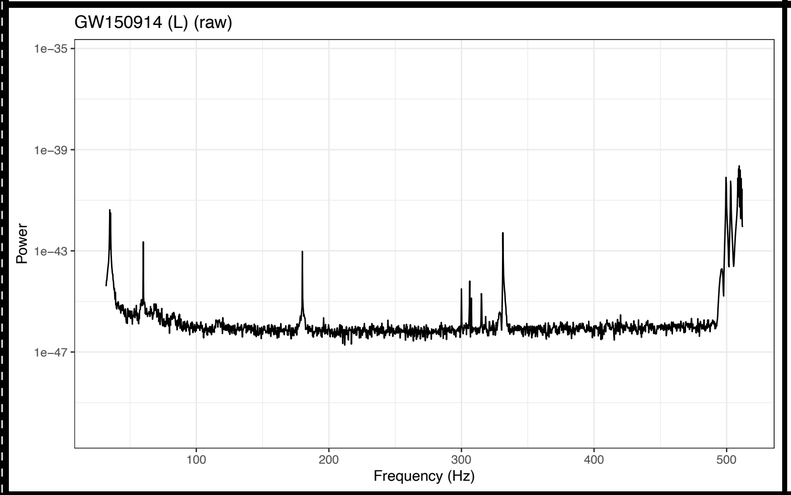

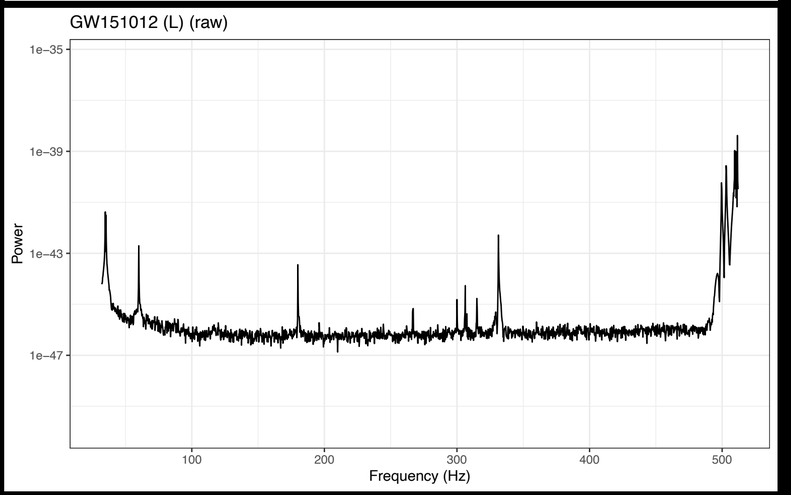

ARIMA

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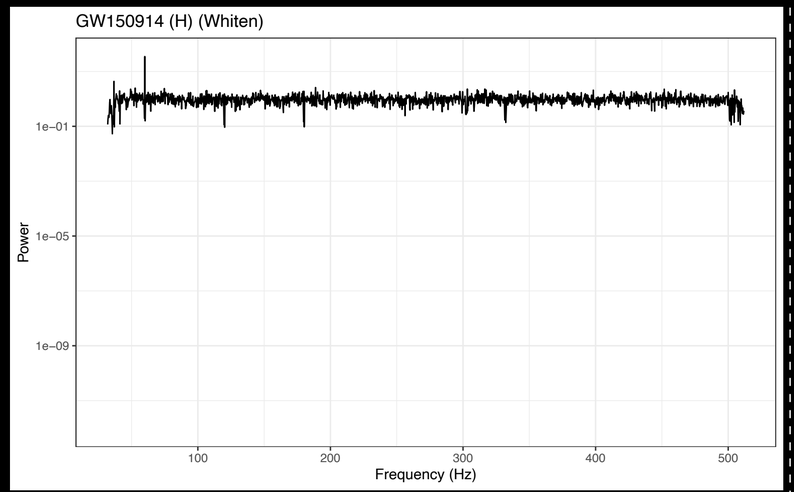
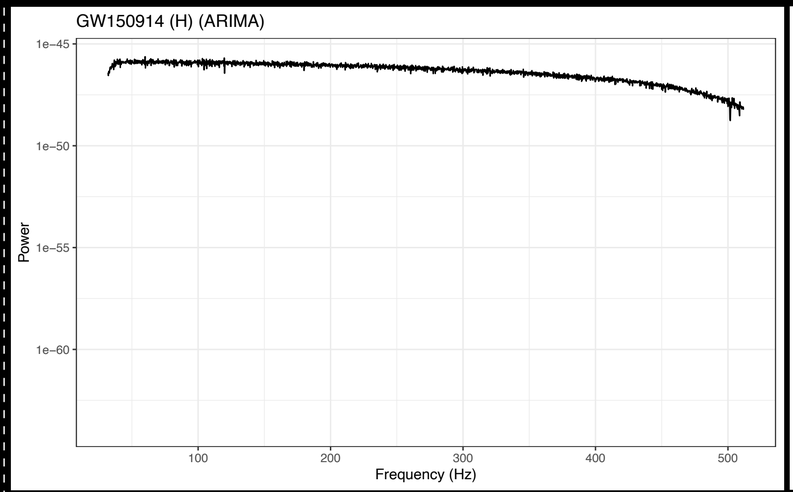
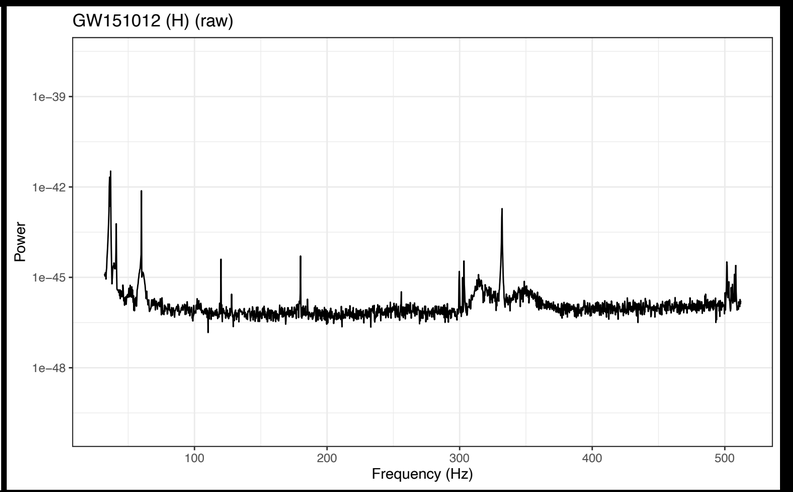
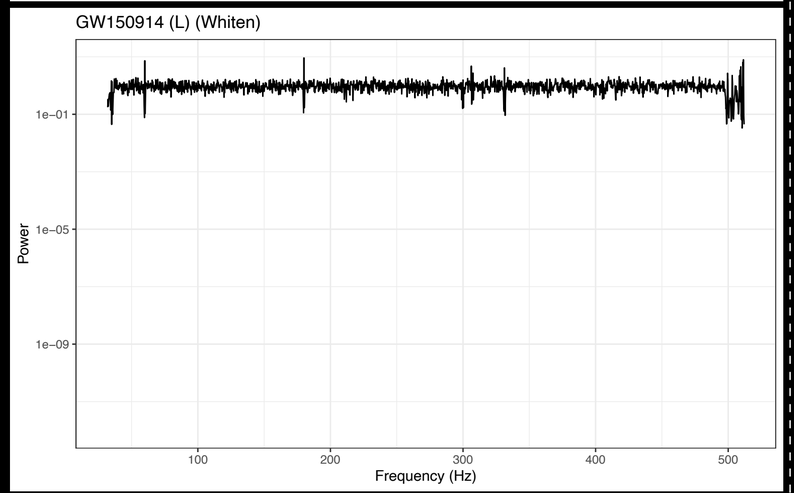
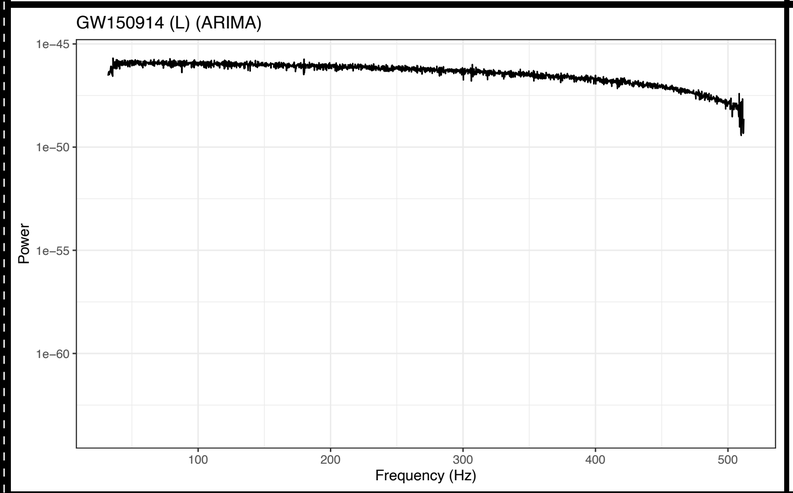
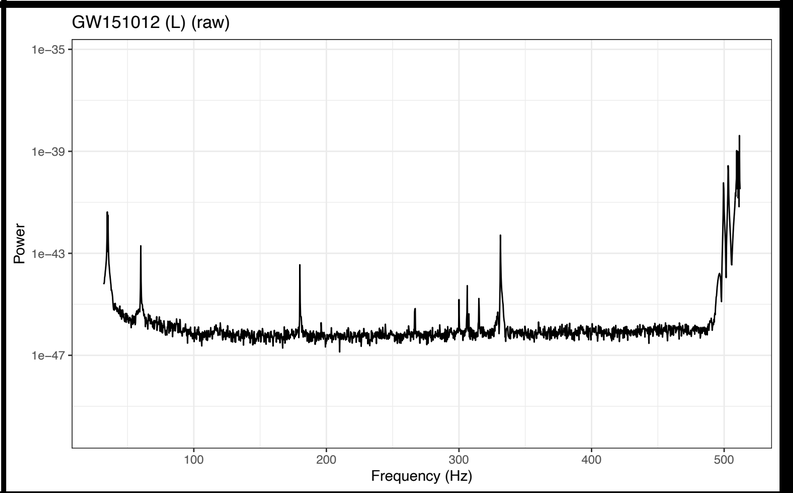












ARIMA

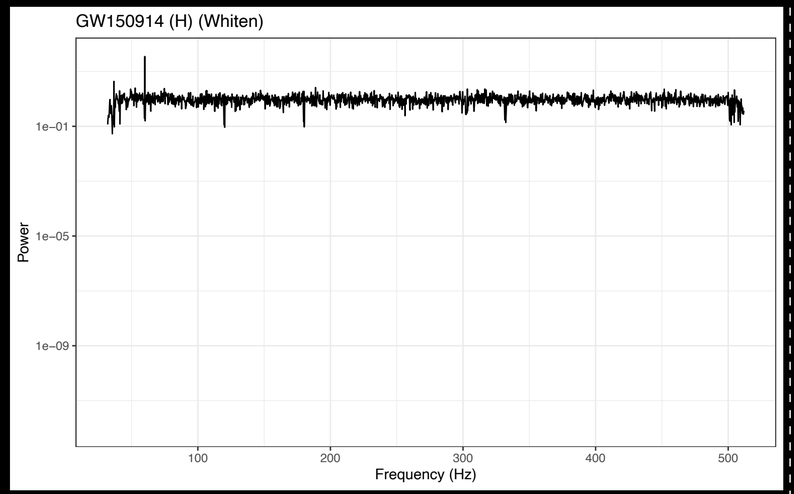
GW150914
(spectral whitening)

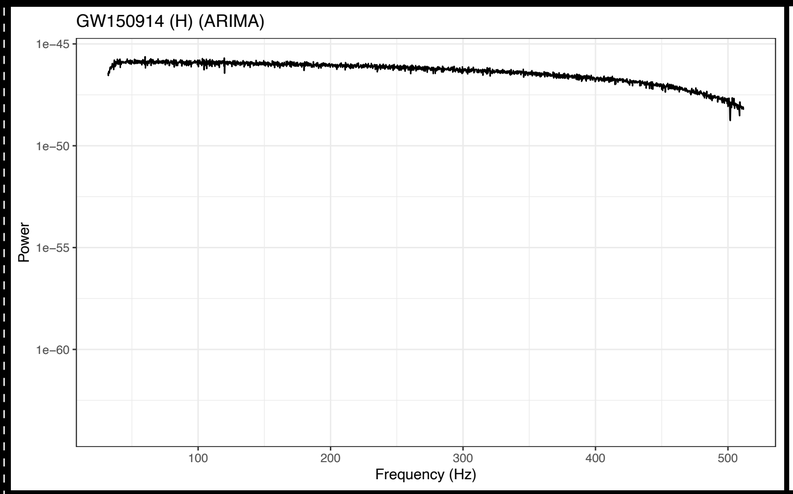
GW150914

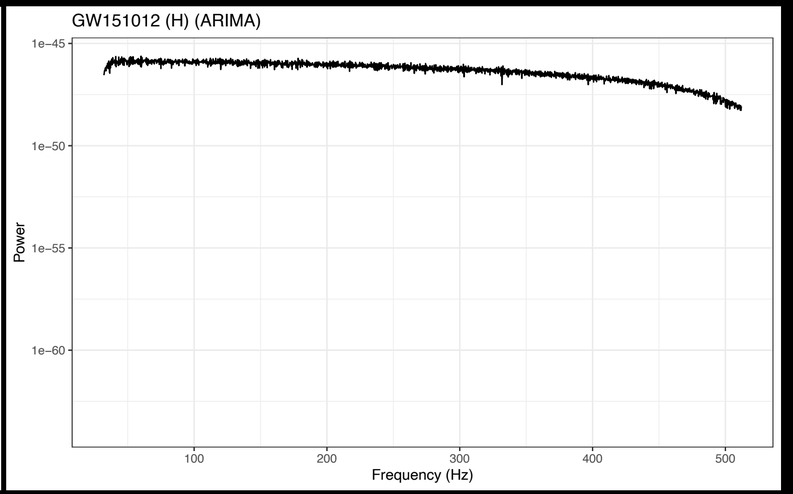
GW151012

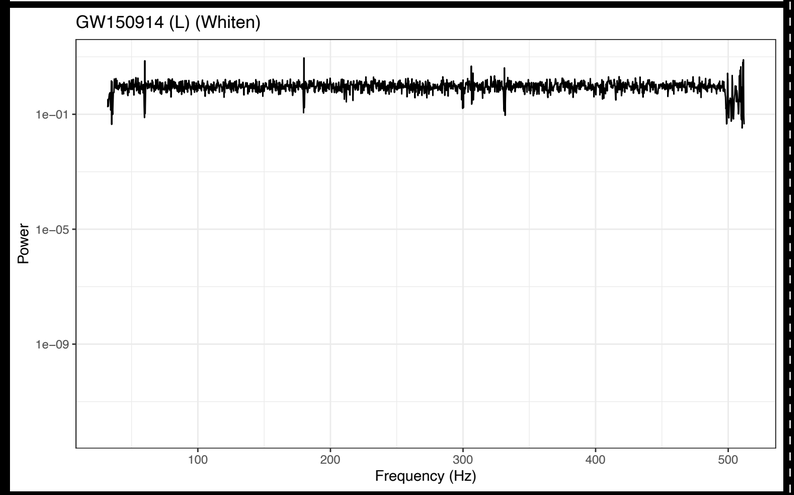
Hanford

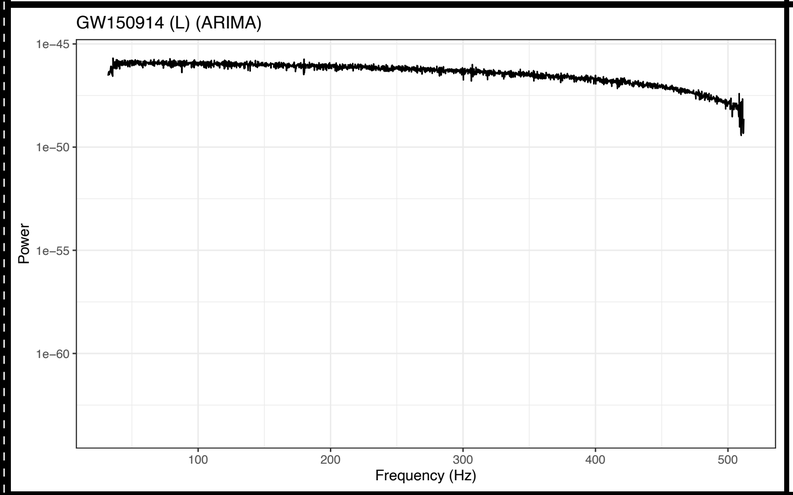
Livingston

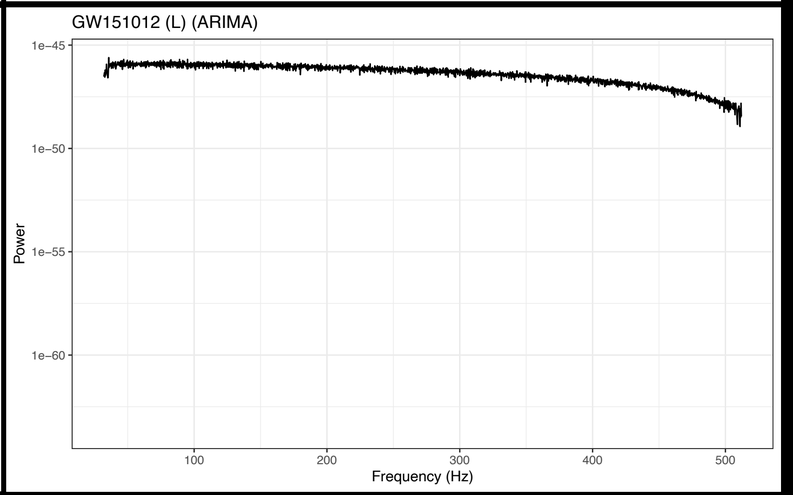








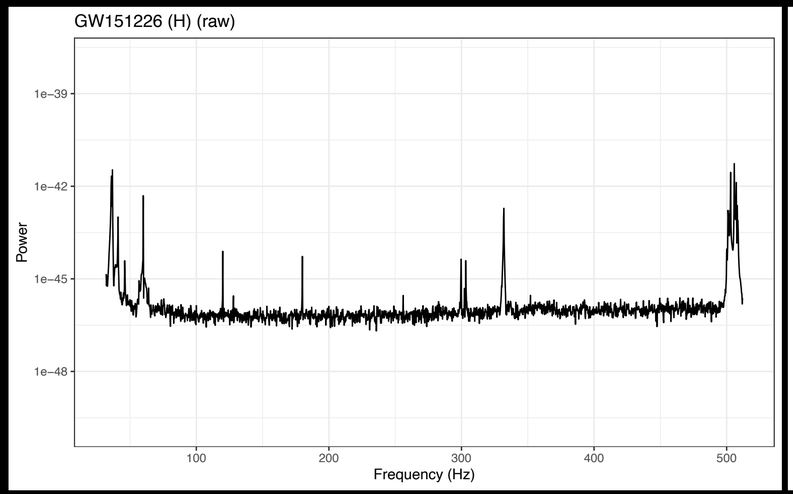


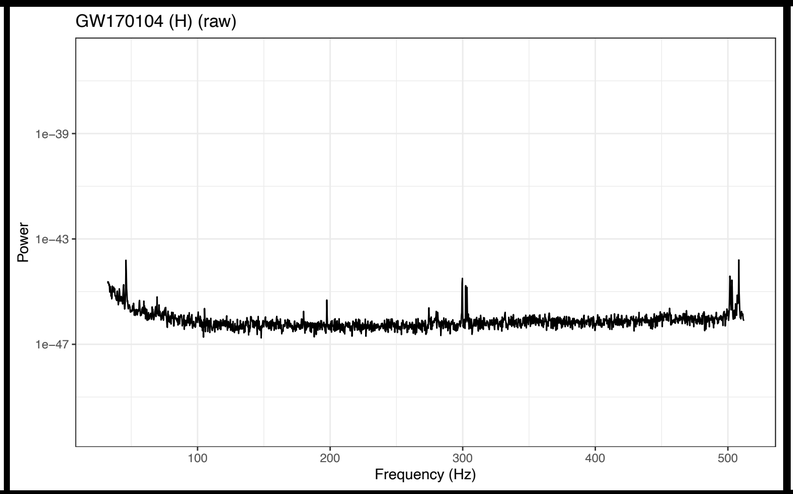


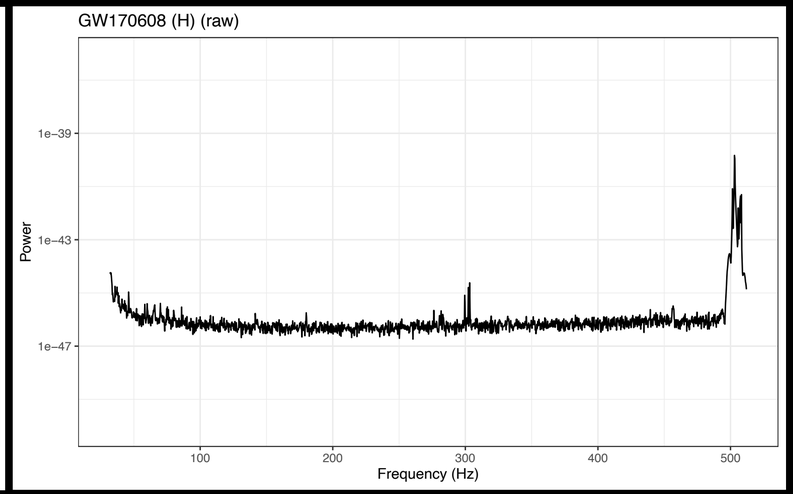
ARIMA

GW151226 GW170104 GW170608

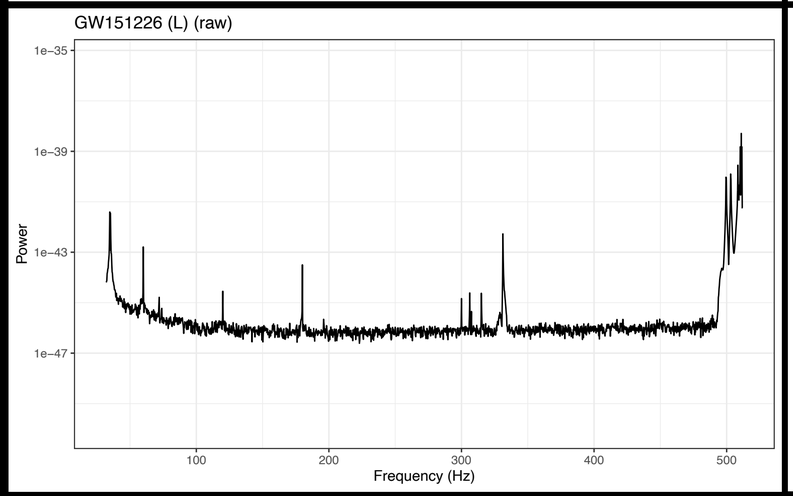
Hanford

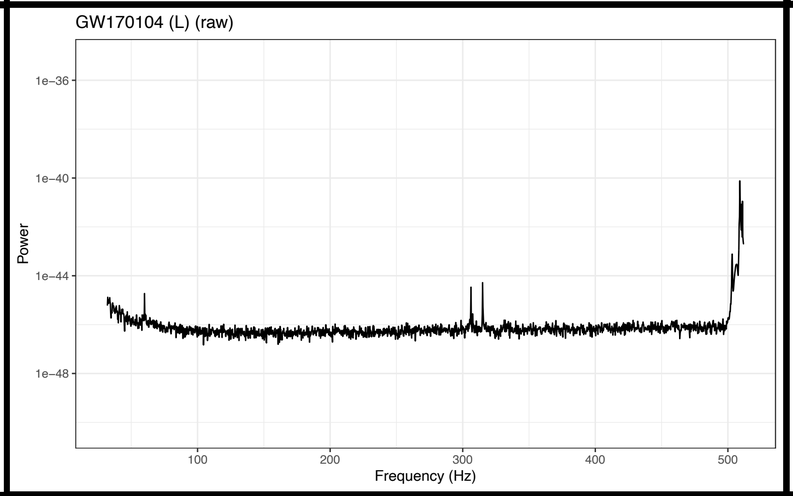


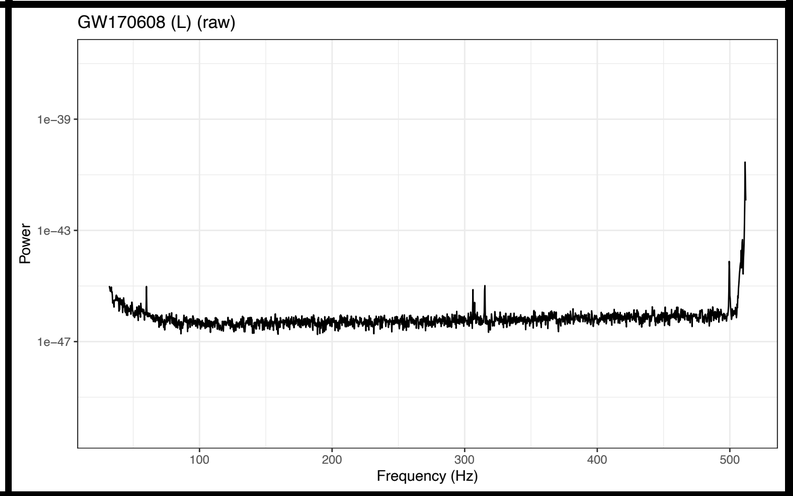




Livingston

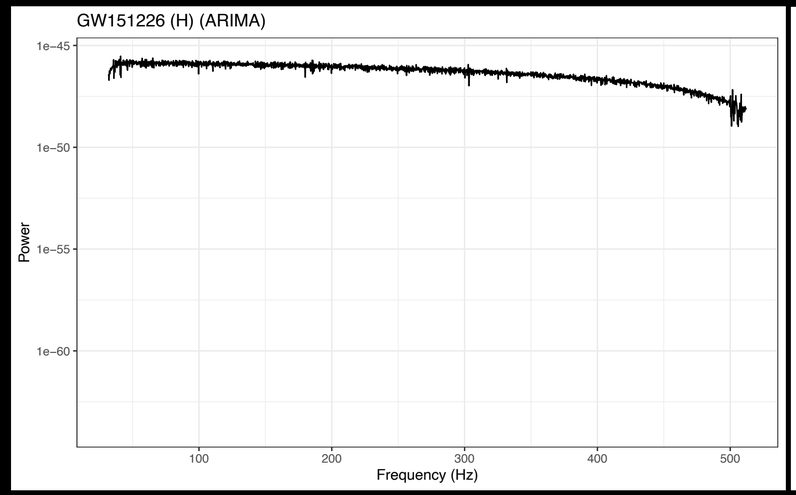


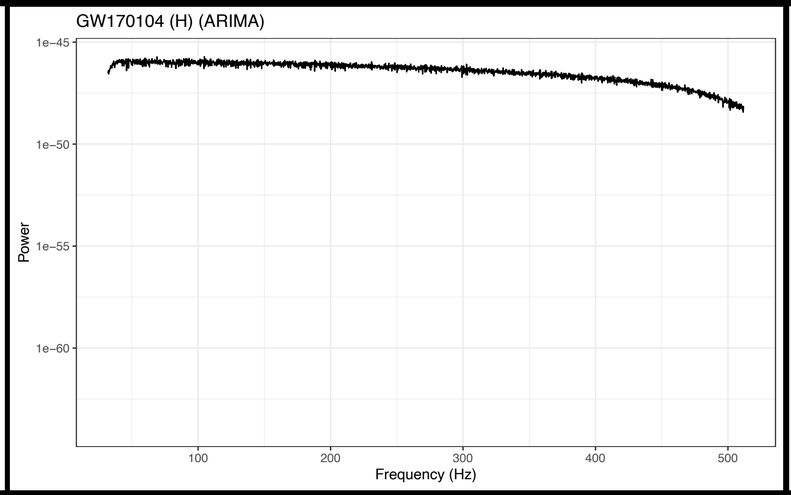


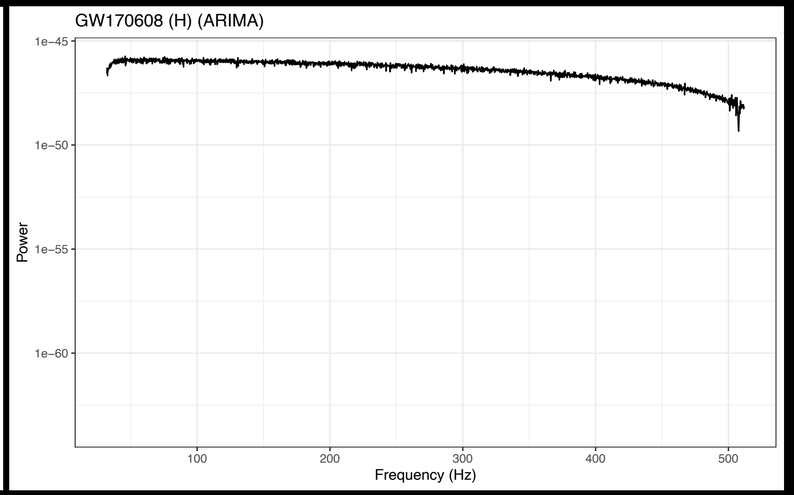


GW151226 GW170104 GW170608

Hanford







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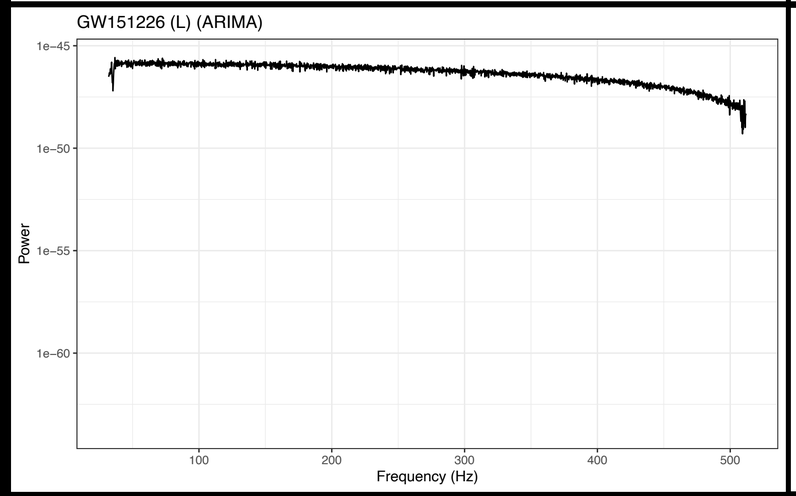
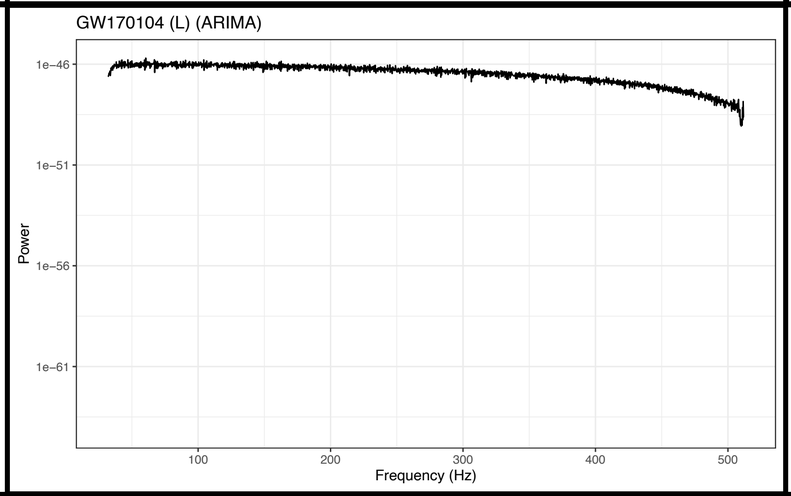
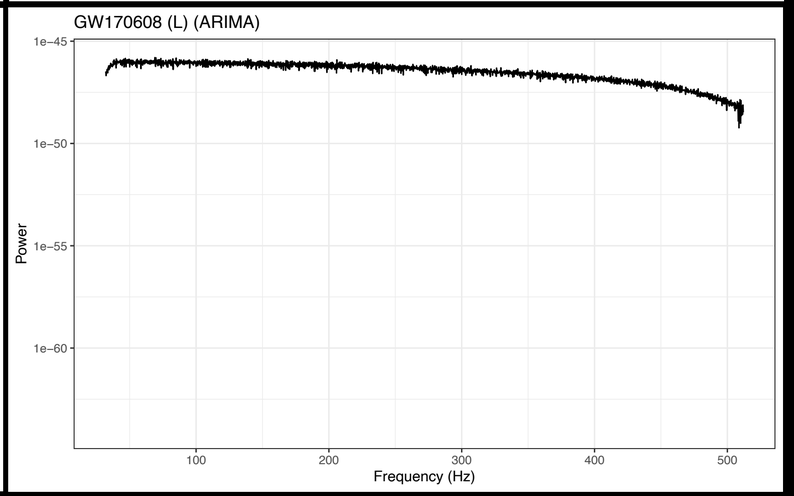
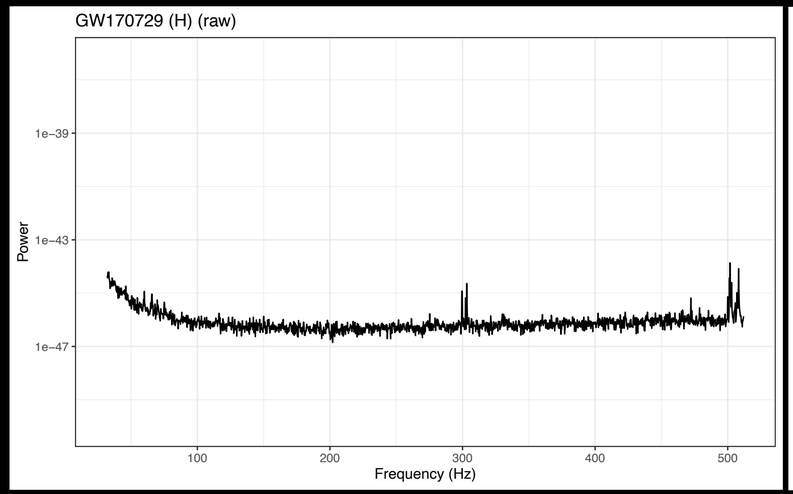


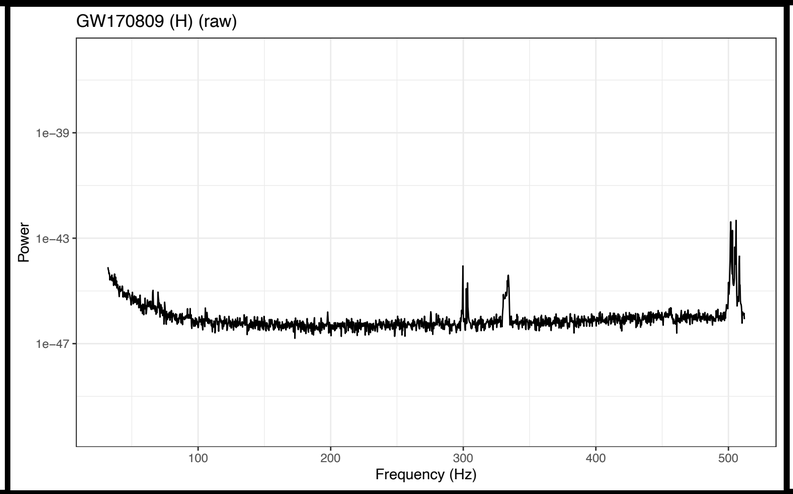


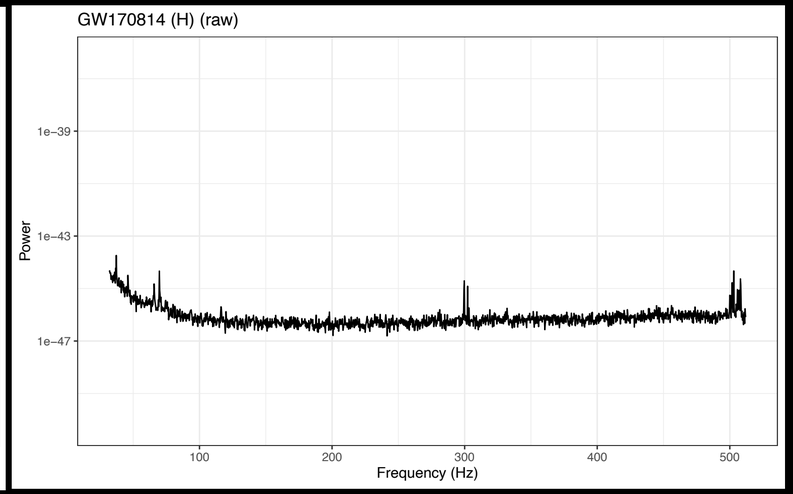


GW170729 GW170809 GW170814

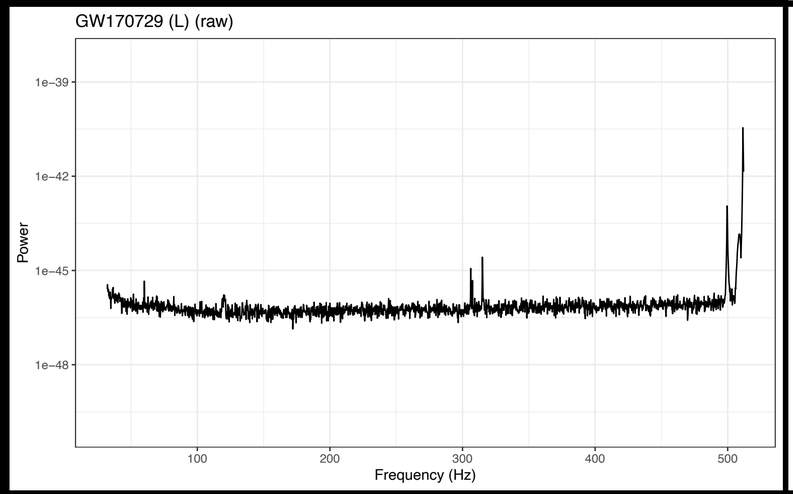
Hanford

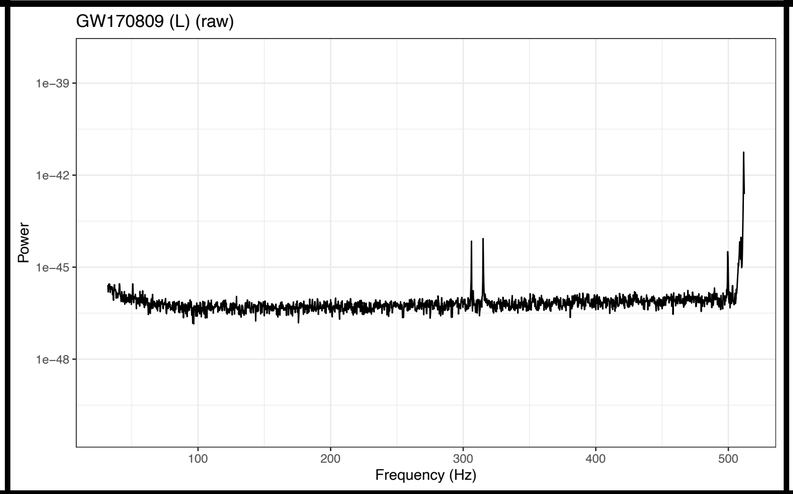


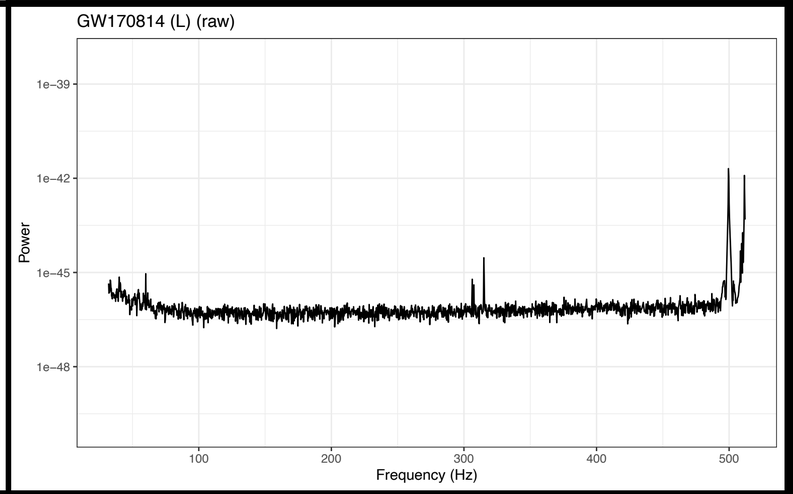




Livingston

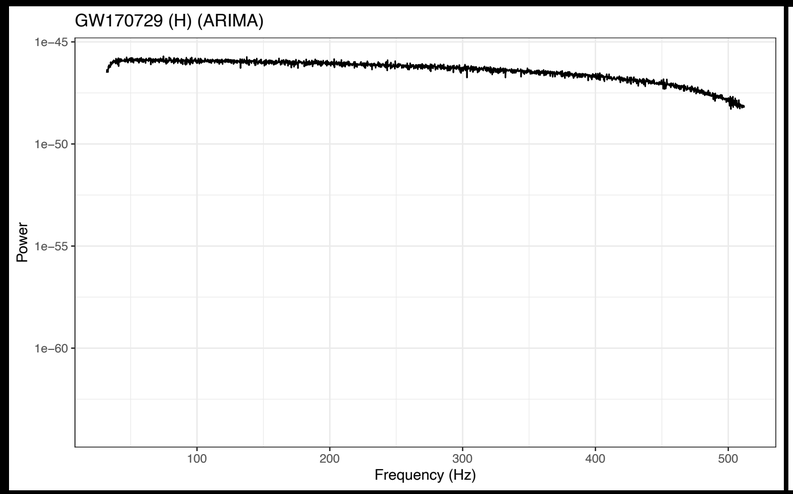


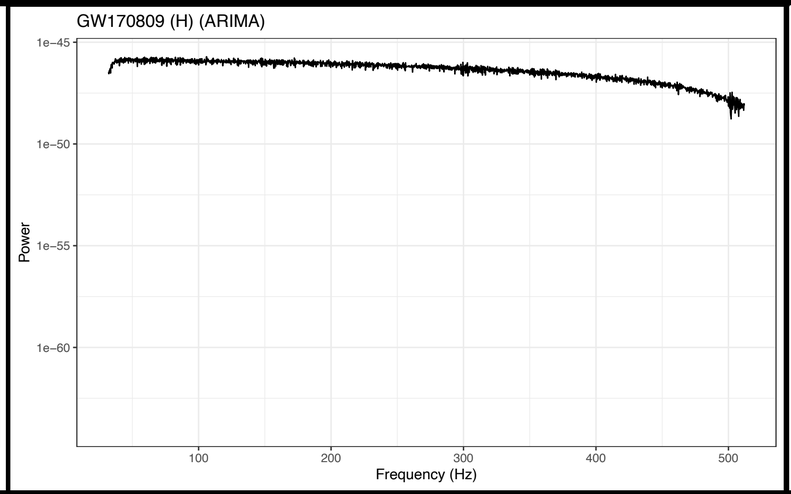


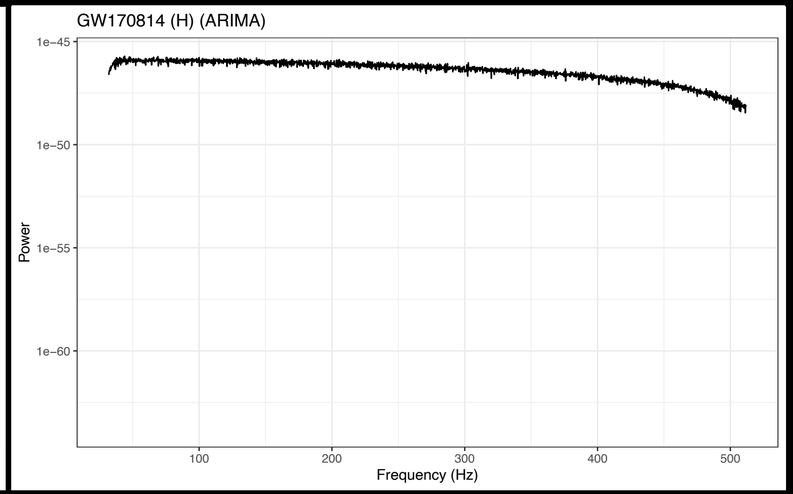


GW170729 GW170809 GW170814

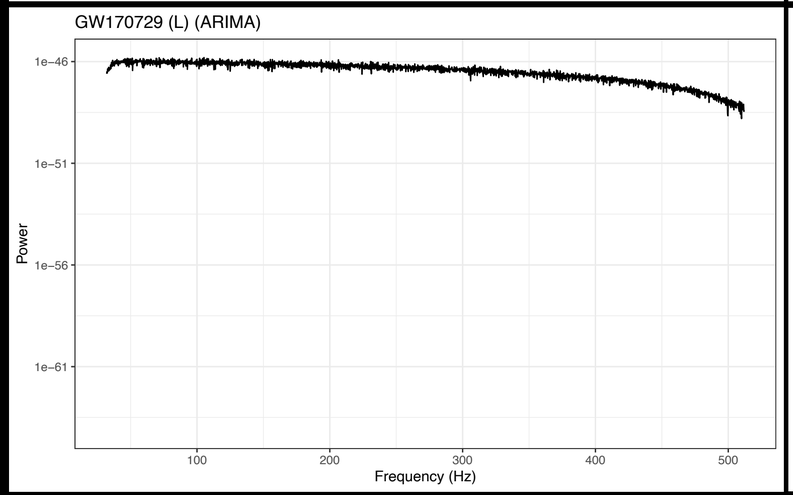
Hanford

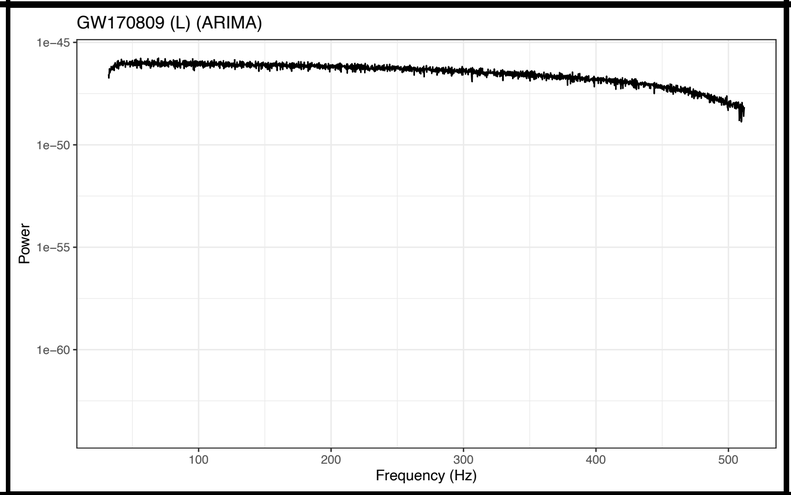


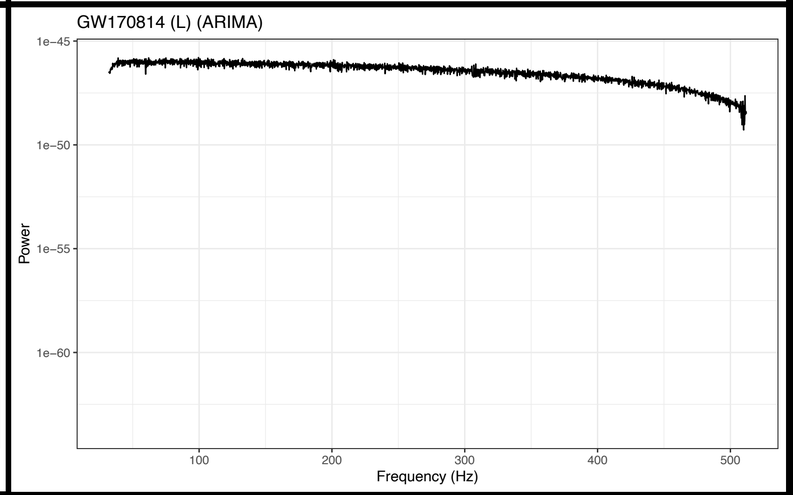




Livingston

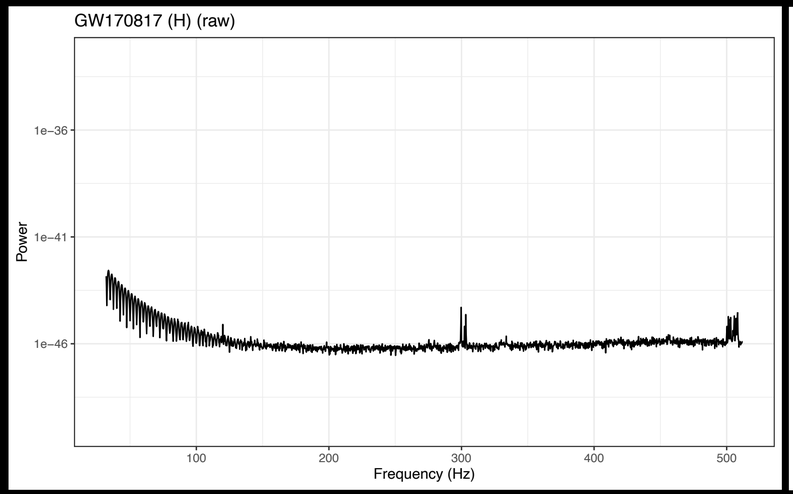


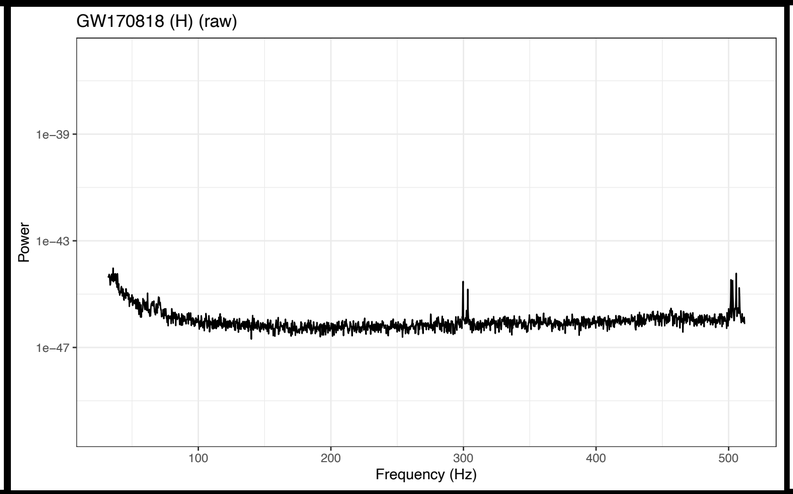


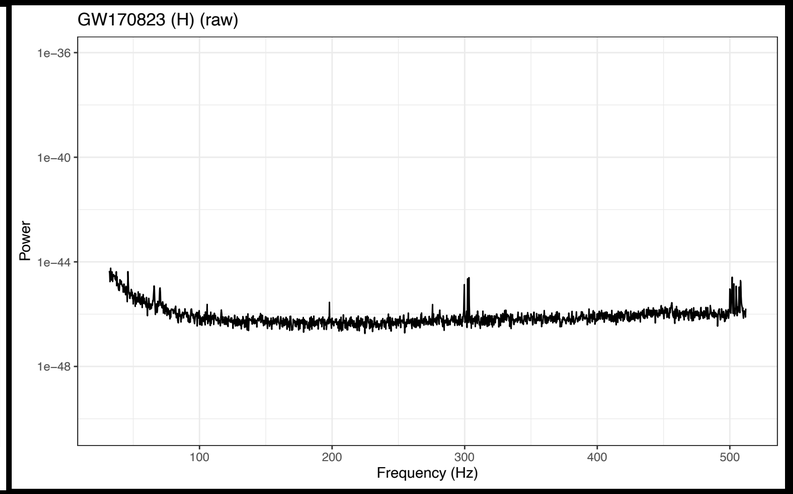


GW170817 GW170818 GW170823

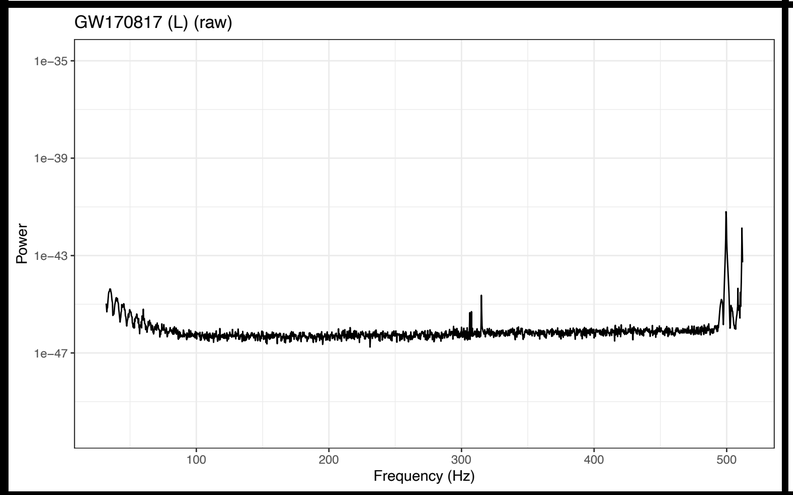
Hanford

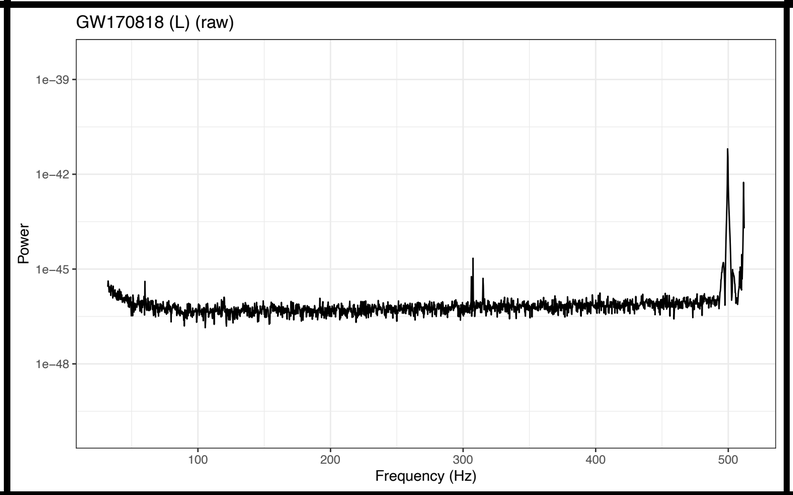


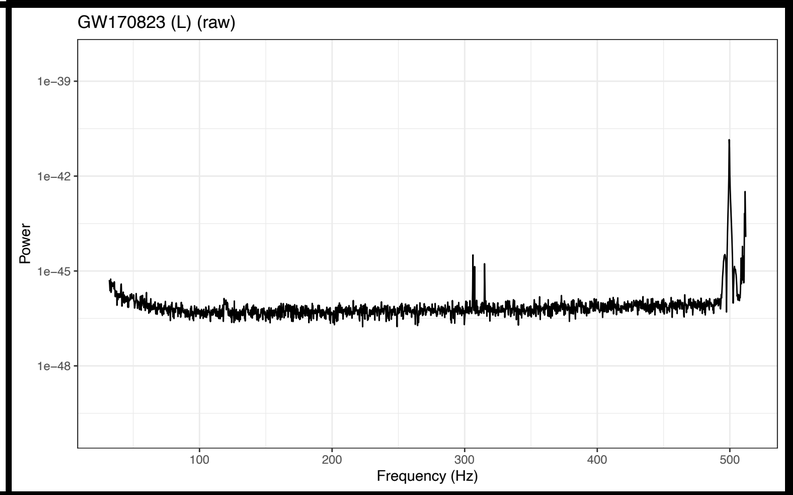




Livingston

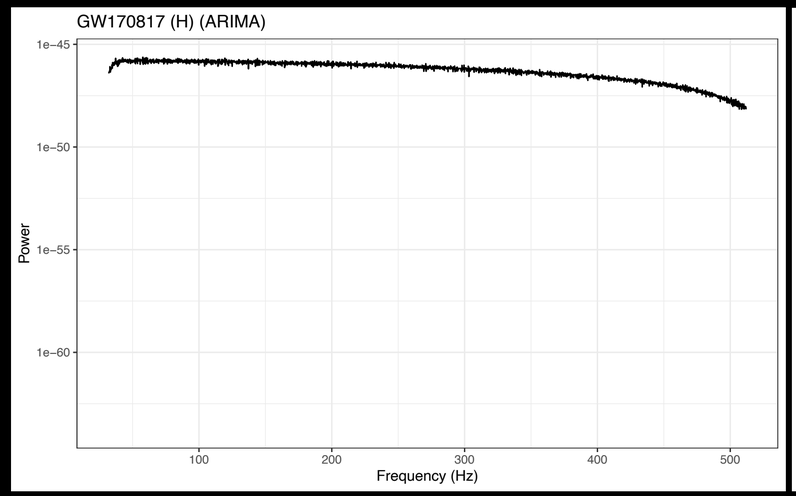


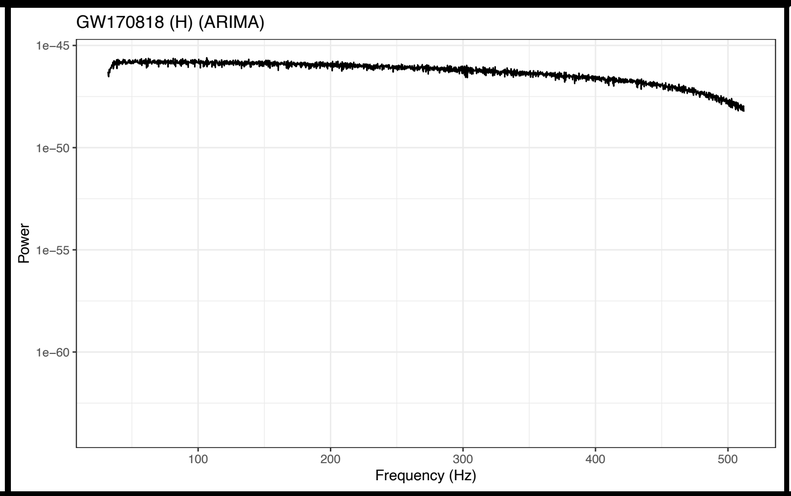


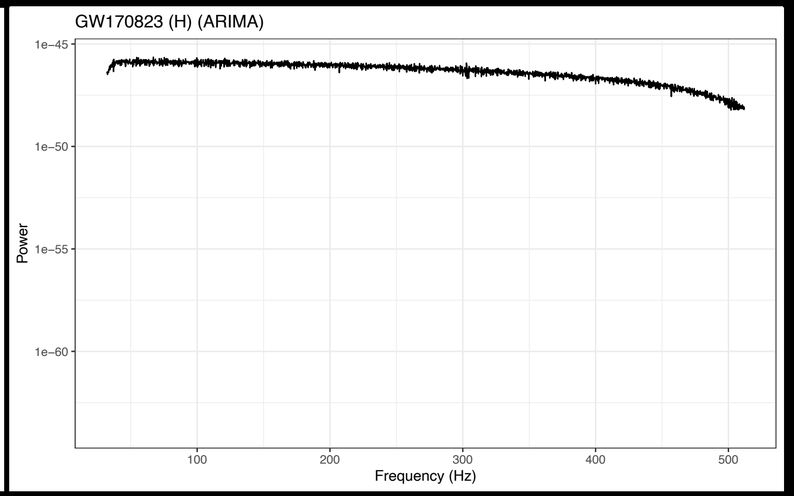


GW170817 GW170818 GW170823

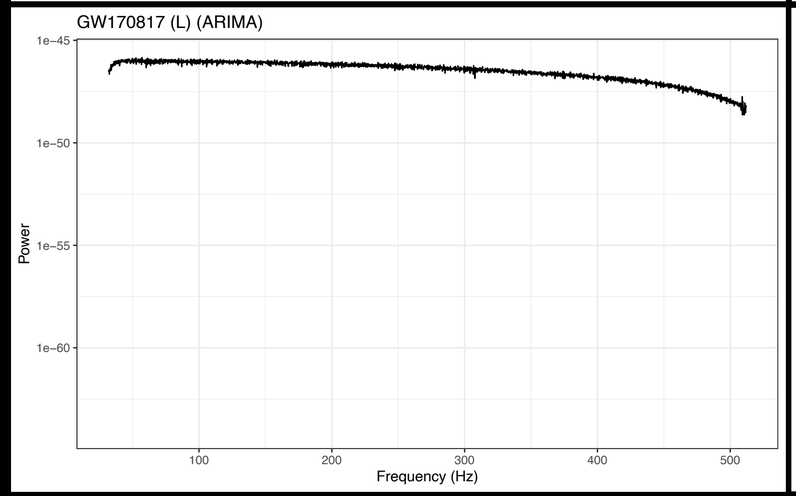
Hanford

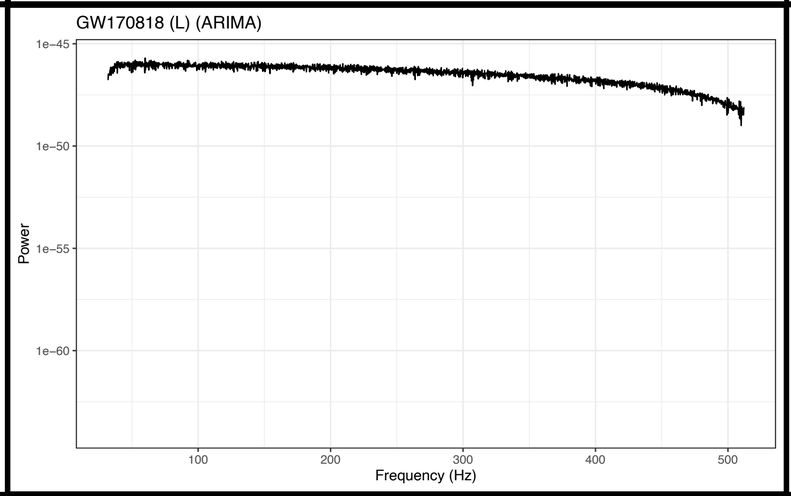


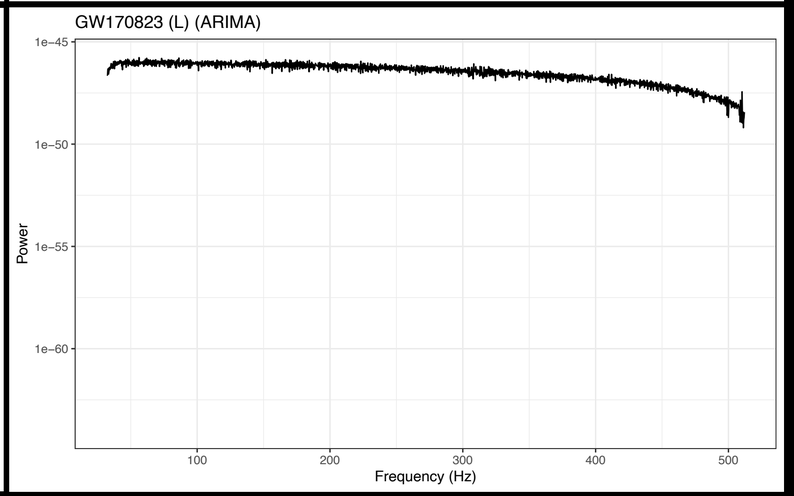




Livingston



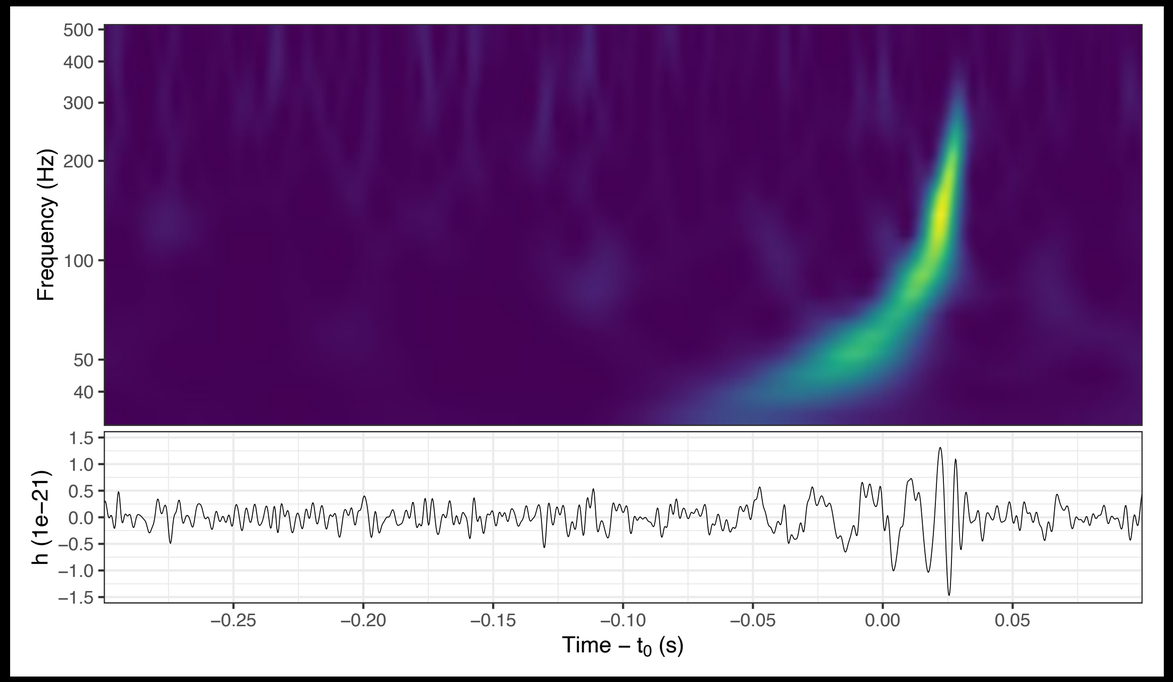




2) Spectro- & Oscillogram

GW150914

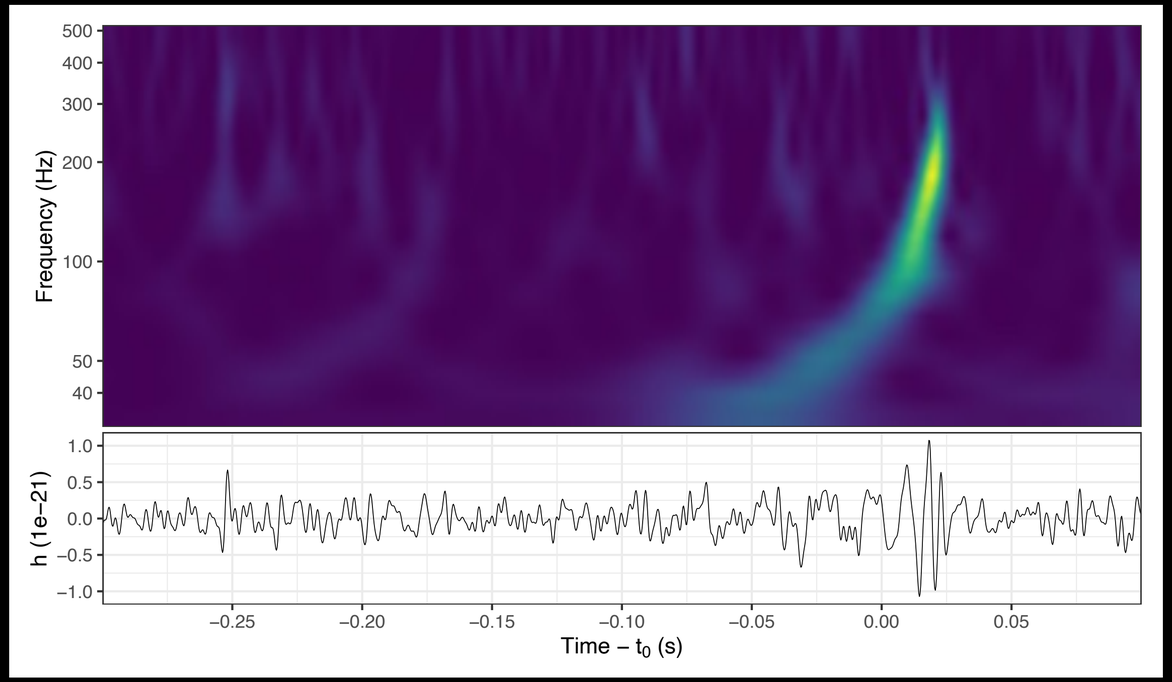
Hanford



$(p, q, d) = (7931, 7, 2)$

$(fl, fu) = (32, 512)$

Livingston

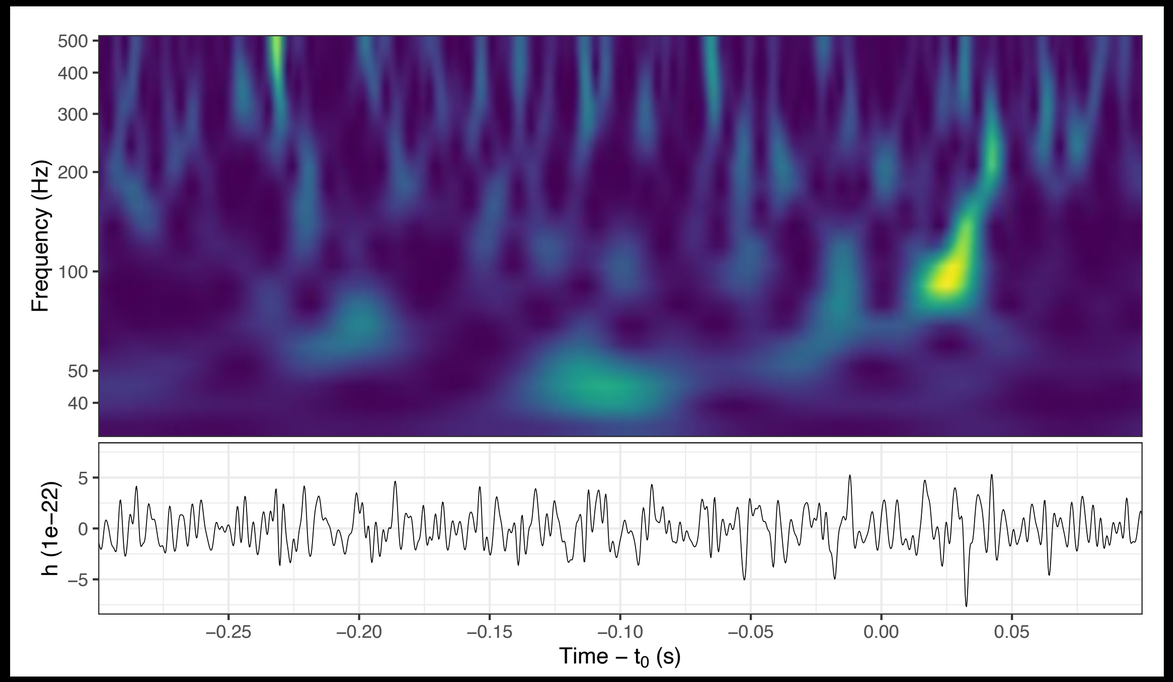


$(p, q, d) = (5354, 7, 2)$

$(fl, fu) = (32, 512)$

GW151012

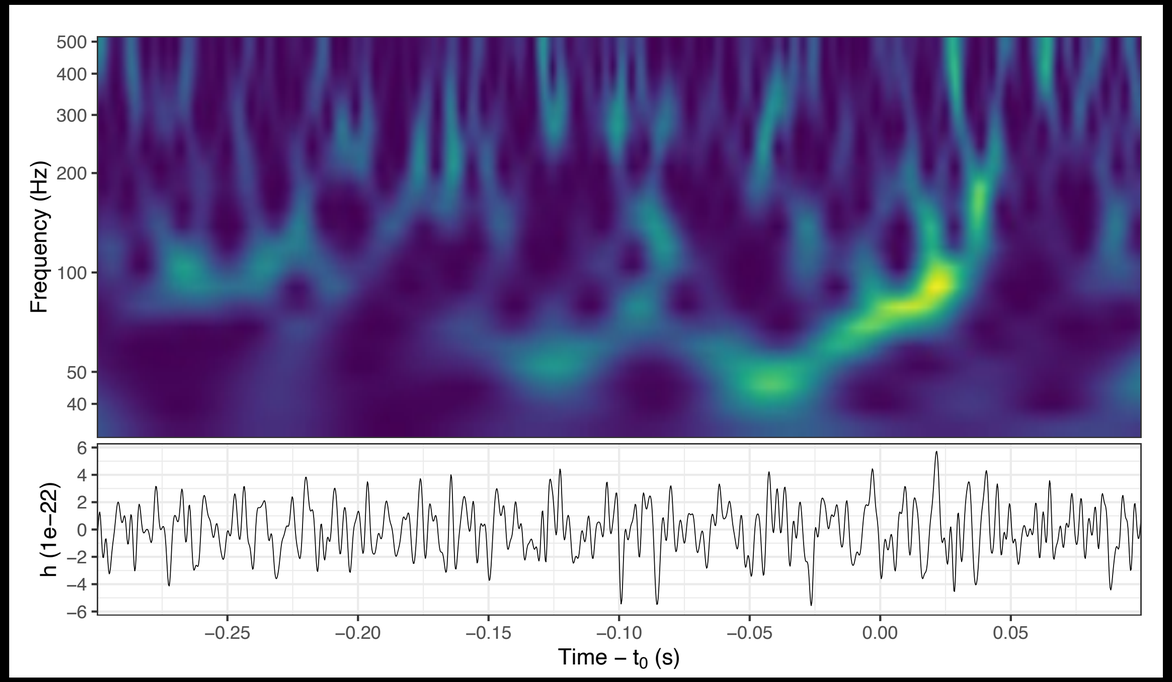
Hanford



$(p, q, d) = (5519, 7, 2)$

$(fl, fu) = (32, 512)$

Livingston



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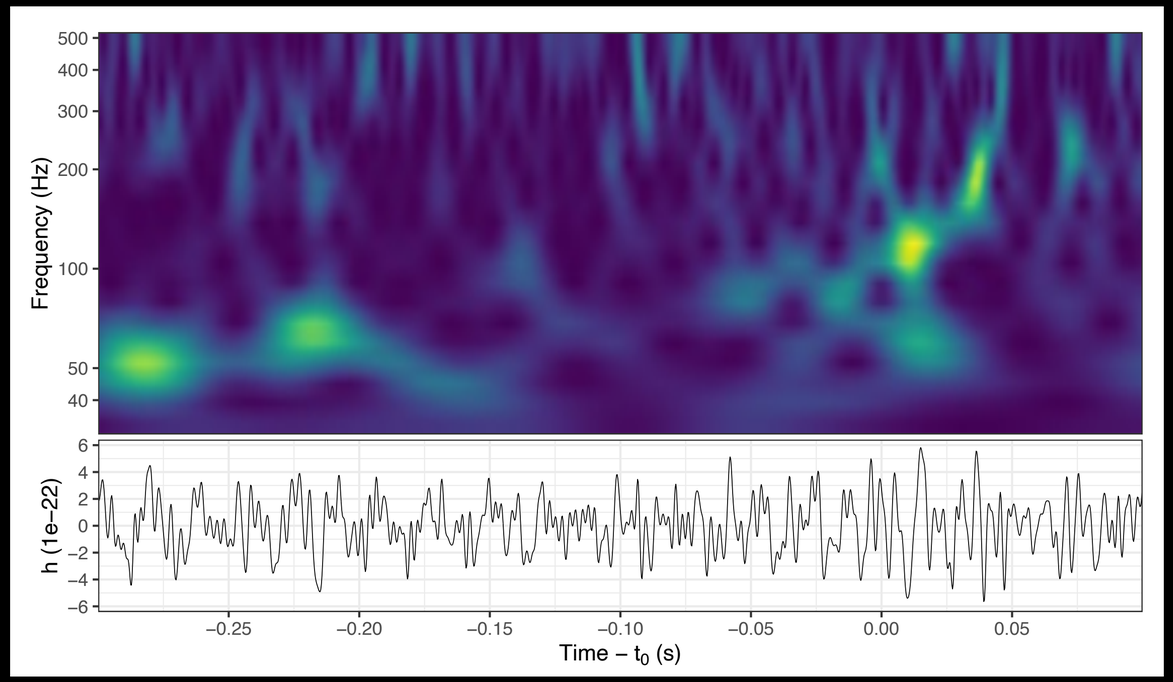
$(fl, fu) = (32, 512)$

GW151226

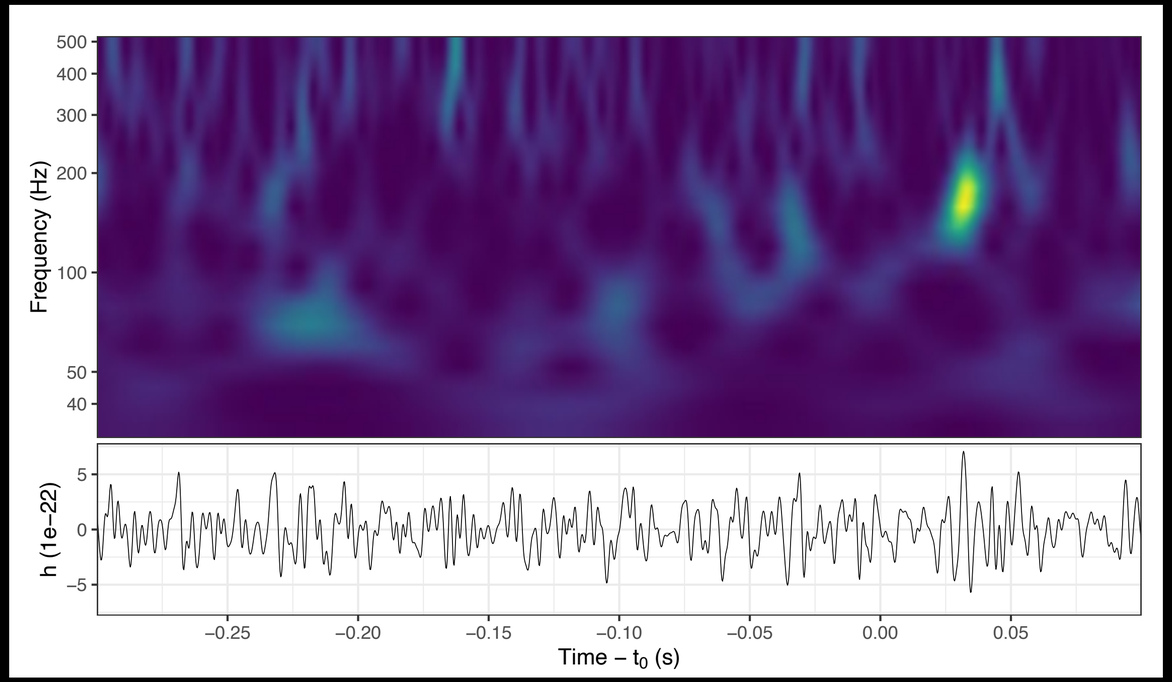
Hanford



$(p, q, d) = (5822, 7, 2)$

$(fl, fu) = (32, 512)$

Livingston

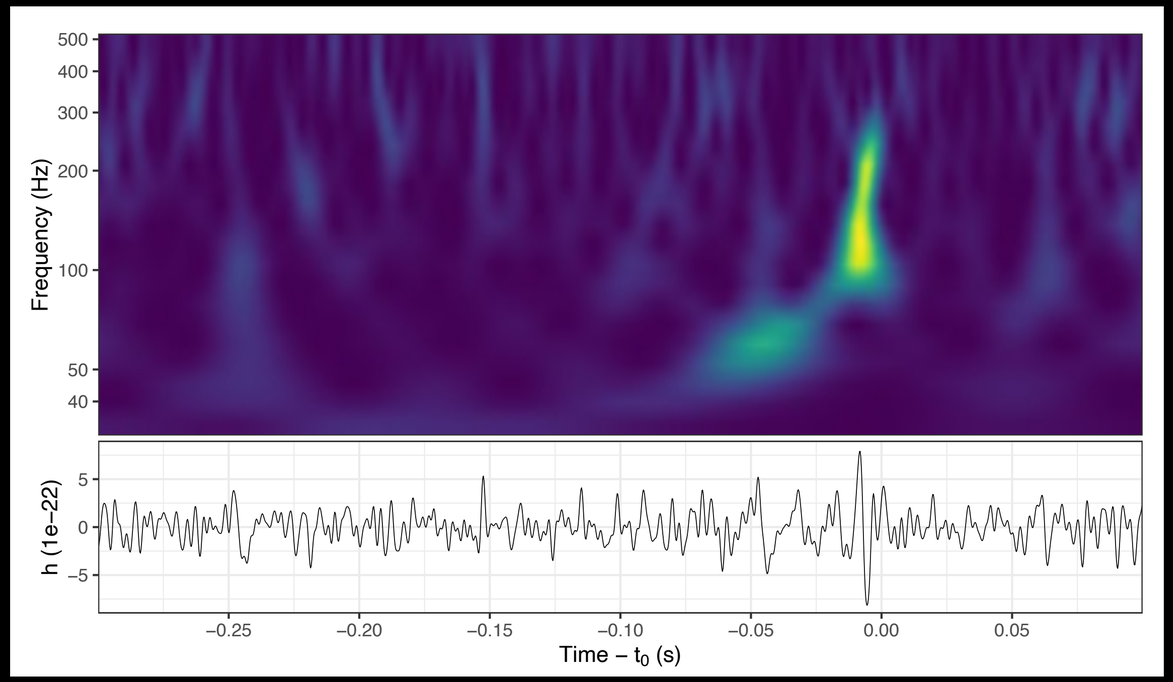


$(p, q, d) = (5213, 7, 2)$

$(fl, fu) = (32, 512)$

GW170104

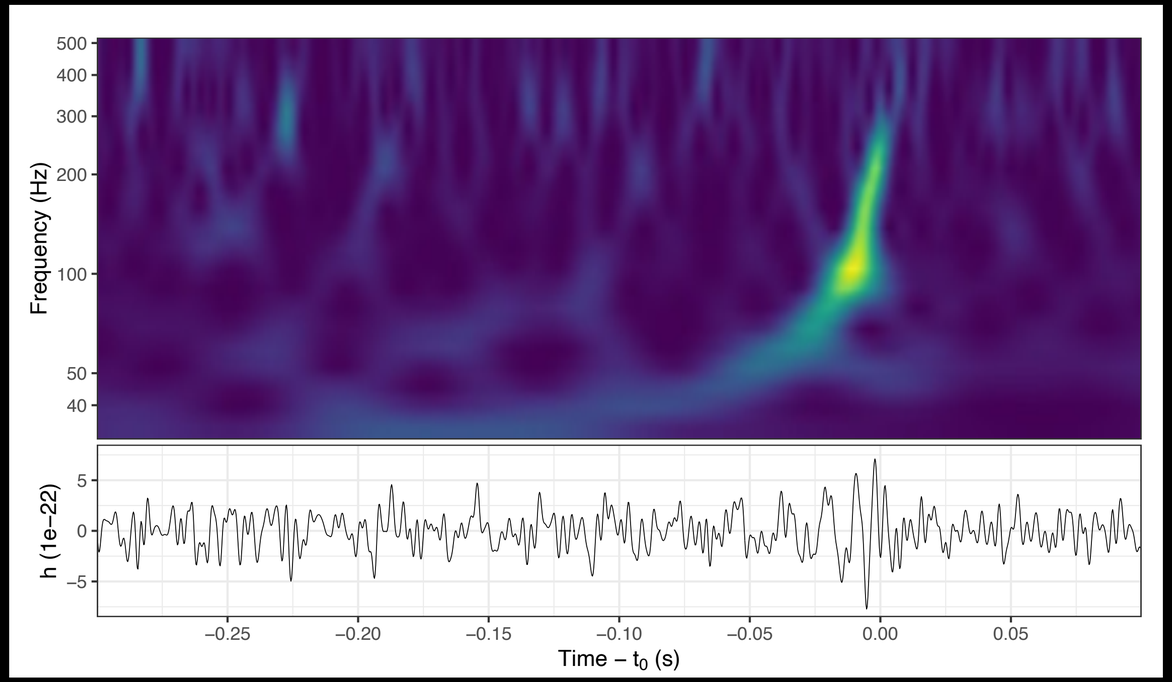
Hanford



$(p, q, d) = (5963, 7, 2)$

$(fl, fu) = (32, 512)$

Livingston



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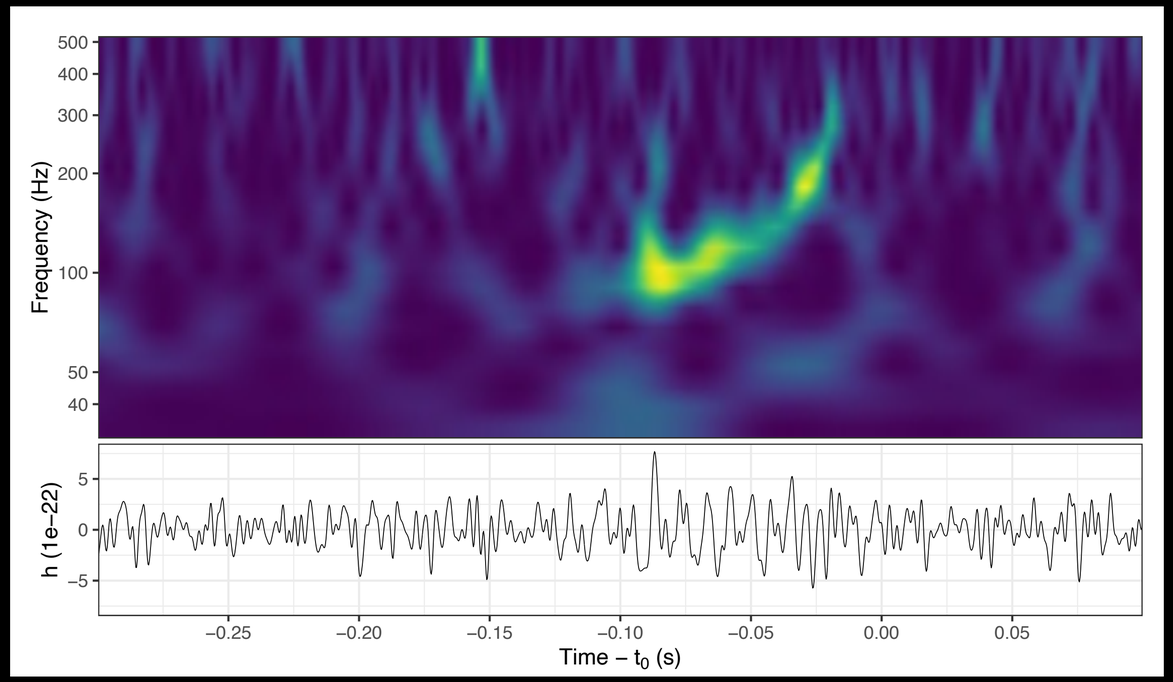
GW170608

Hanford



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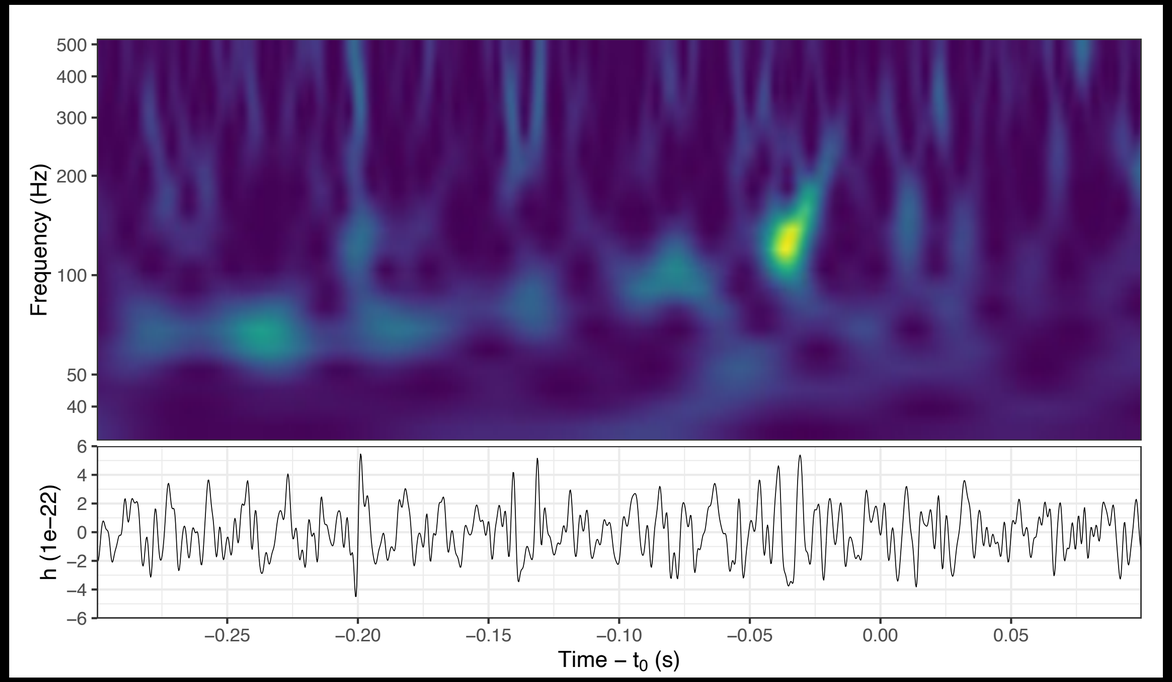
Livingston



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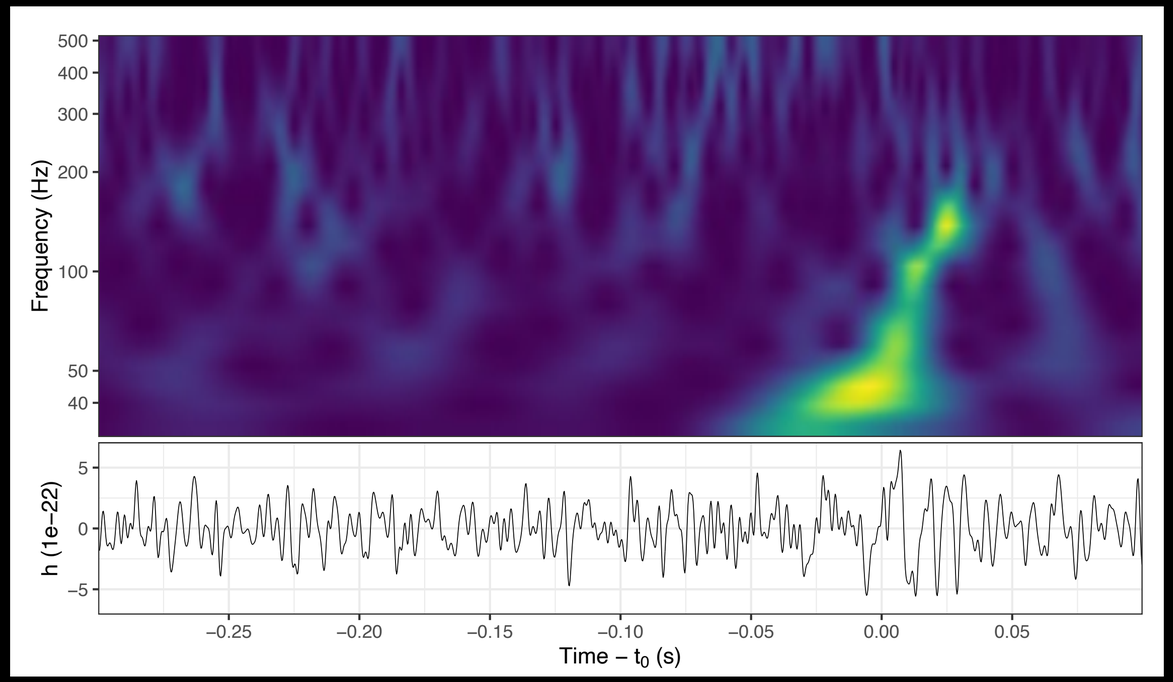
$(fl, fu) = (32, 512)$

GW170729

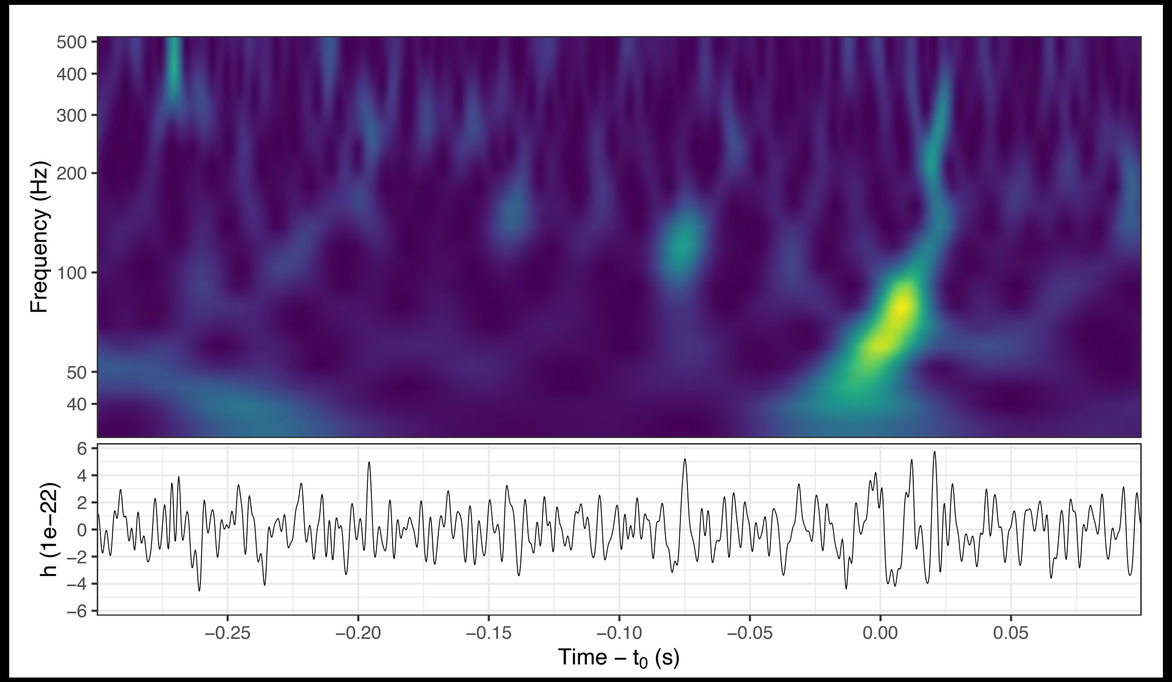
Hanford



$(p, q, d) = (7318, 7, 2)$

$(fl, fu) = (32, 512)$

Livingston

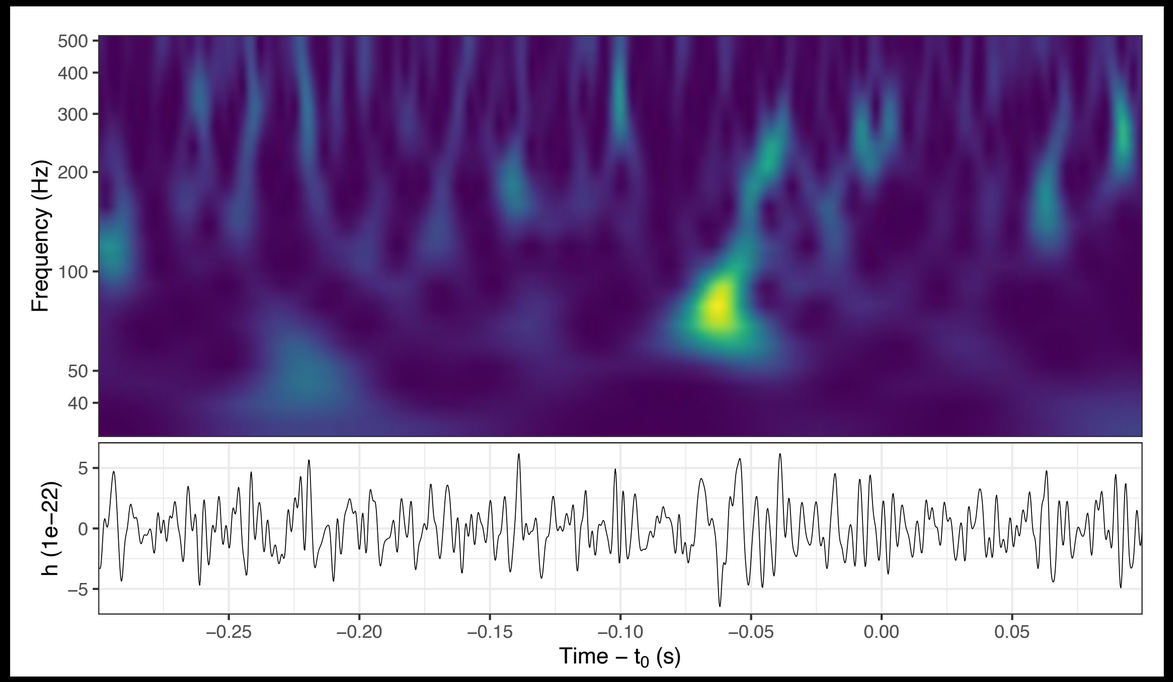


$(p, q, d) = (3501, 7, 2)$

$(fl, fu) = (32, 512)$

GW170809

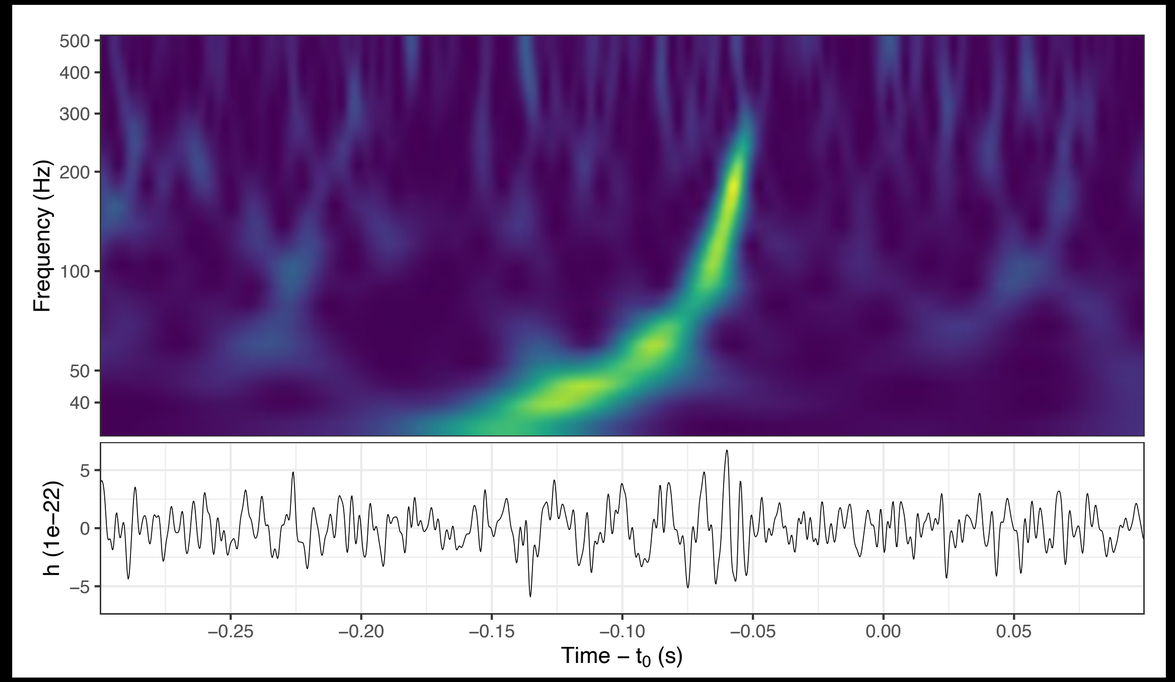
Hanford



$(p, q, d) = (5446, 7, 2)$

$(fl, fu) = (32, 512)$

Livingston

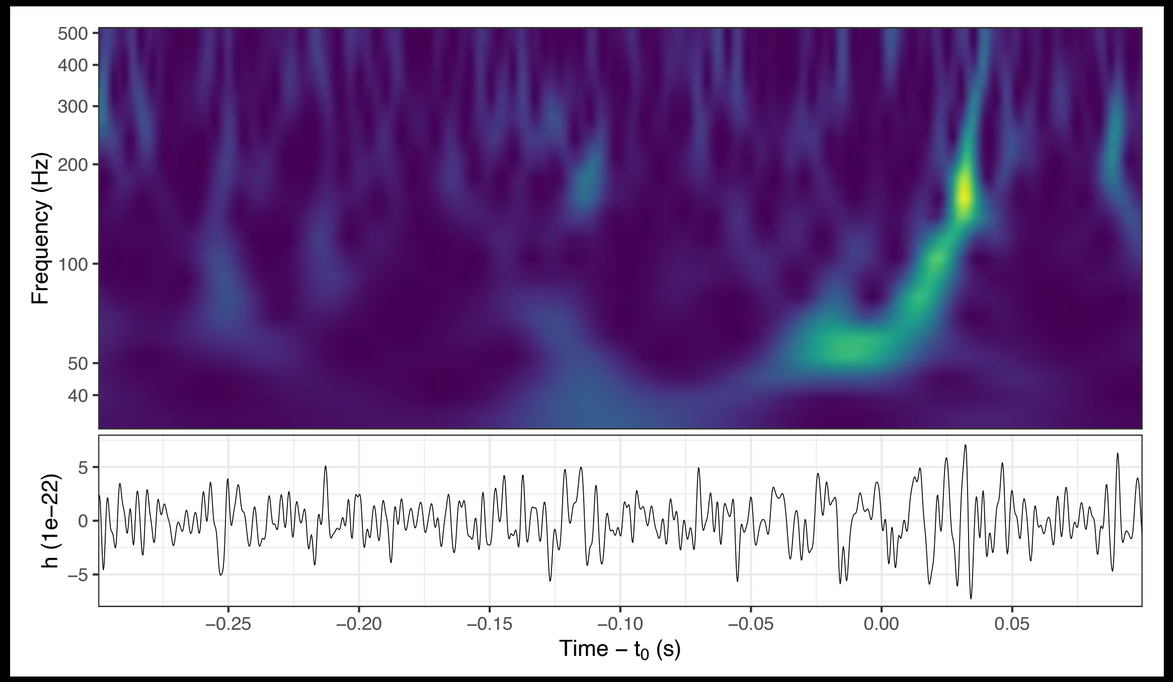


$(p, q, d) = (3512, 7, 2)$

$(fl, fu) = (32, 512)$

GW170814

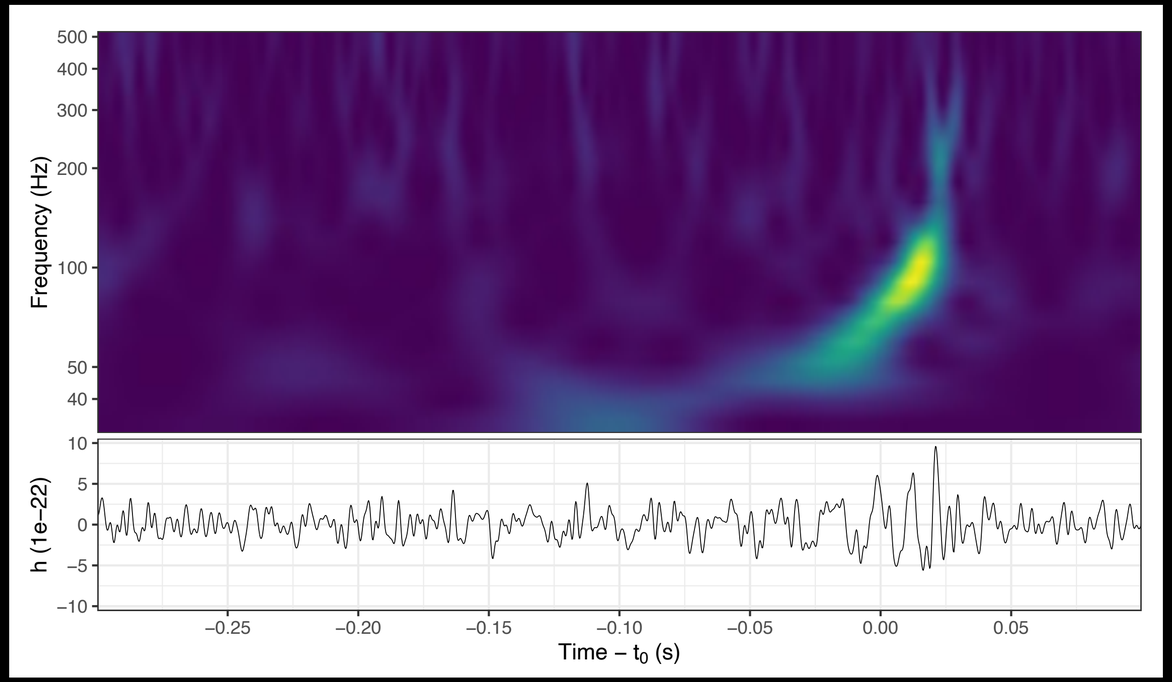
Hanford



$(p, q, d) = (5553, 7, 2)$

$(fl, fu) = (32, 512)$

Livingston

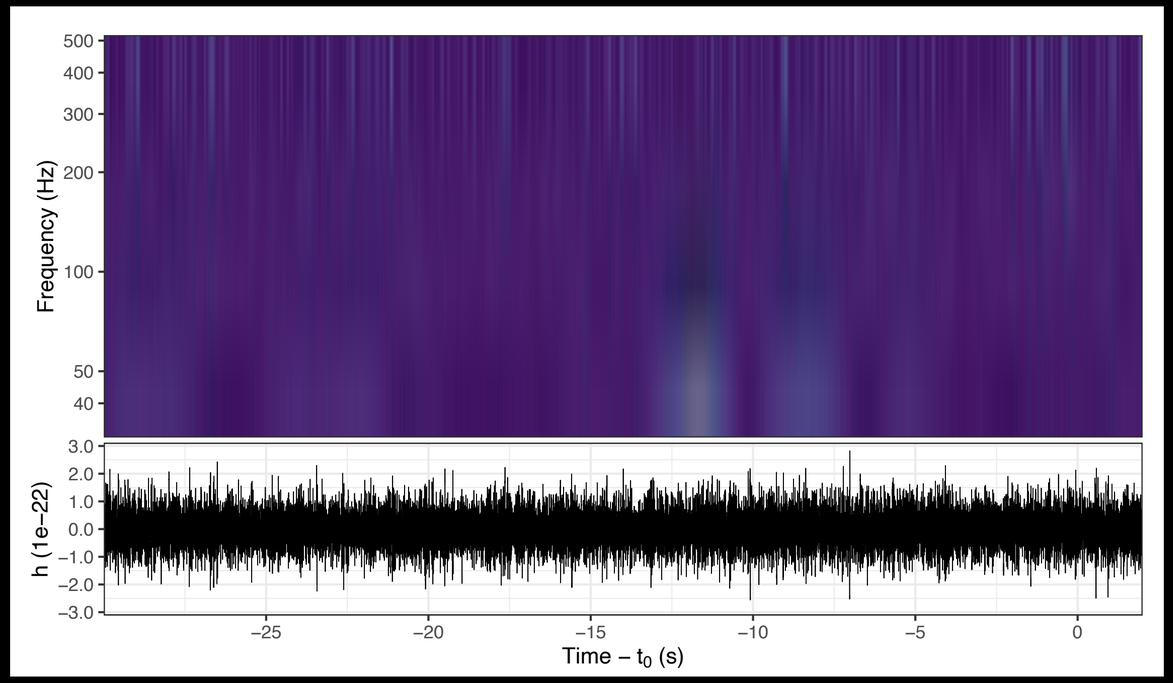


$(p, q, d) = (3492, 7, 2)$

$(fl, fu) = (32, 512)$

GW170817

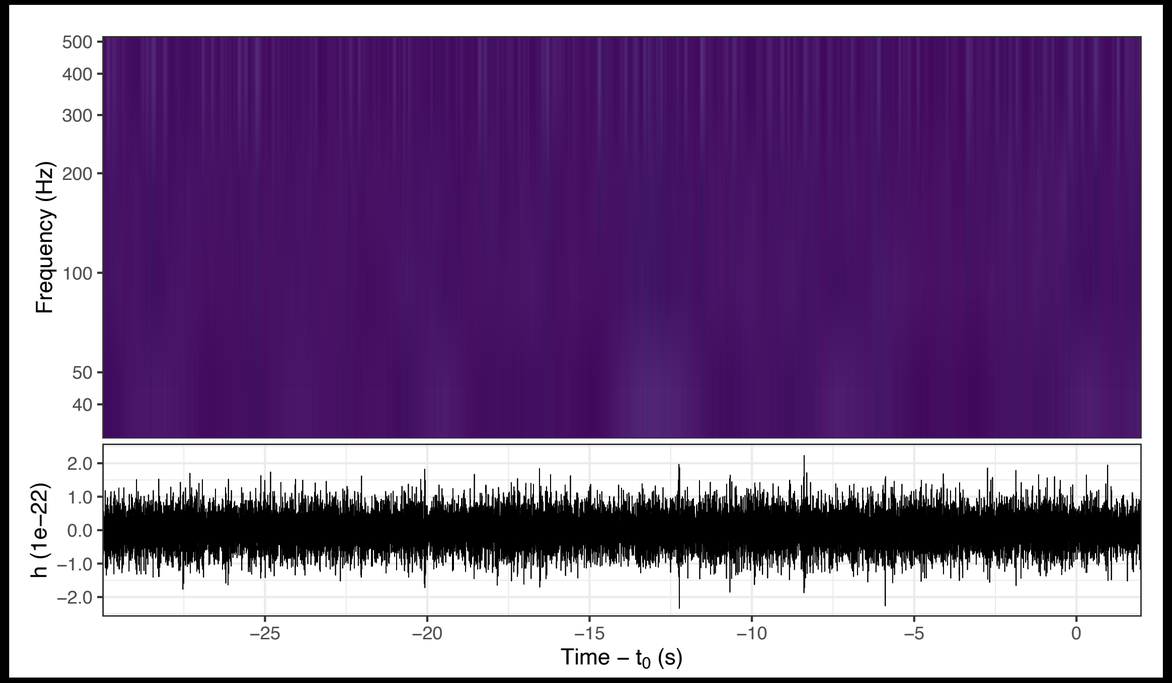
Hanford



$(p, q, d) = (5791, \mathbf{37}, 2)$

$(fl, fu) = (32, 512)$

Livingston

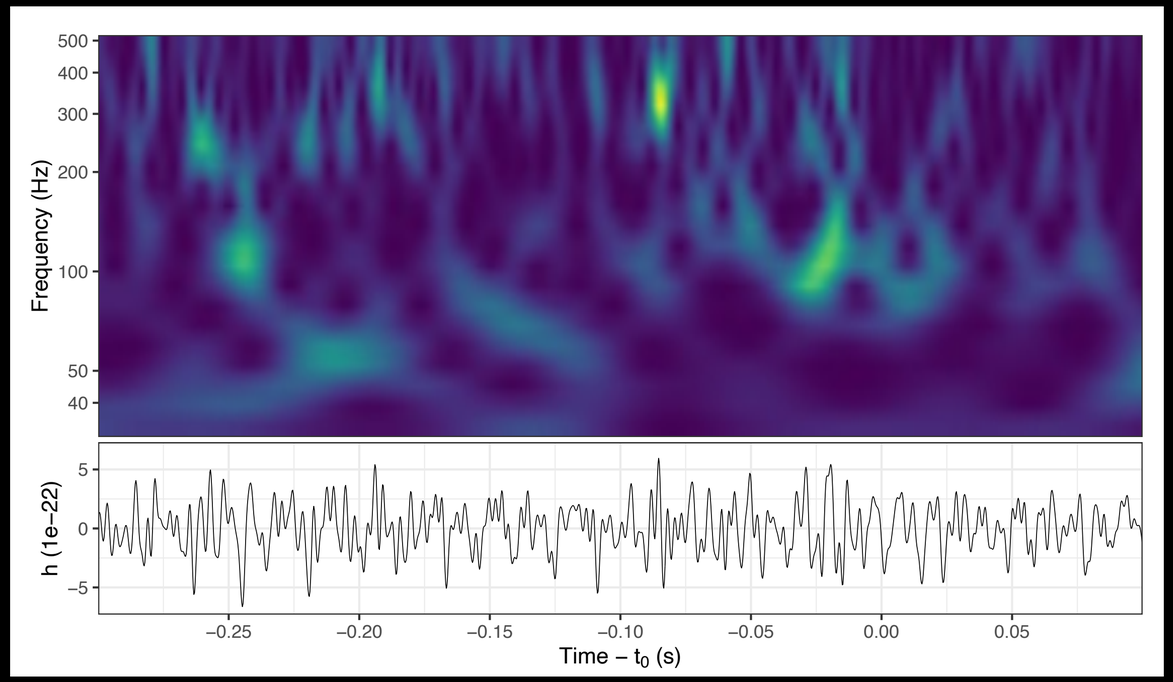


$(p, q, d) = (4841, \mathbf{37}, 2)$

$(fl, fu) = (32, 512)$

GW170818

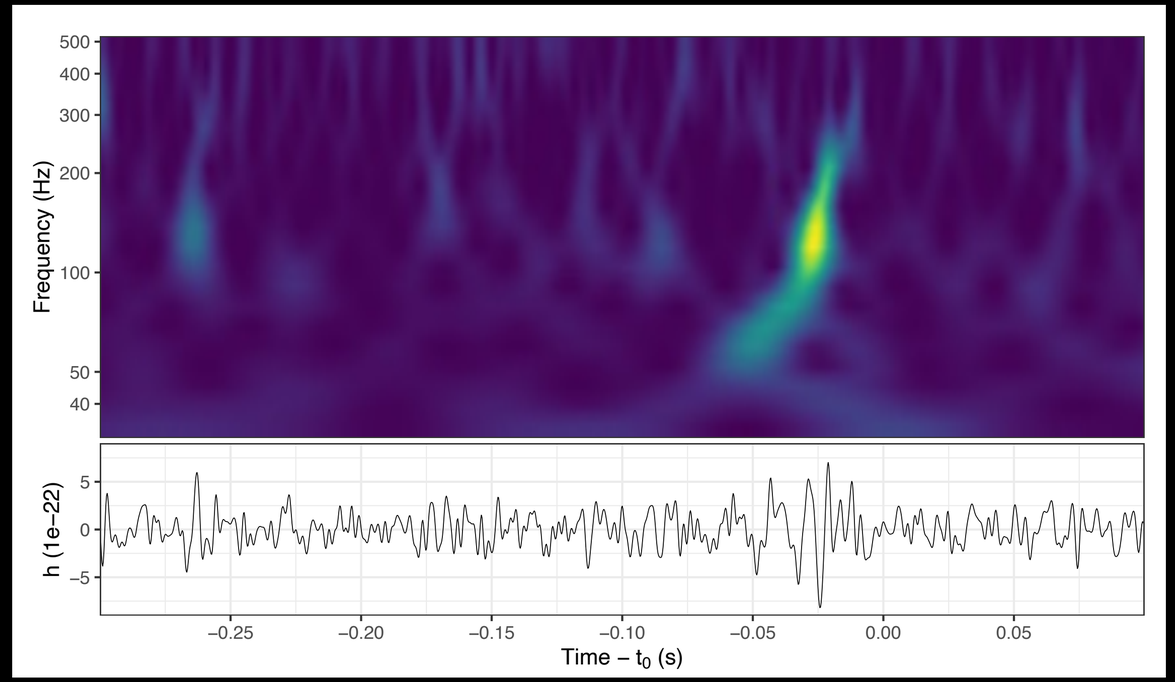
Hanford



$(p, q, d) = (5504, 7, 2)$

$(fl, fu) = (32, 512)$

Livingston

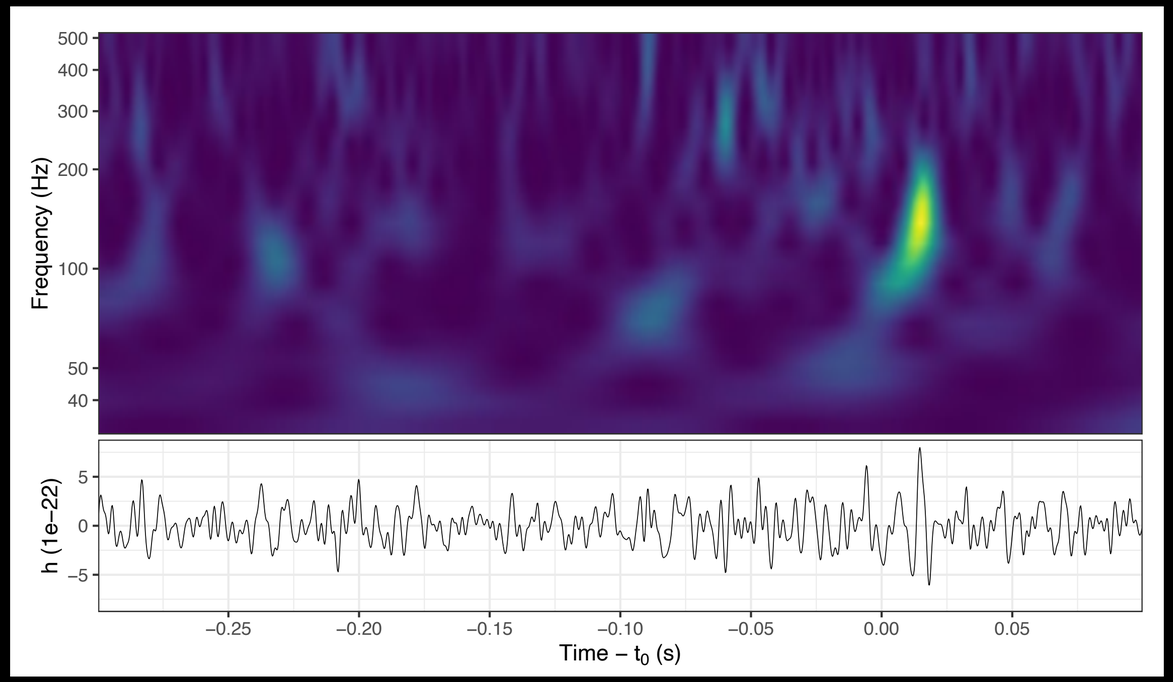


$(p, q, d) = (3467, 7, 2)$

$(fl, fu) = (32, 512)$

GW170823

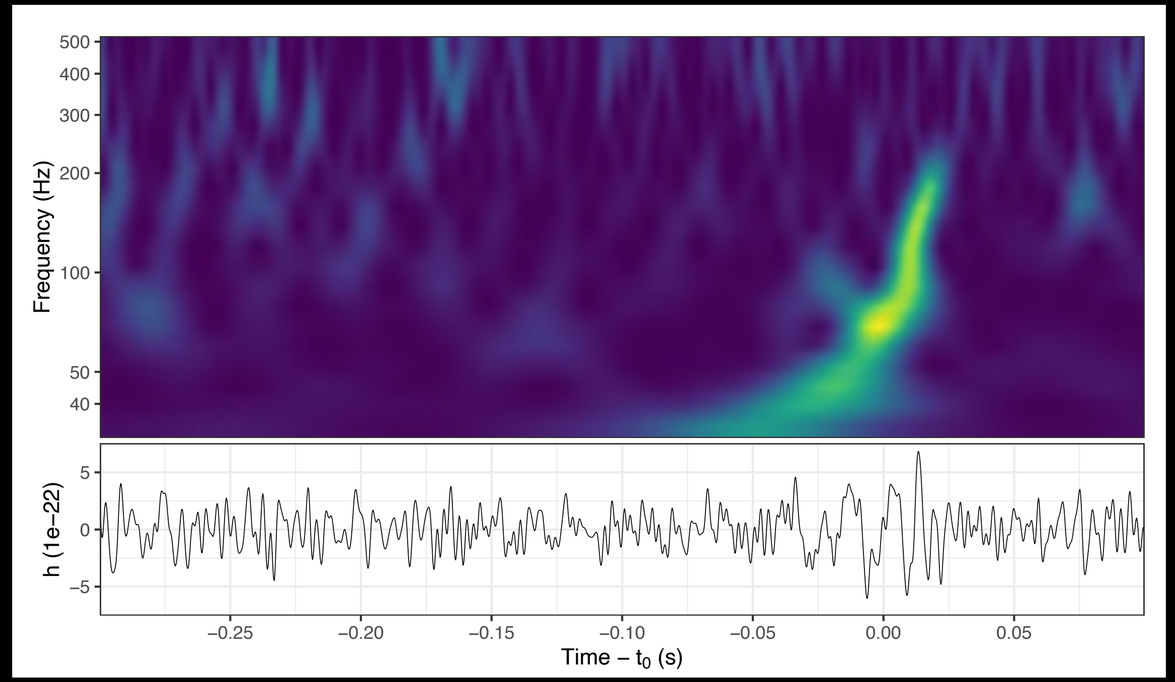
Hanford



$(p, q, d) = (4893, 7, 2)$

$(fl, fu) = (32, 512)$

Livingston



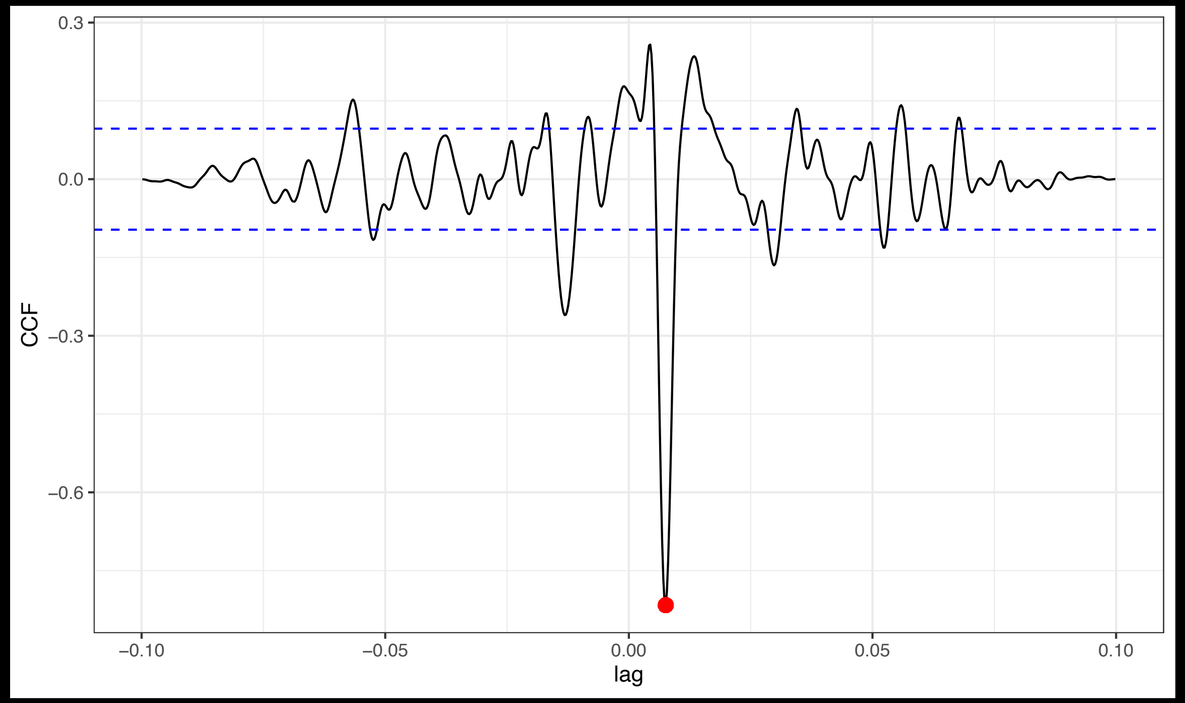
$(p, q, d) = (3449, 7, 2)$

$(fl, fu) = (32, 512)$

3) Cross-correlation & Combination

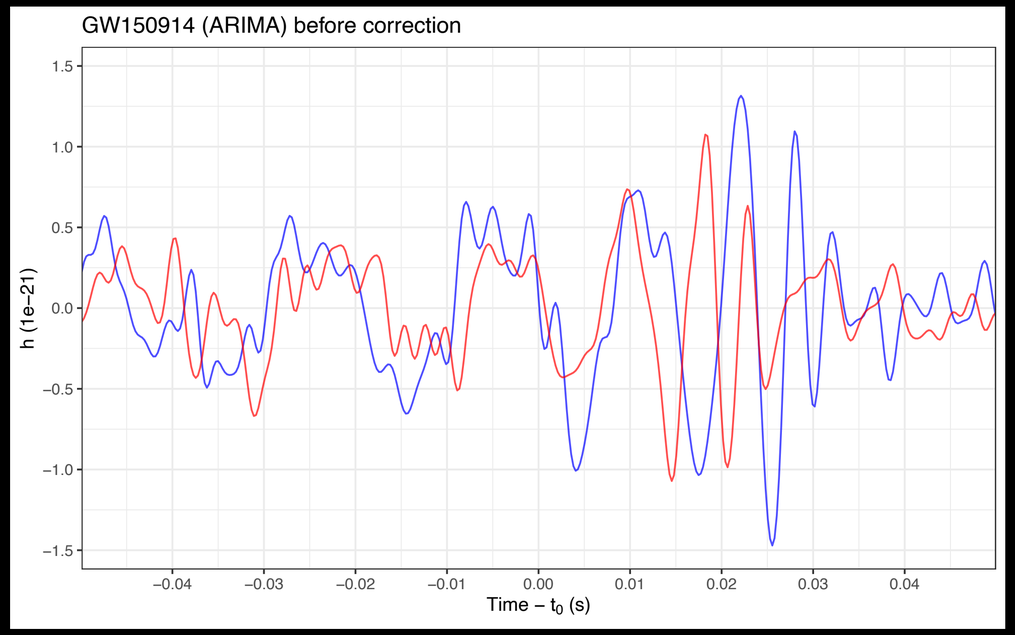
GW150914

CCF

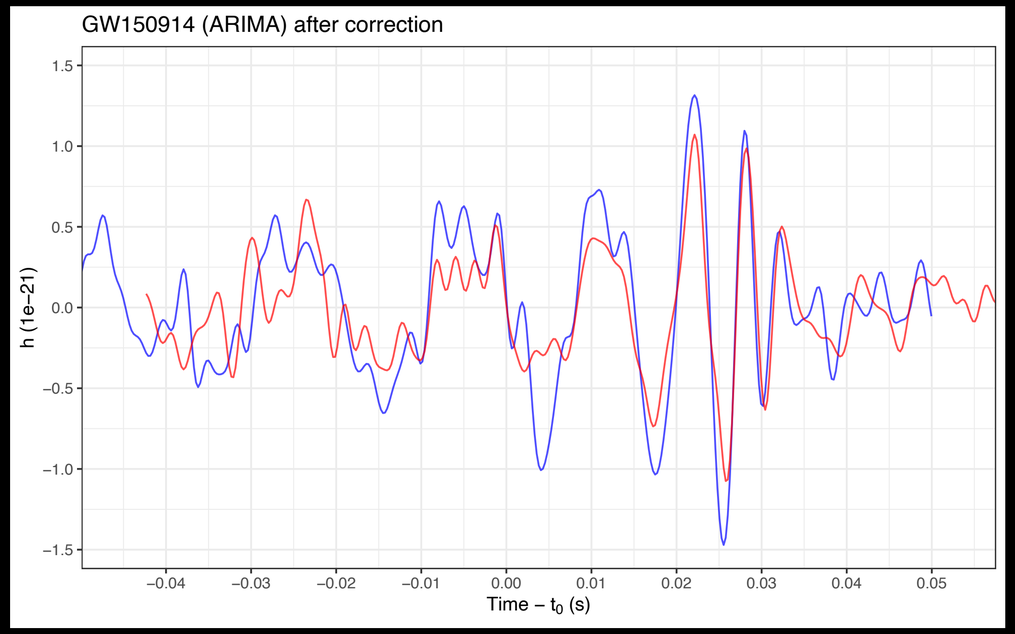


Window = ± 0.05 s
C = −0.816
Delay = +7.57 ms
First = Livingston

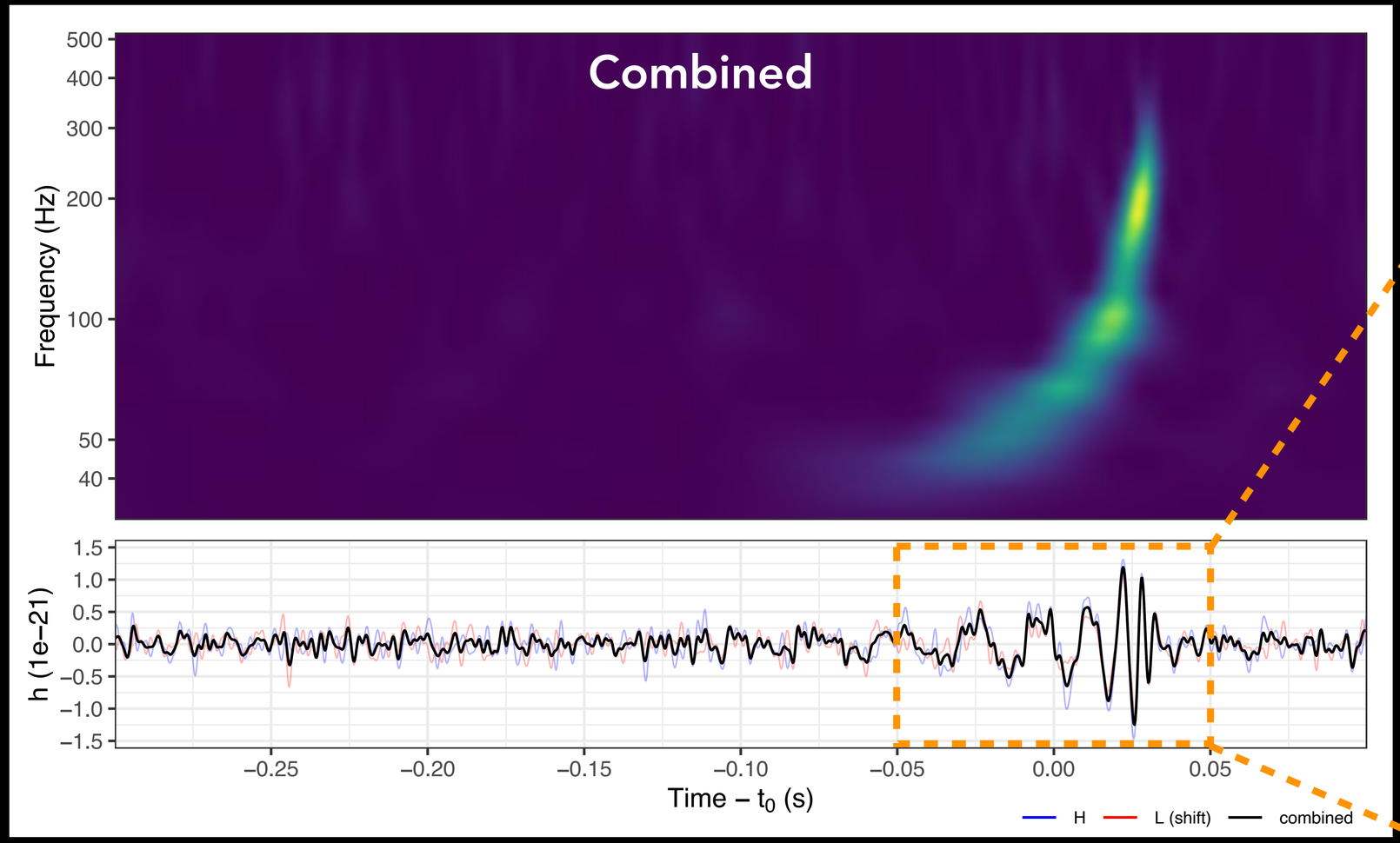
Before

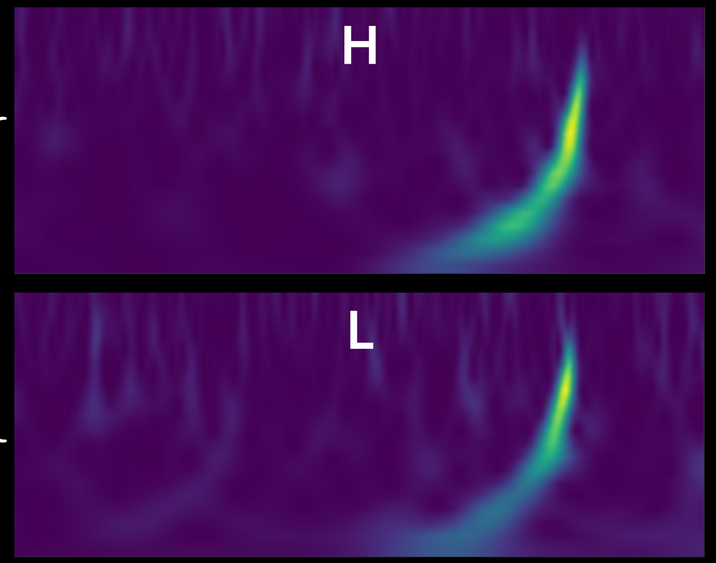


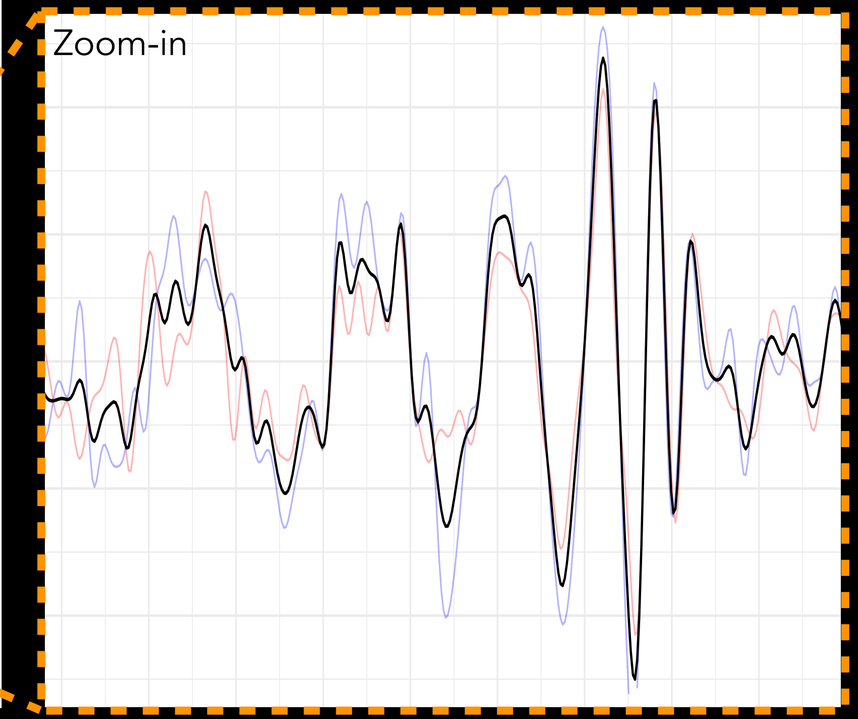
After



GW150914

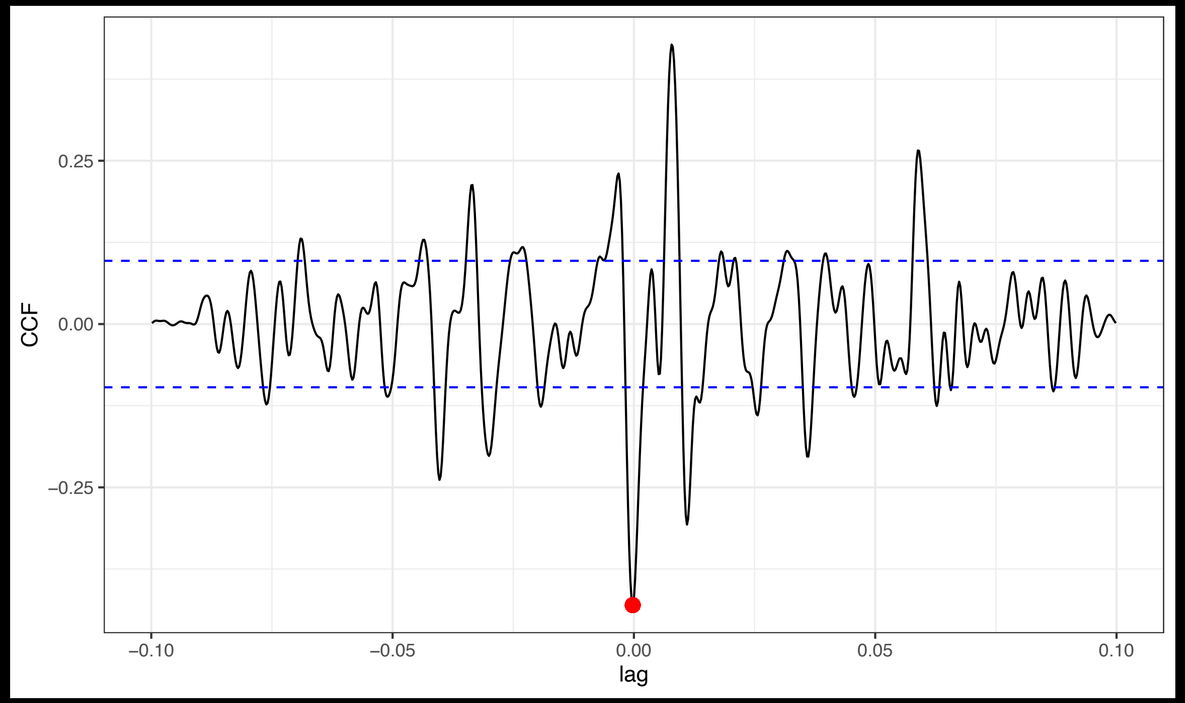

Combined
500
400
300
200
100
50
40
Frequency (Hz)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
h (1e−21)
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t0 (s)
H
L (shift)
combined


H
L


Zoom-in

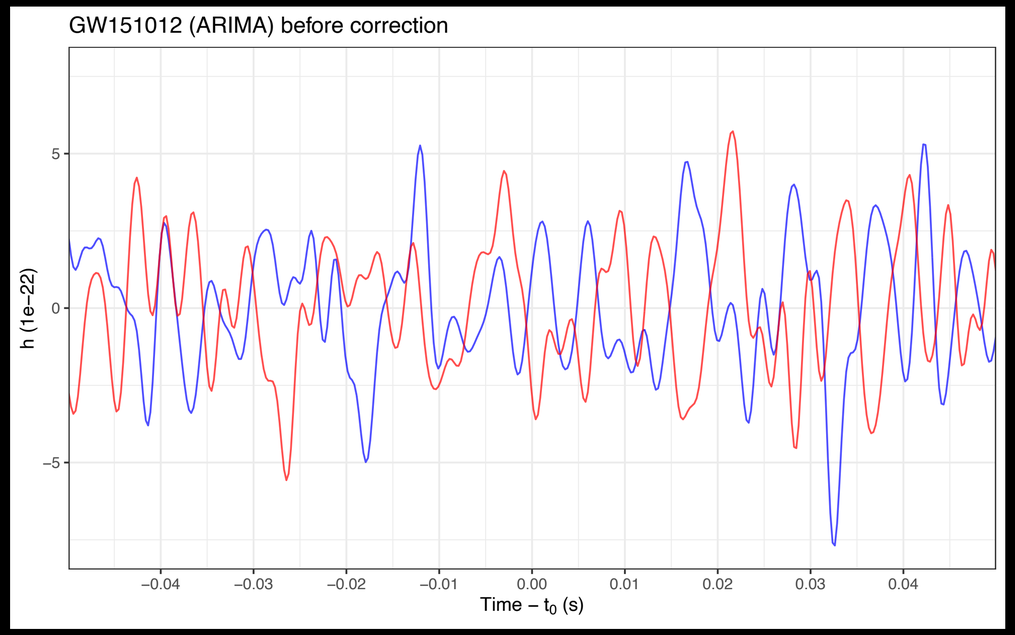
GW151012

CCF

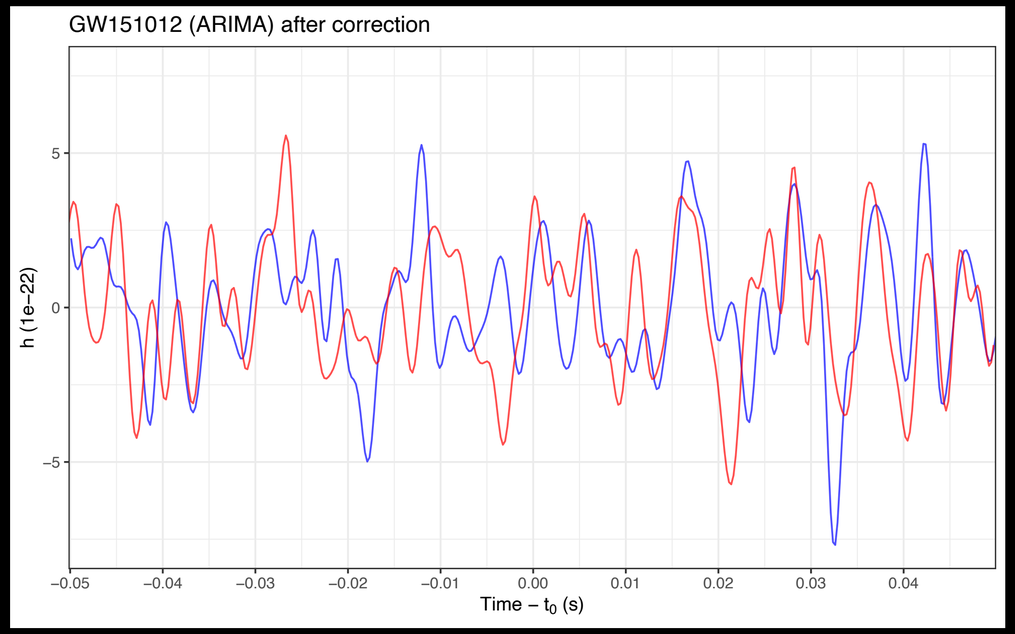


Window = ± 0.05 s
C = −0.43
Delay = −0.244 ms
First = Hanford

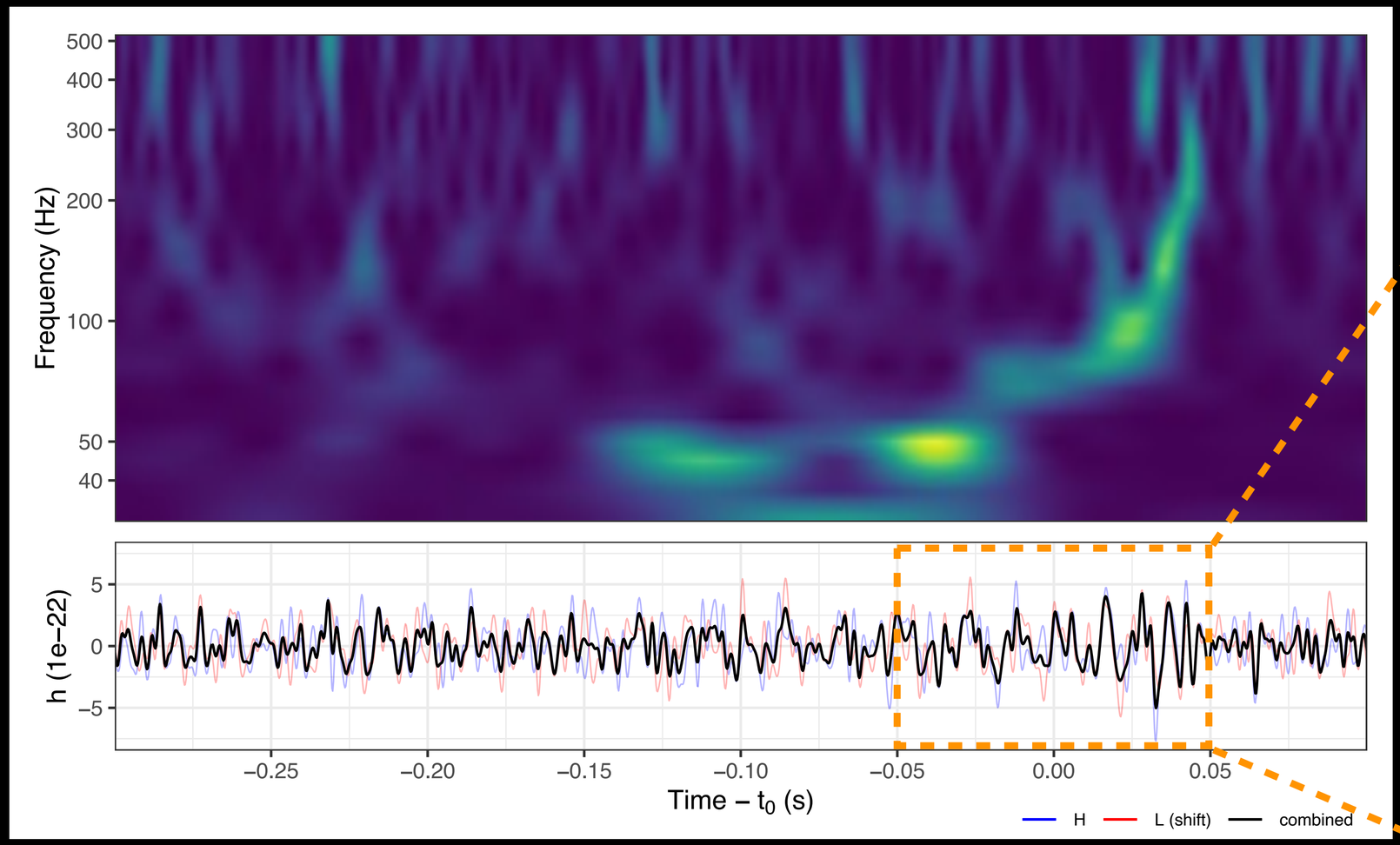
Before

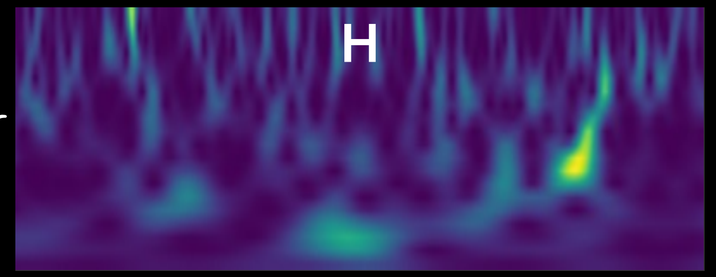


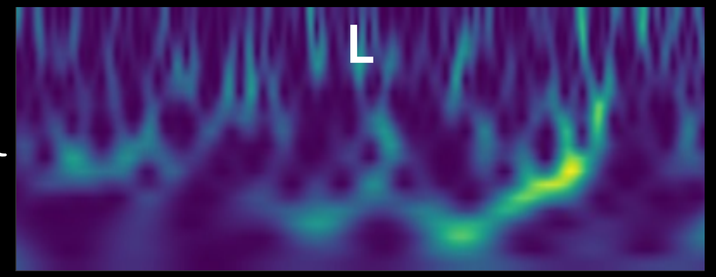
After

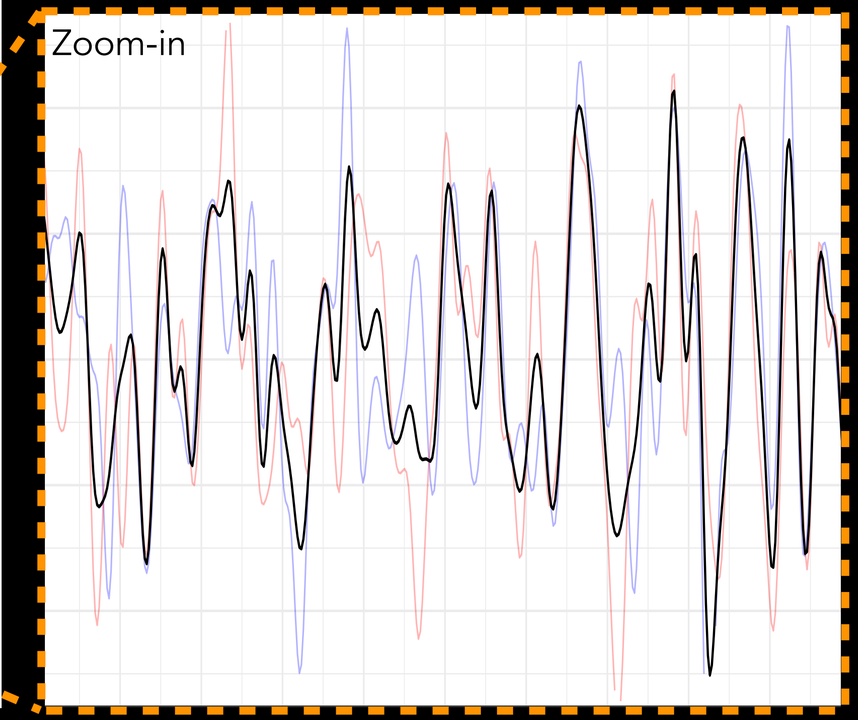


GW151012


Combined
500
400
300
200
100
50
40
Frequency (Hz)
5
0
−5
h (1e−22)
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t0 (s)
H
L (shift)
combined

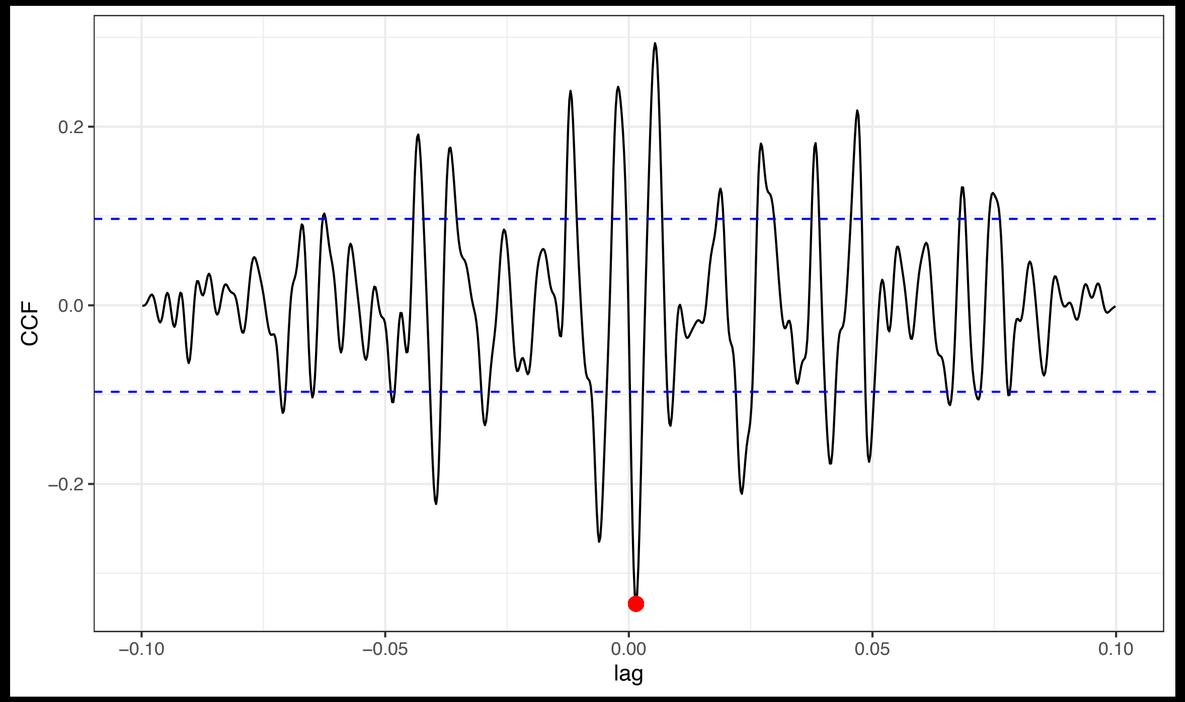

H


L


Zoom-in

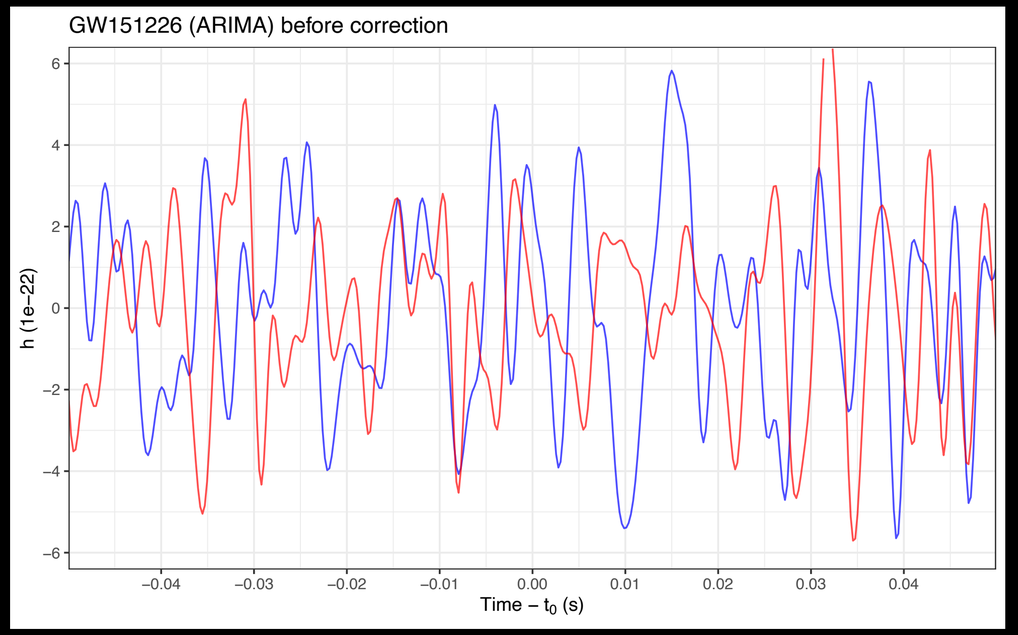
GW151226

CCF

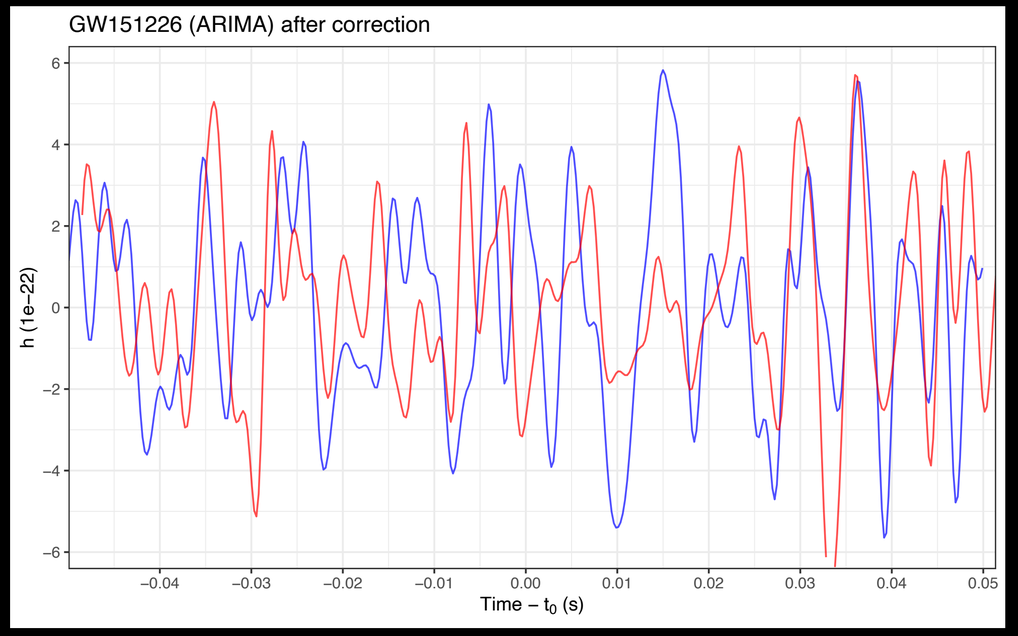


Window = ± 0.05 s
C = −0.334
Delay = +1.46 ms
First = Livingston

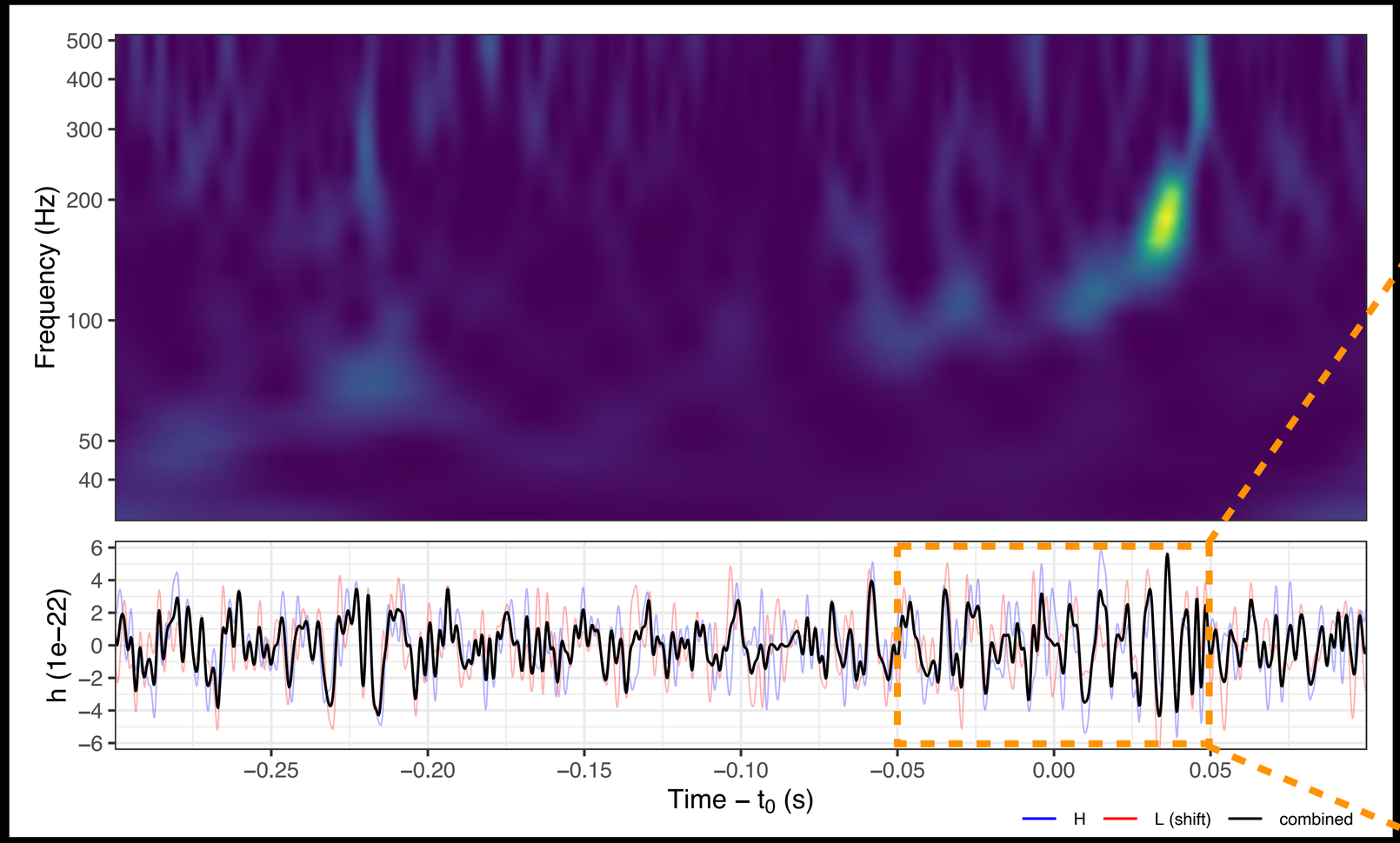
Before

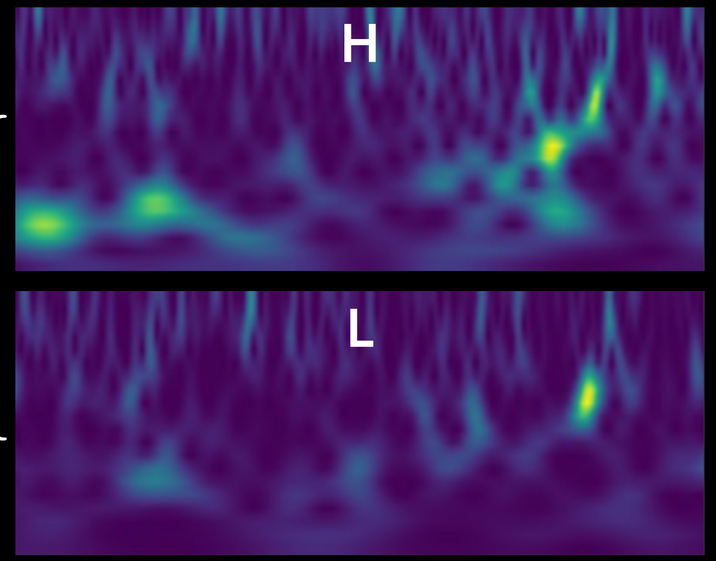


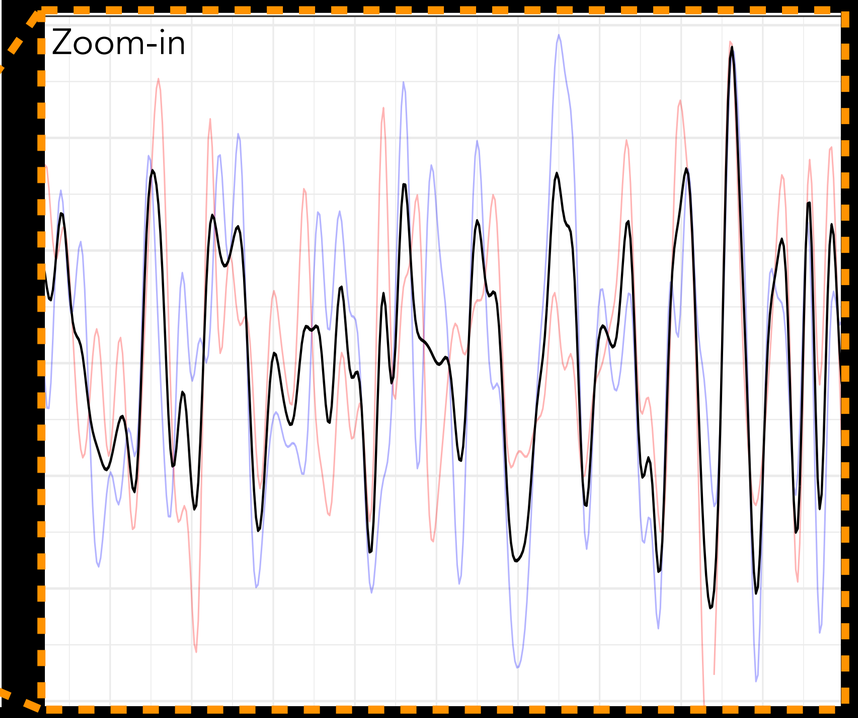
After



GW151226

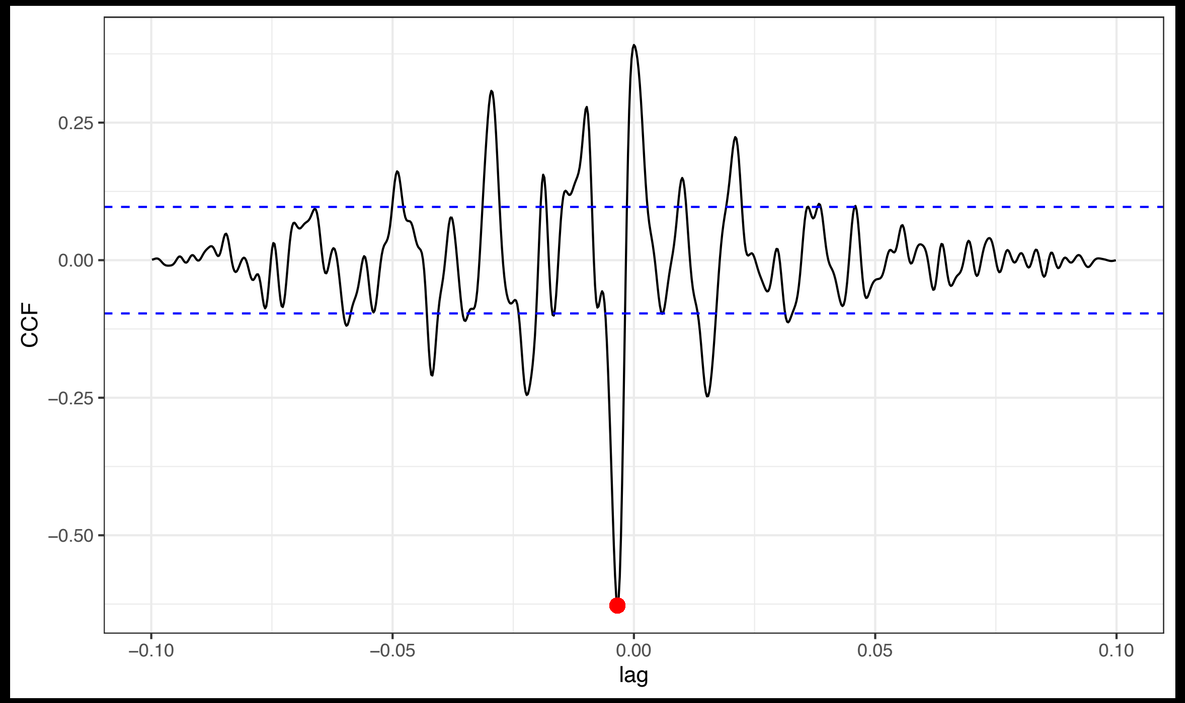

Combined
500
400
300
200
100
50
40
Frequency (Hz)
6
4
2
0
−2
−4
−6
h (1e−22)
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t0 (s)
H
L (shift)
combined


H
L


Zoom-in

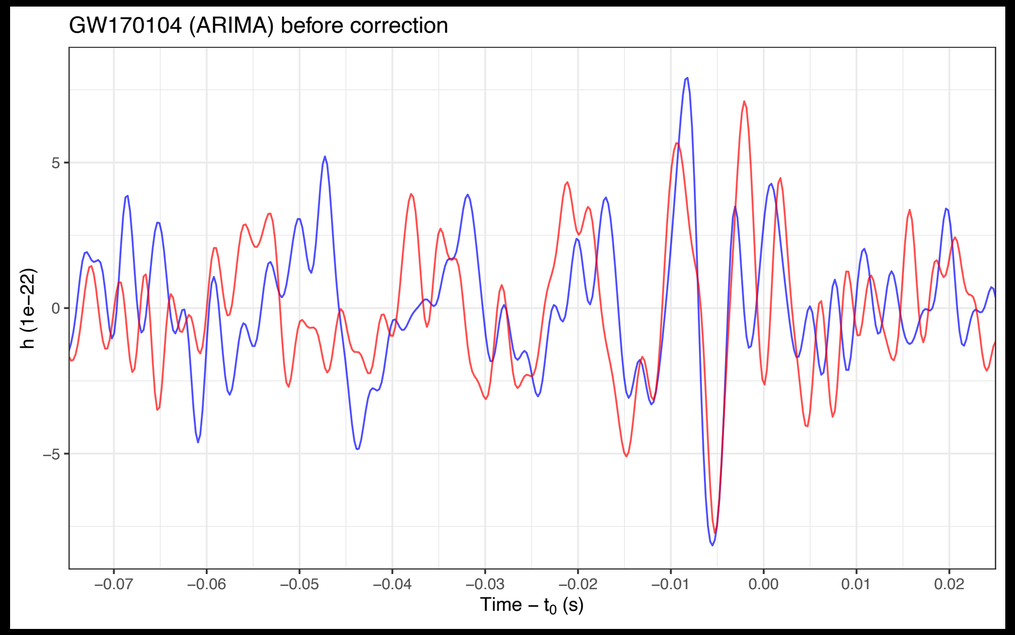
GW170104

CCF

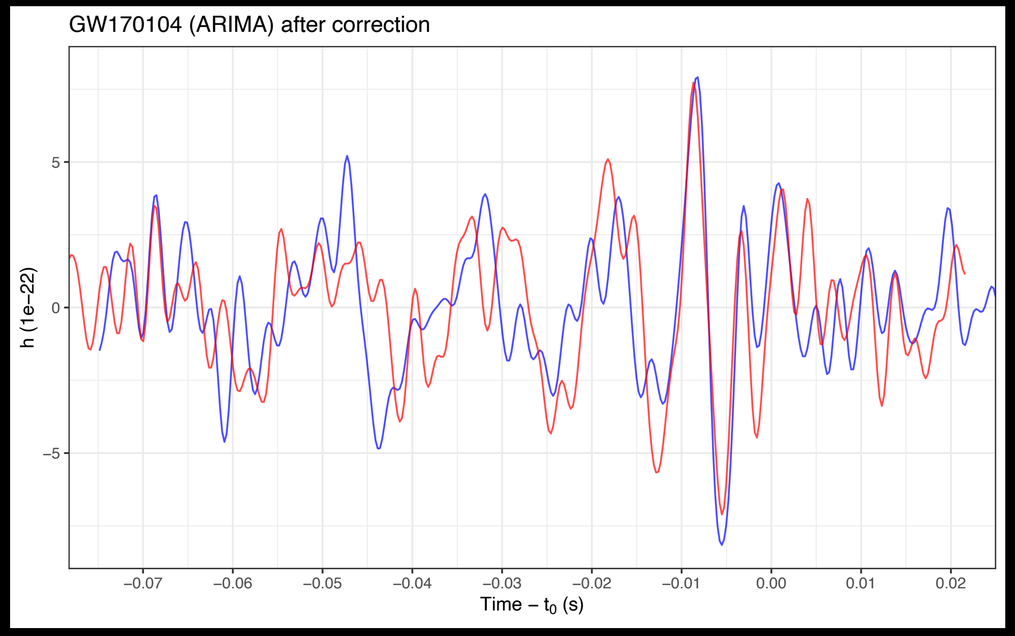


Window = (−0.075, 0.025) s
C = −0.628
Delay = −3.42 ms
First = Hanford

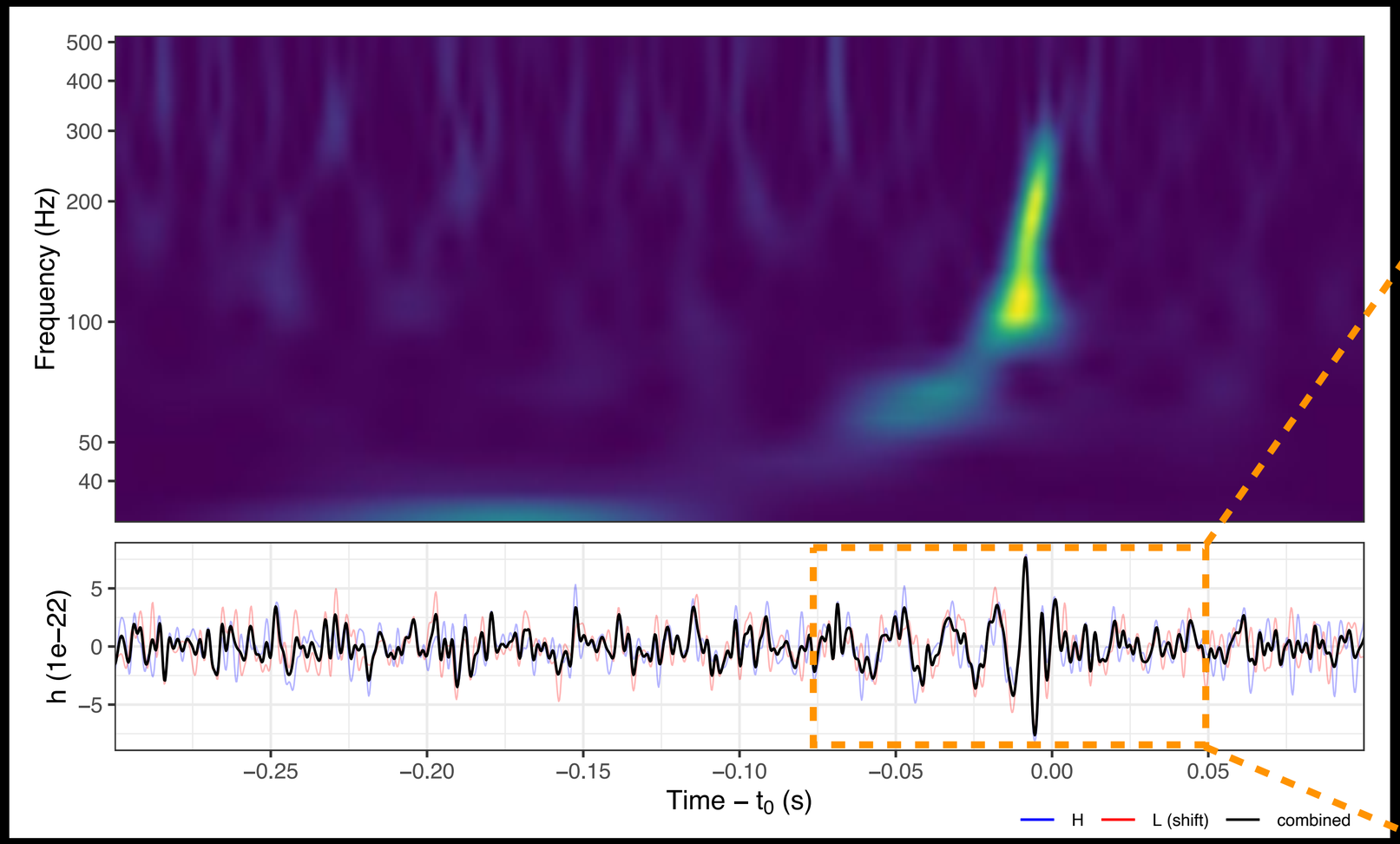
Before

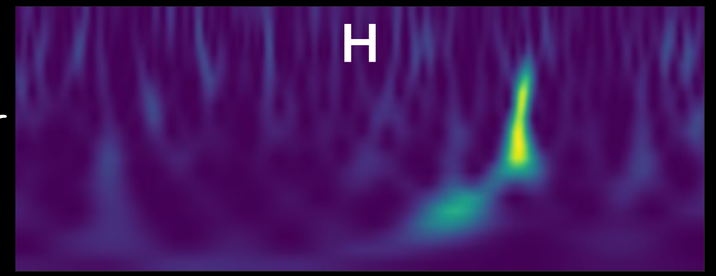


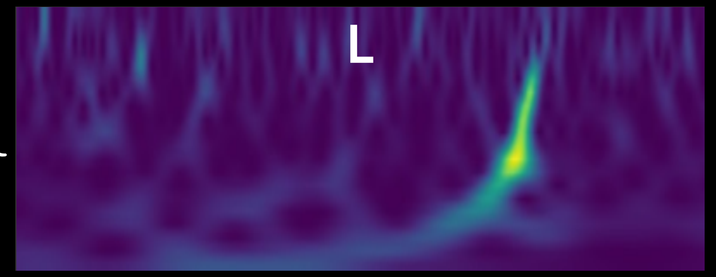
After

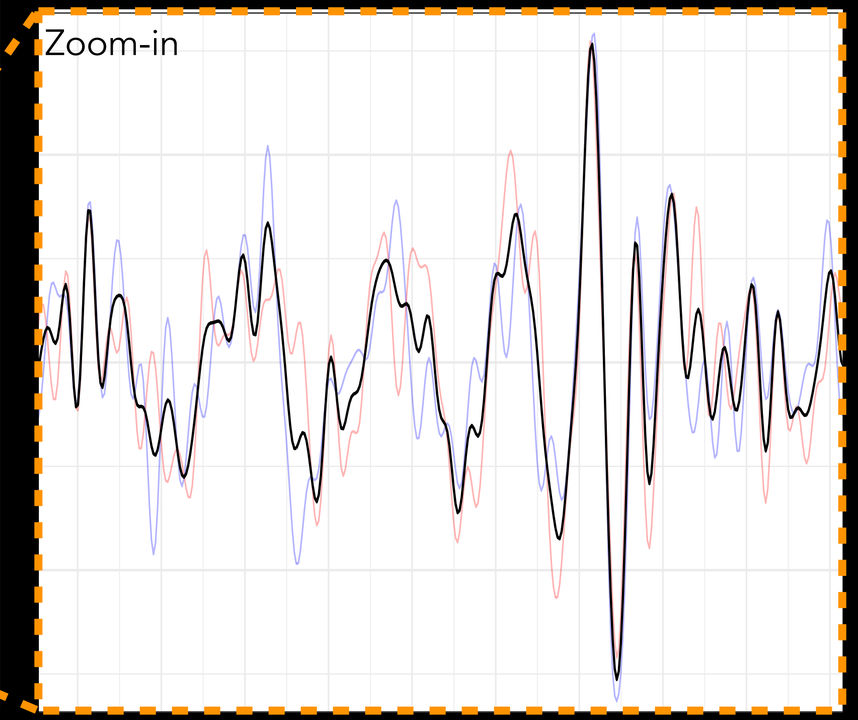


GW170104


Combined
500
400
300
200
100
50
40
Frequency (Hz)
5
0
−5
h (1e−22)
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t0 (s)
H
L (shift)
combined

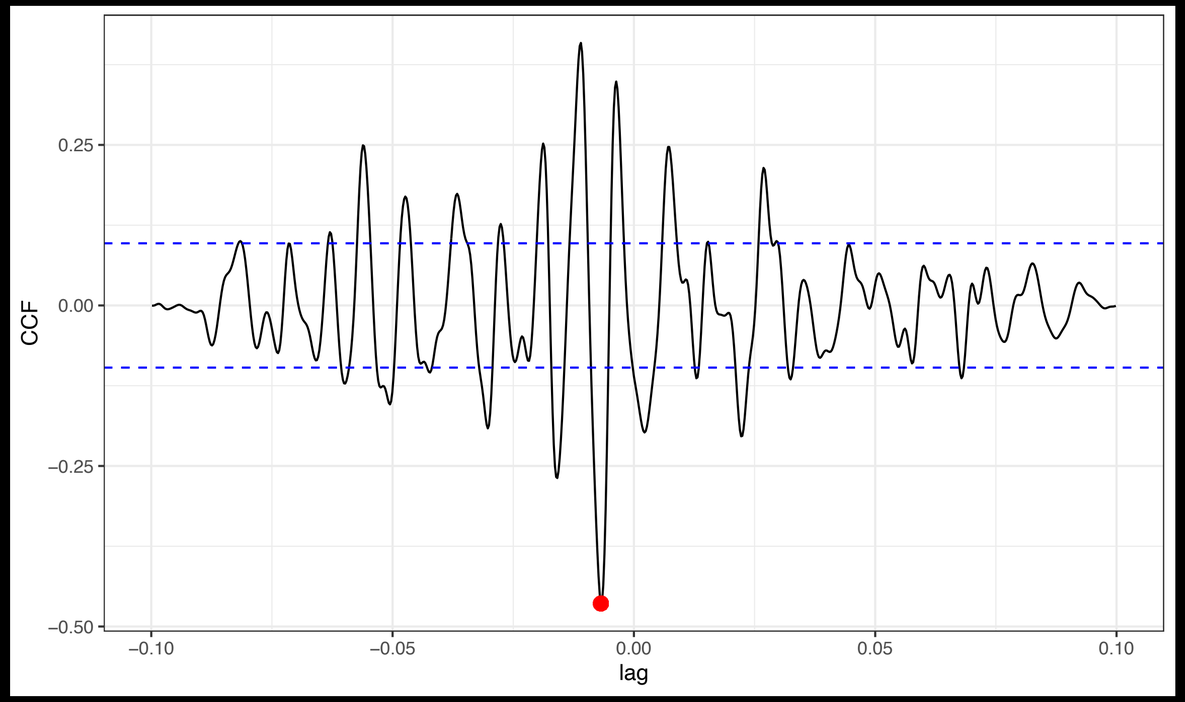

H


L


Zoom-in

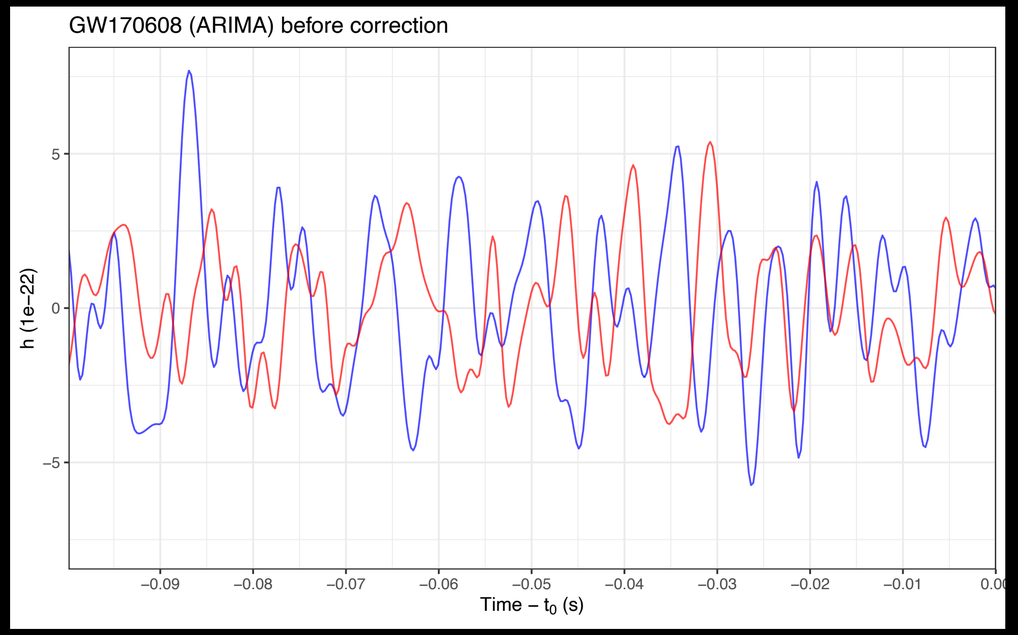
GW170608

CCF

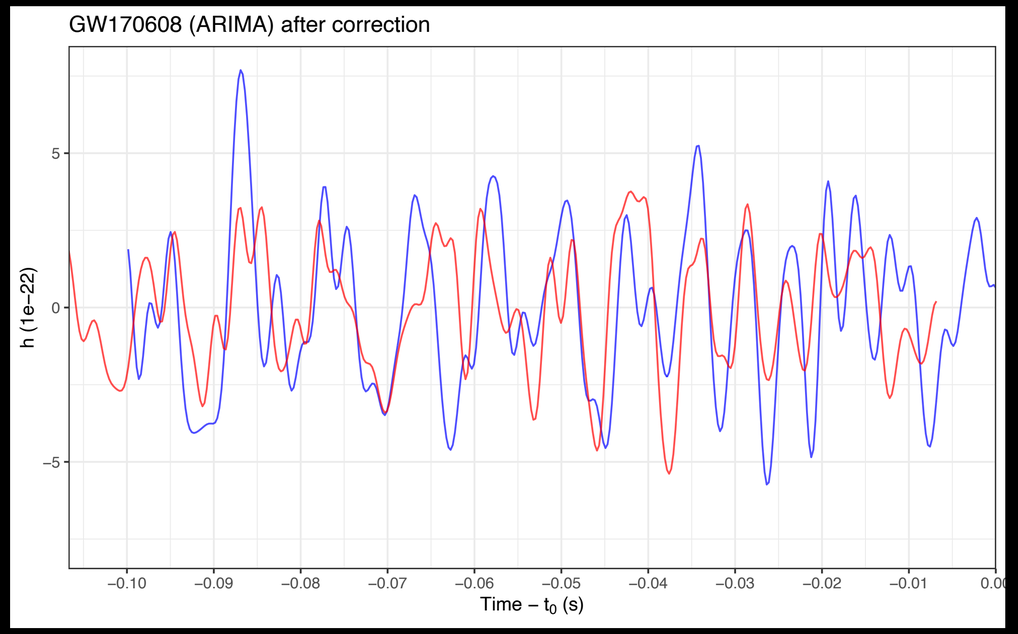


Window = (−0.1, 0) s
C = −0.464
Delay = −6.84 ms
First = Hanford

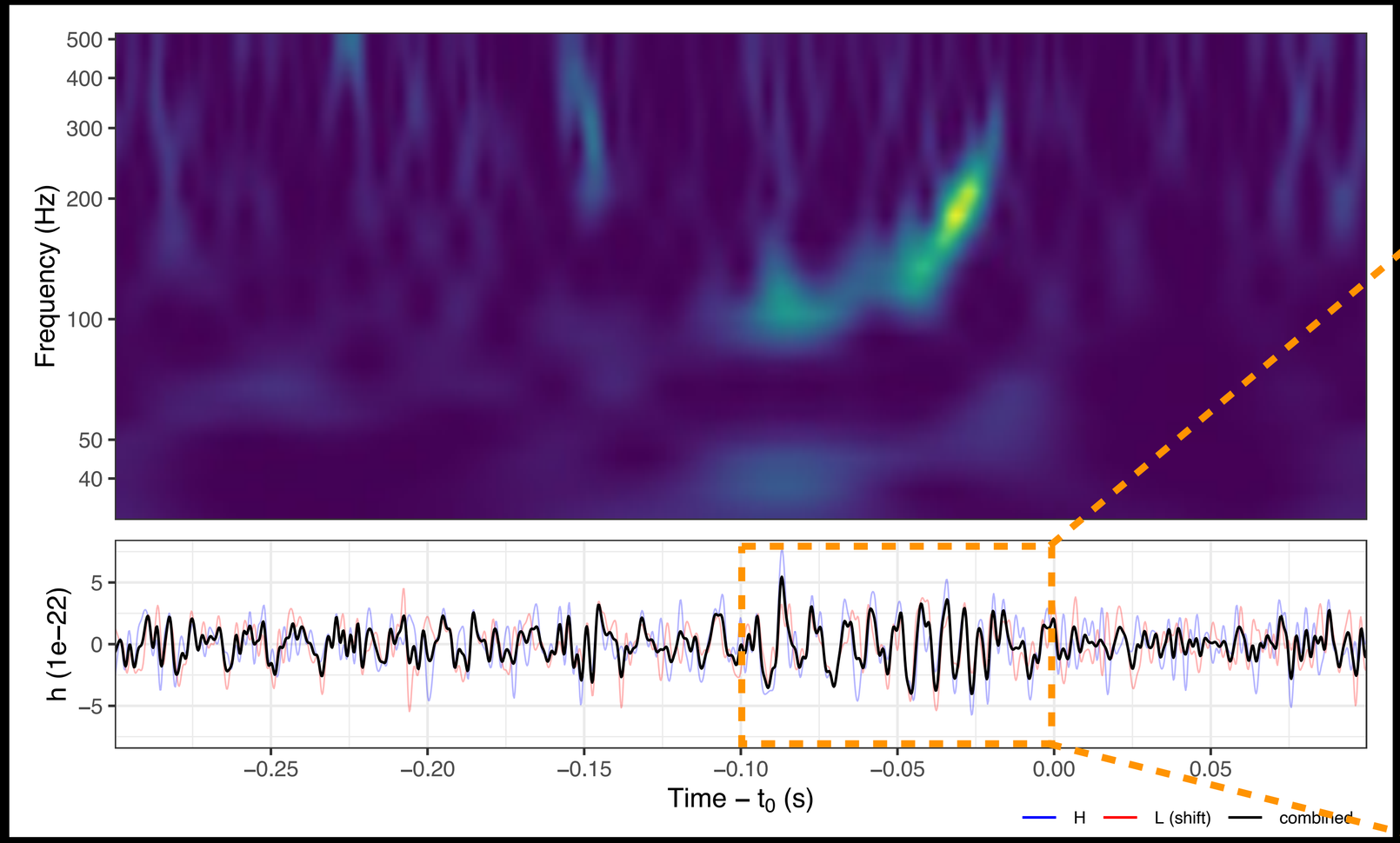
Before

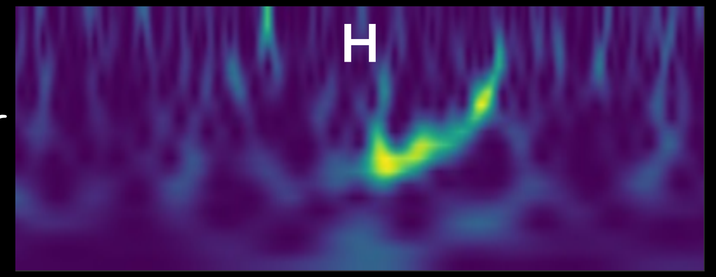


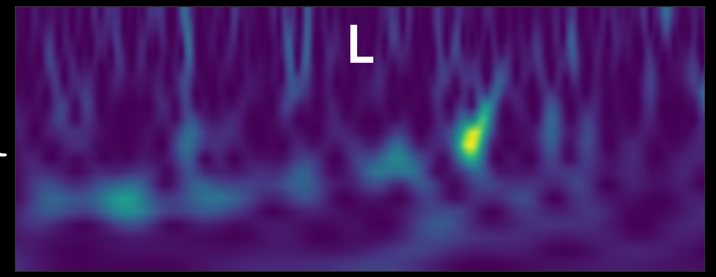
After

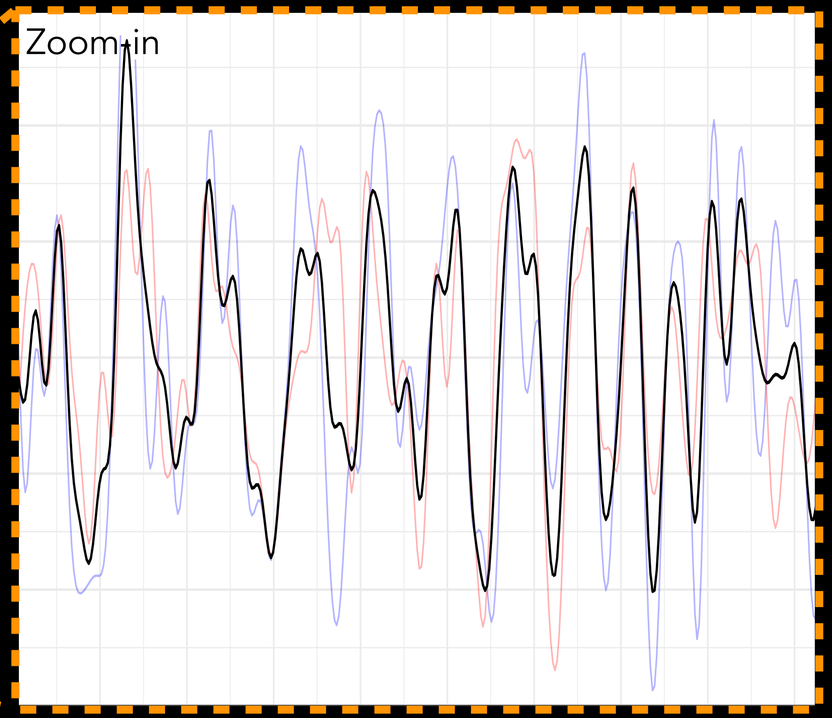


GW170608


Combined
500
400
300
200
100
50
40
Frequency (Hz)
5
0
−5
h (1e−22)
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t0 (s)
H
L (shift)
combined

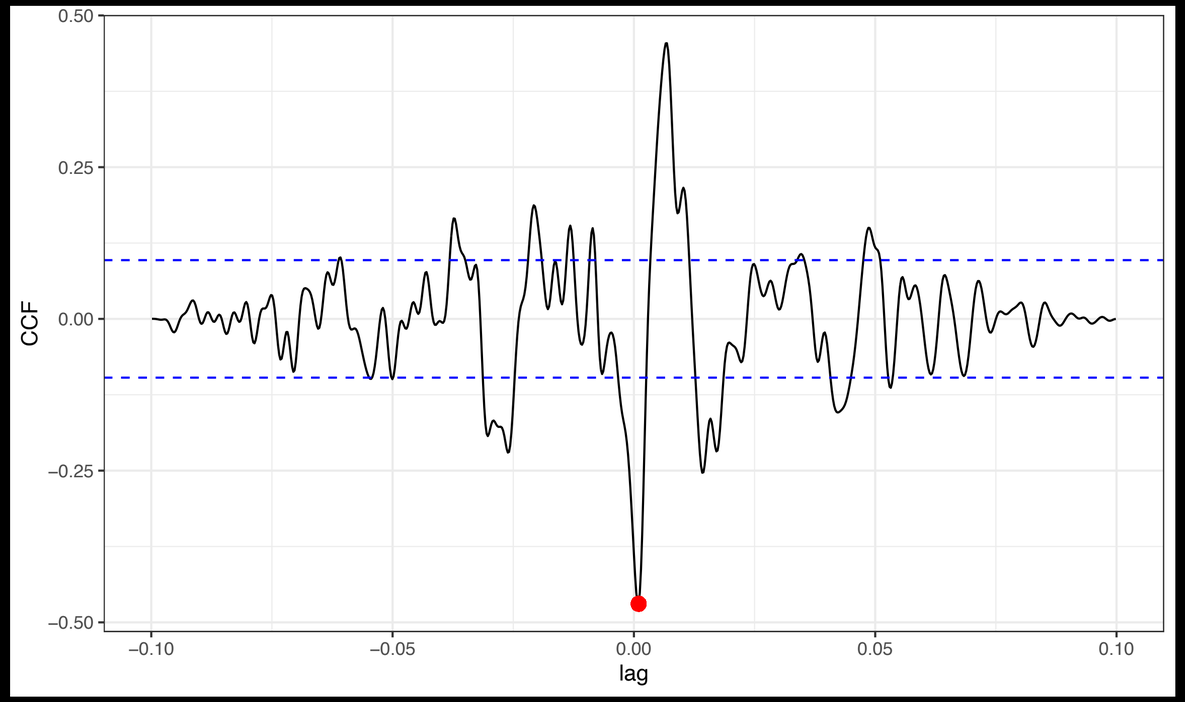

H


L


Zoom-in

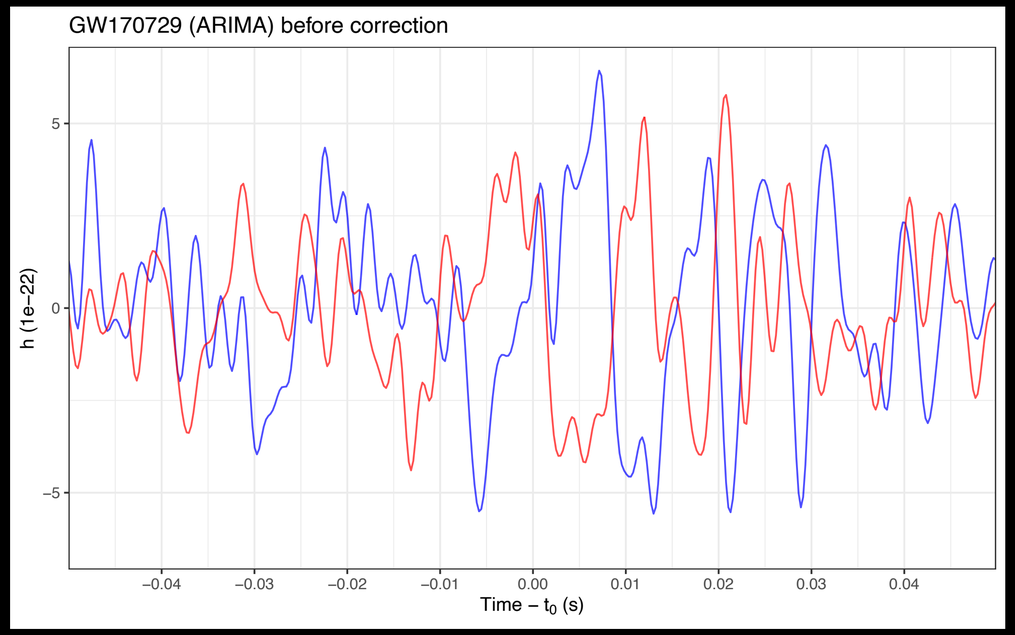
GW170729

CCF

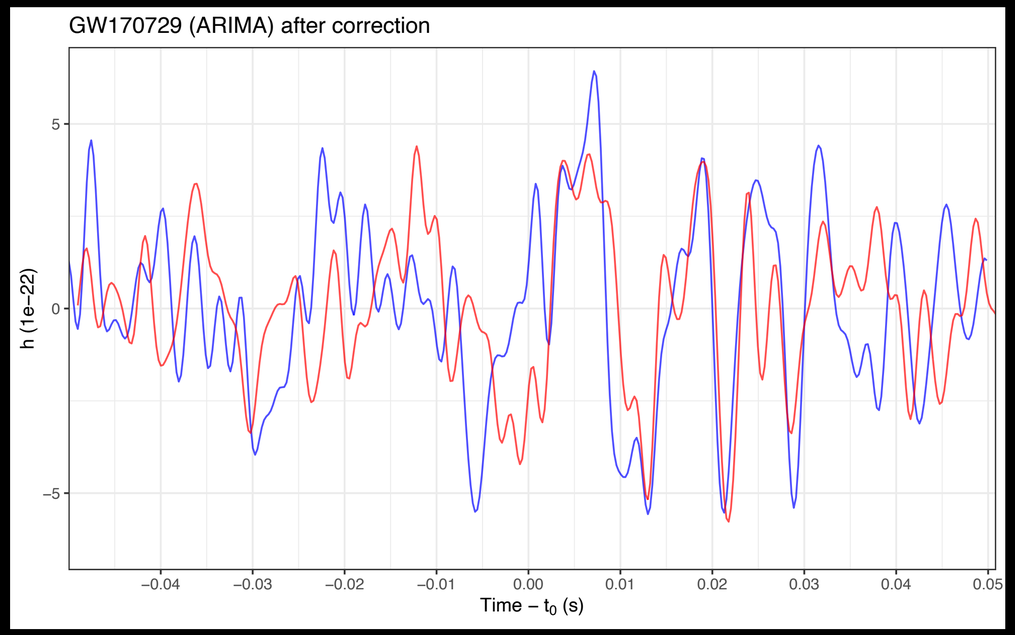


Window = ± 0.05 s
C = −0.469
Delay = +0.98 ms
First = Livingston

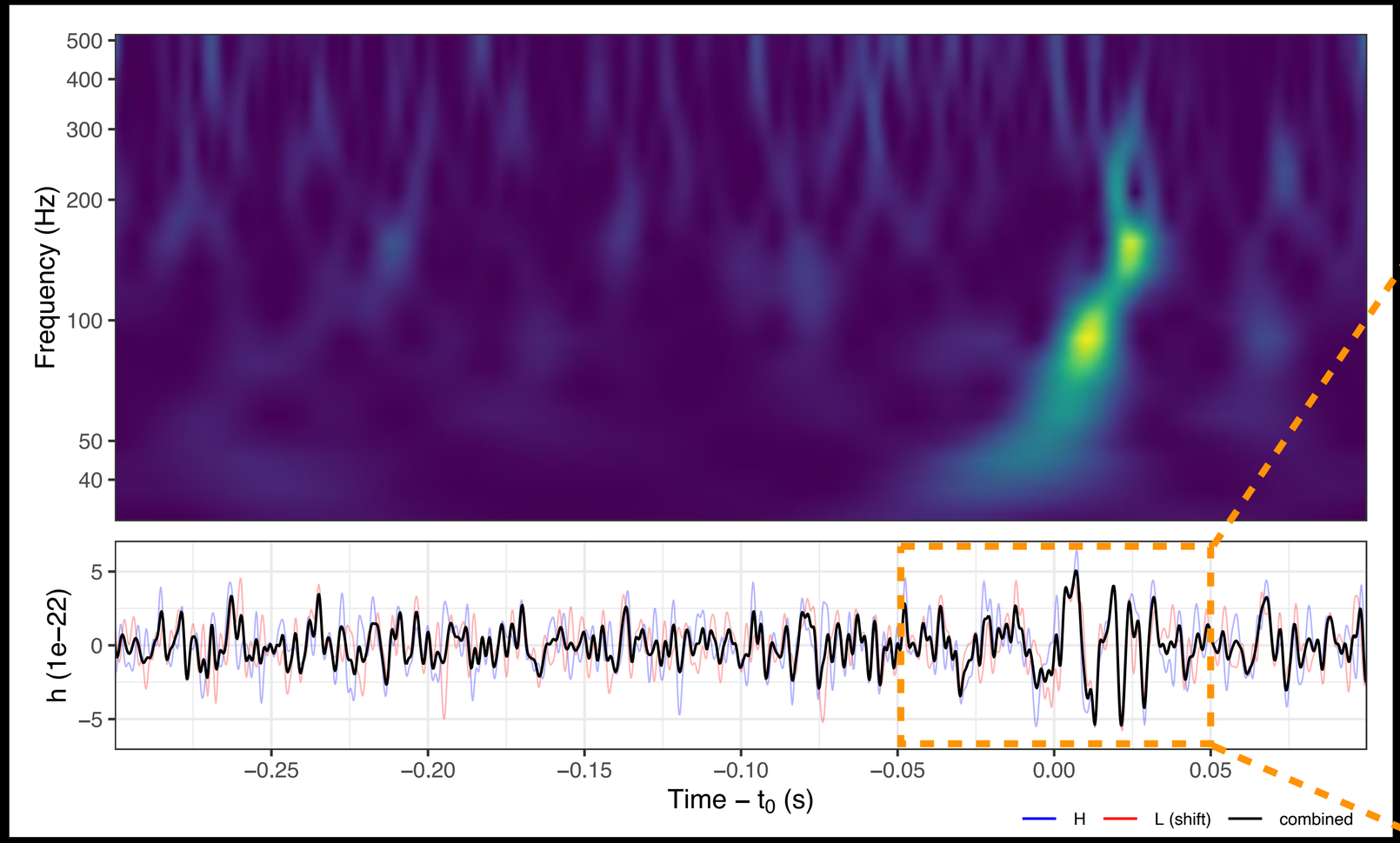
Before

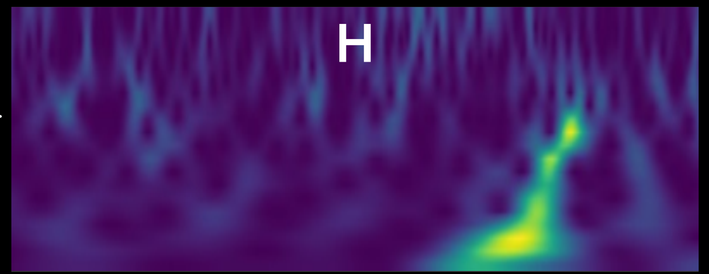


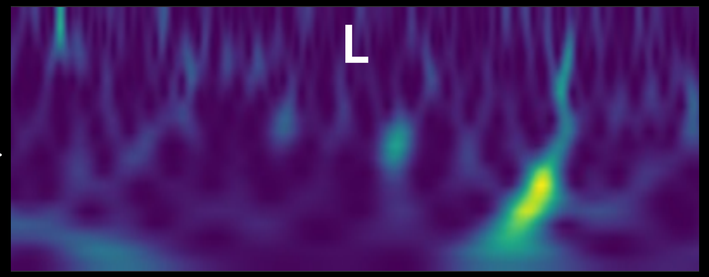
After

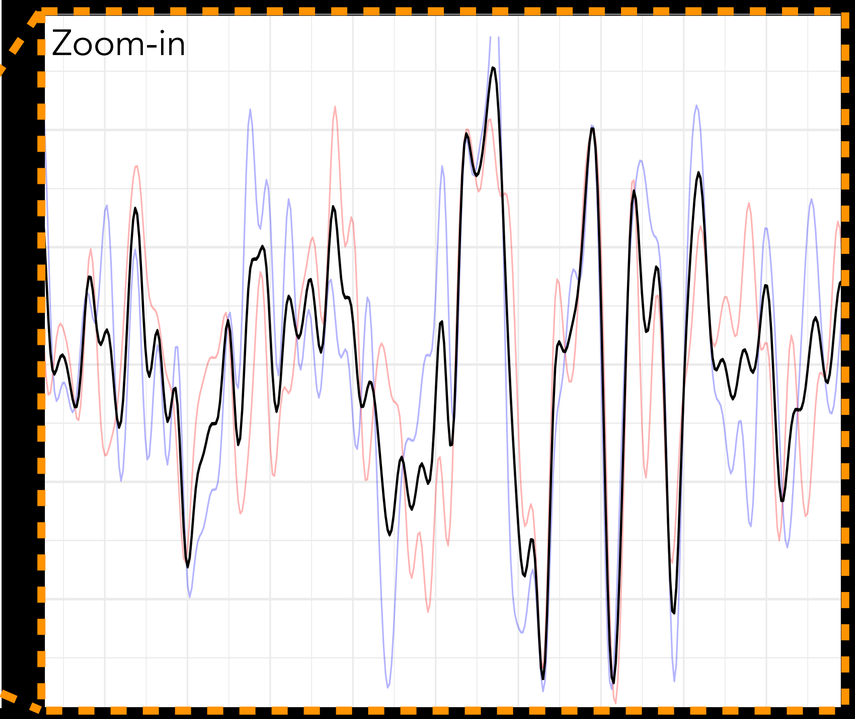


GW170729


Combined
500
400
300
200
100
50
40
Frequency (Hz)
5
0
−5
h (1e−22)
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t0 (s)
H
L (shift)
combined

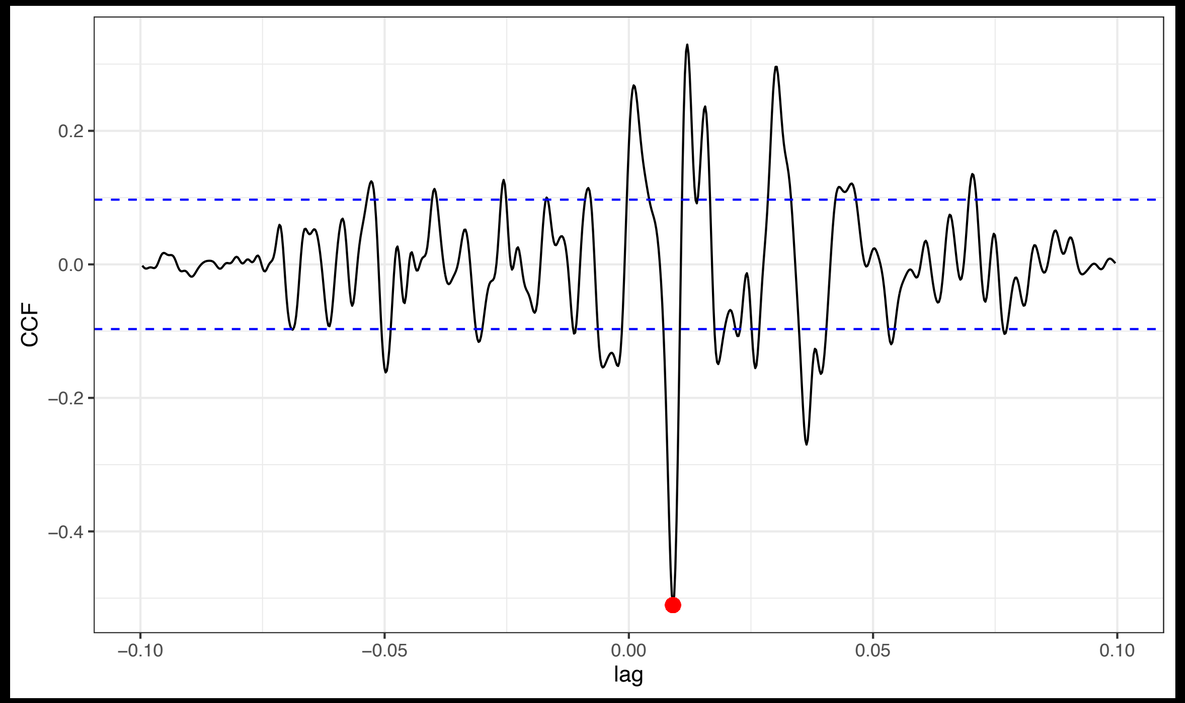

H


L


Zoom-in

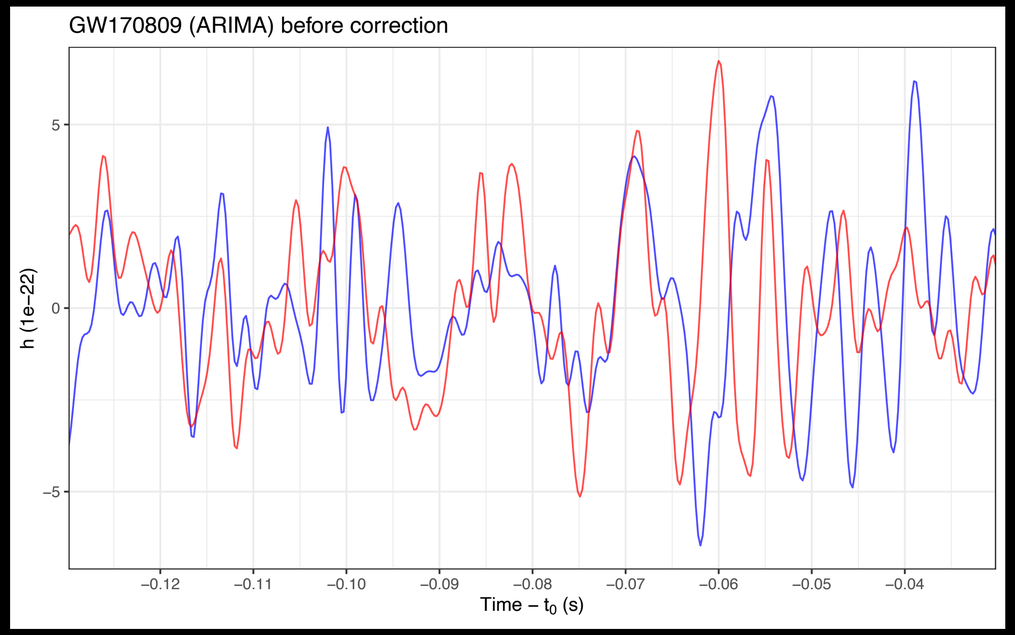
GW170809

CCF

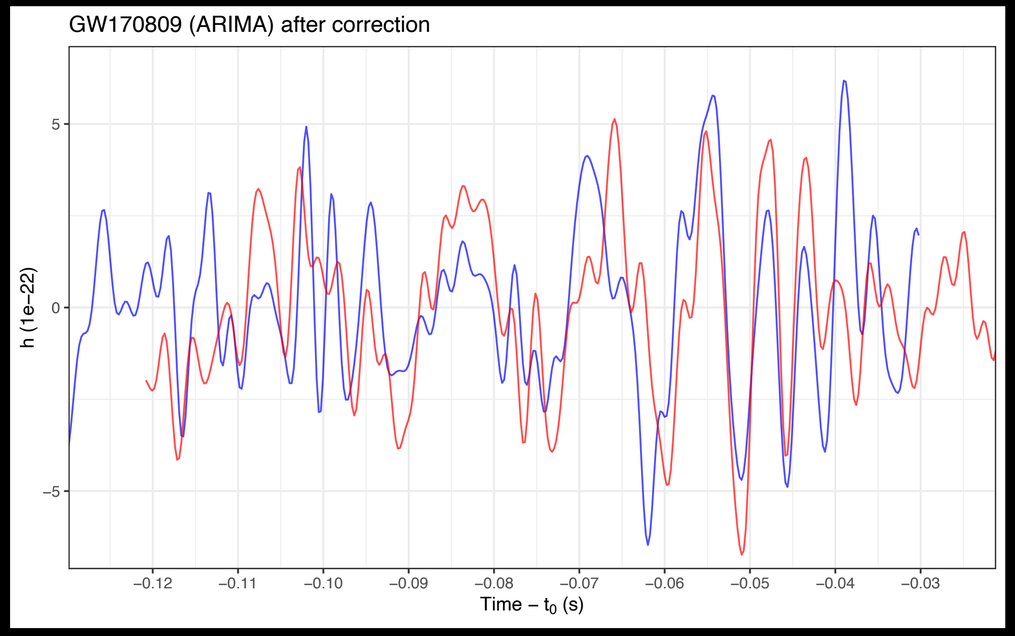


Window = (−0.13, 0.03) s
C = −0.510
Delay = +9.03 ms
First = Livingston

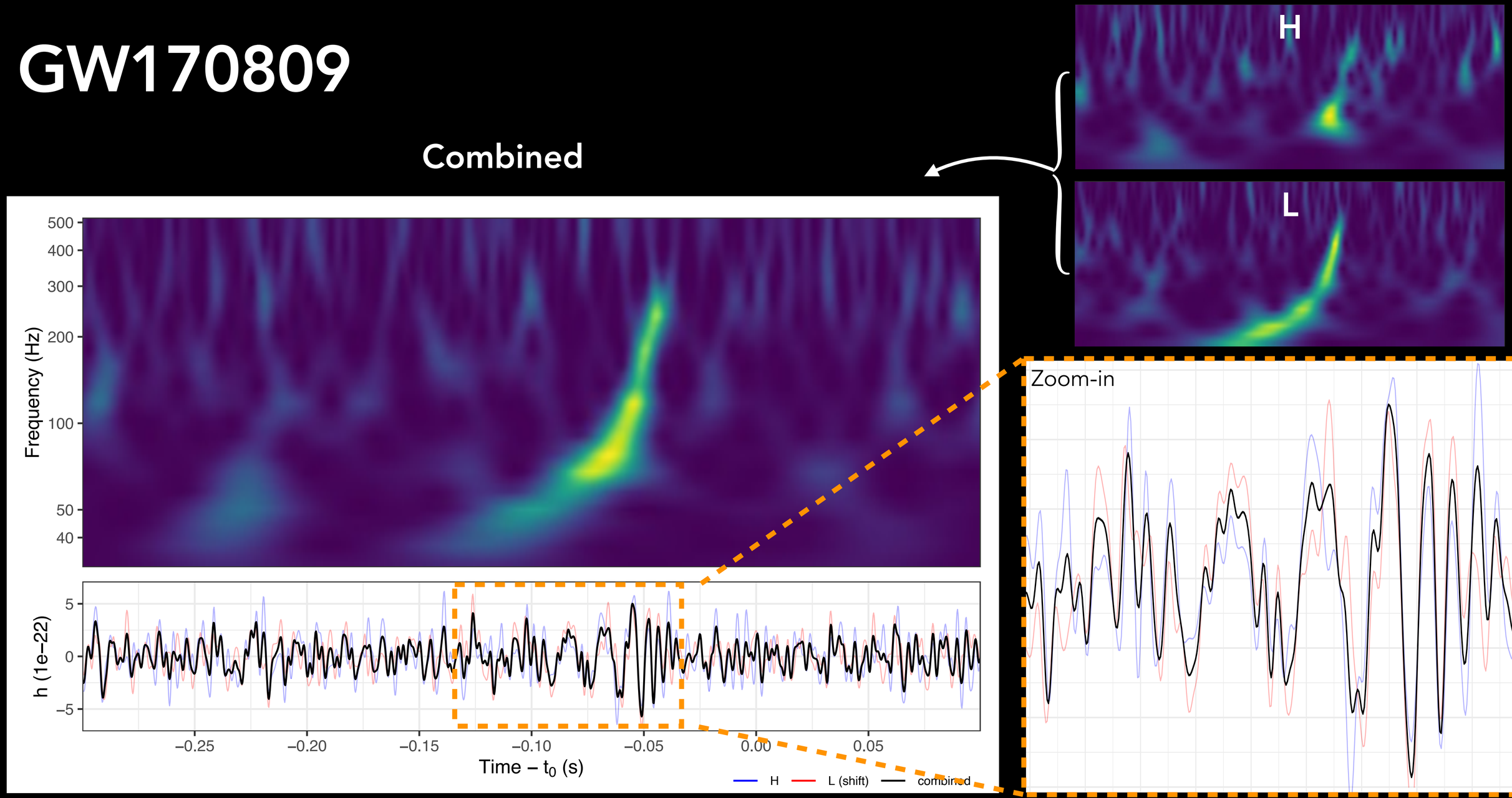
Before



After

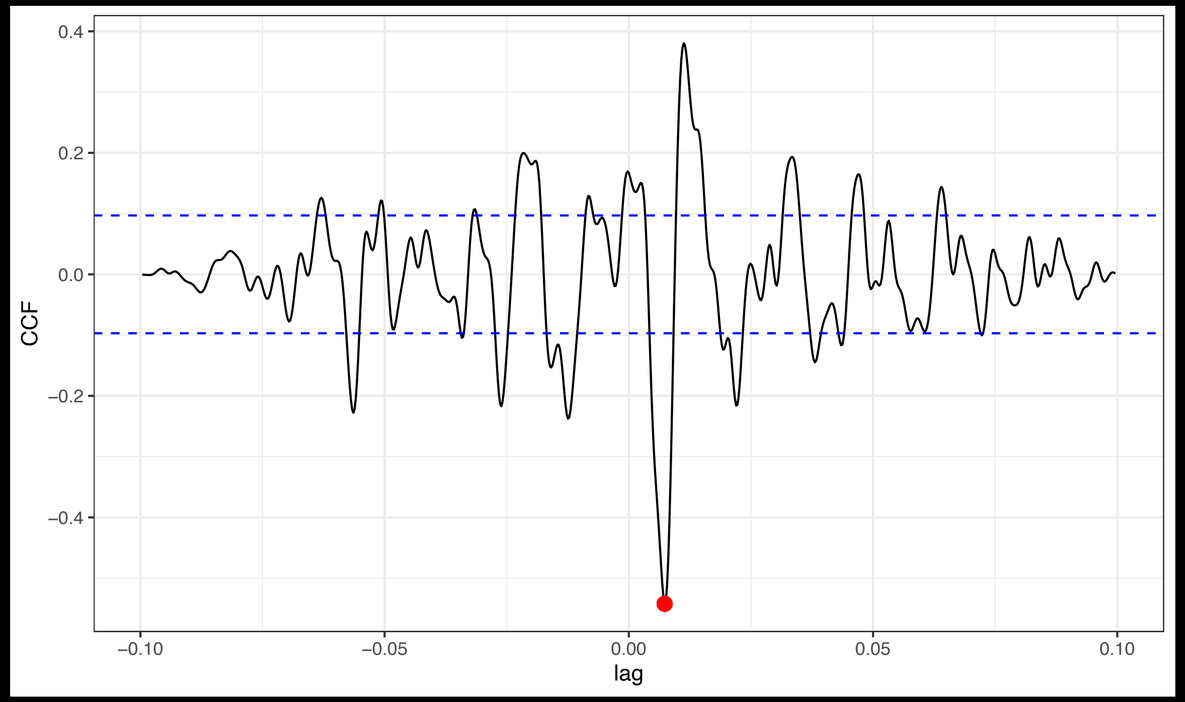


GW170809


Combined
H
L
Zoom-in
Frequency (Hz)
500
400
300
200
100
50
40
h (1e−22)
5
0
−5
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t0 (s)
H
L (shift)
combined

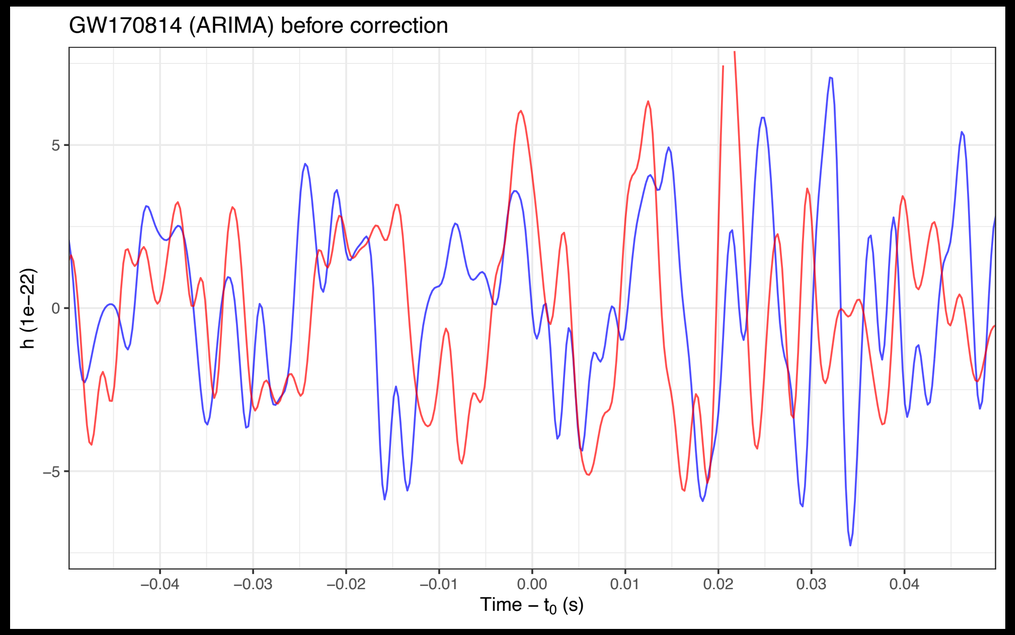
GW170814

CCF

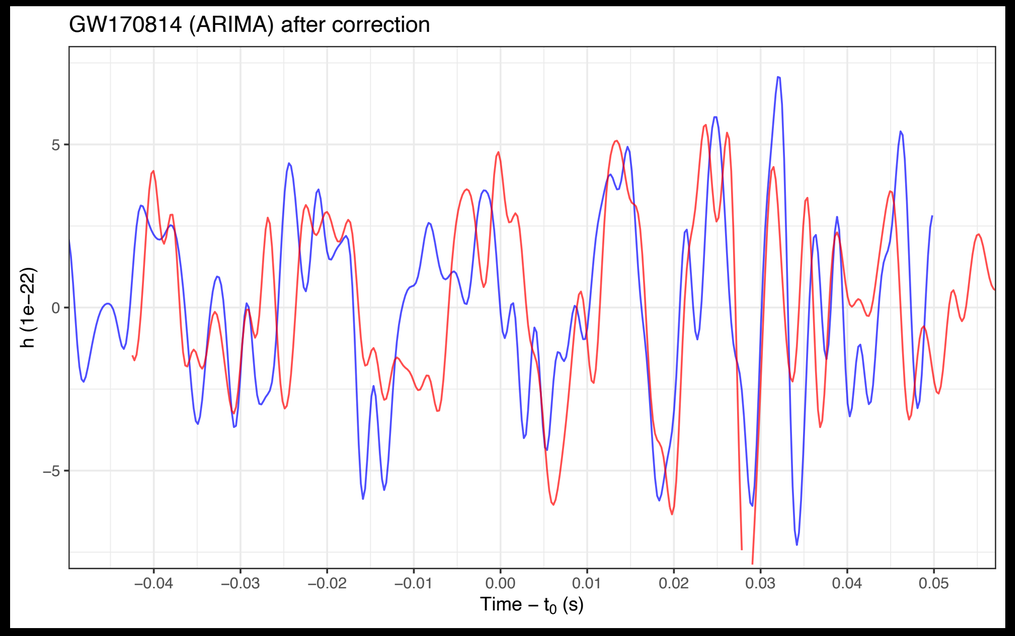


Window = ± 0.05 s
C = −0.542
Delay = +7.32 ms
First = Livingston

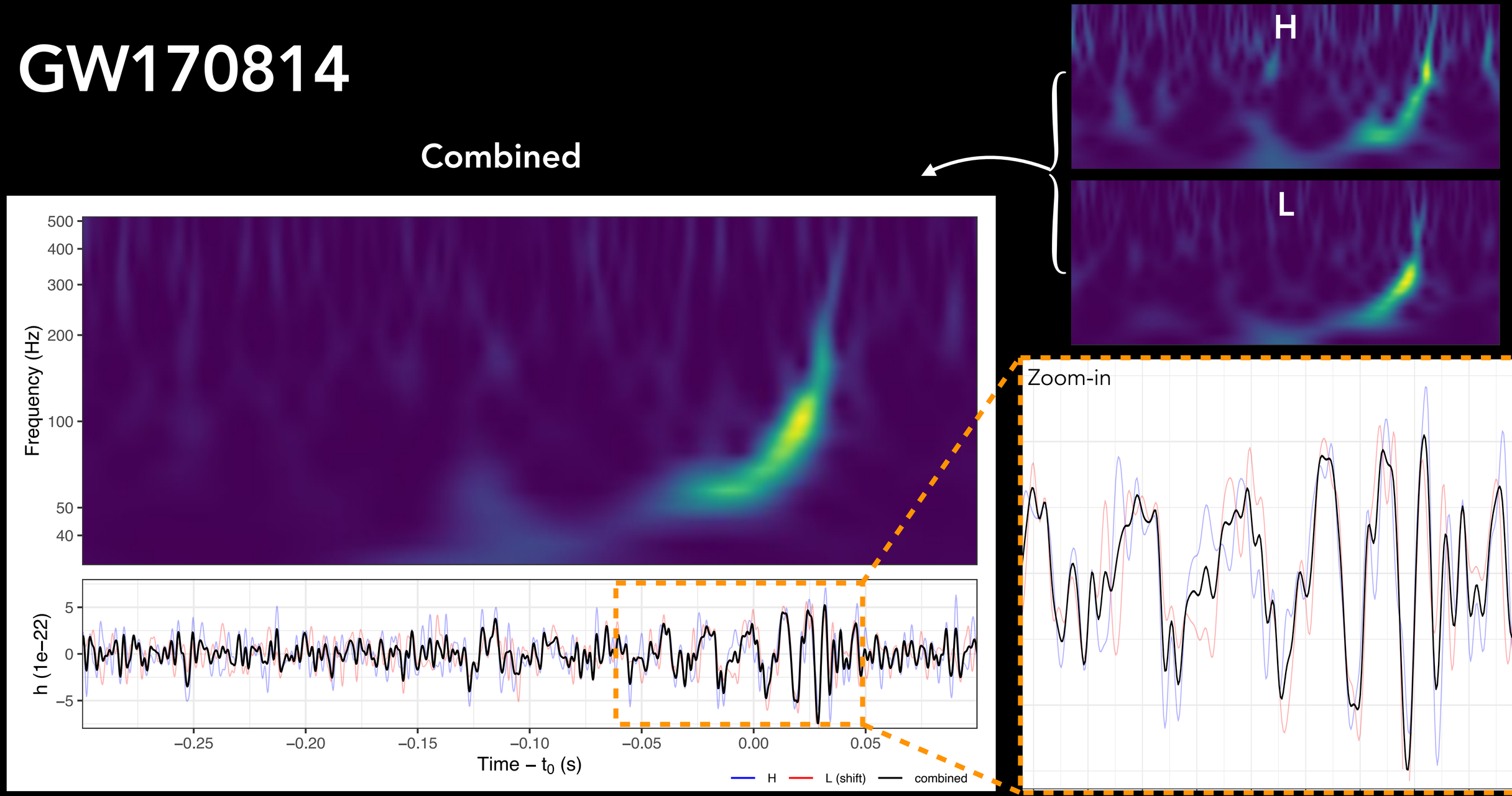
Before



After

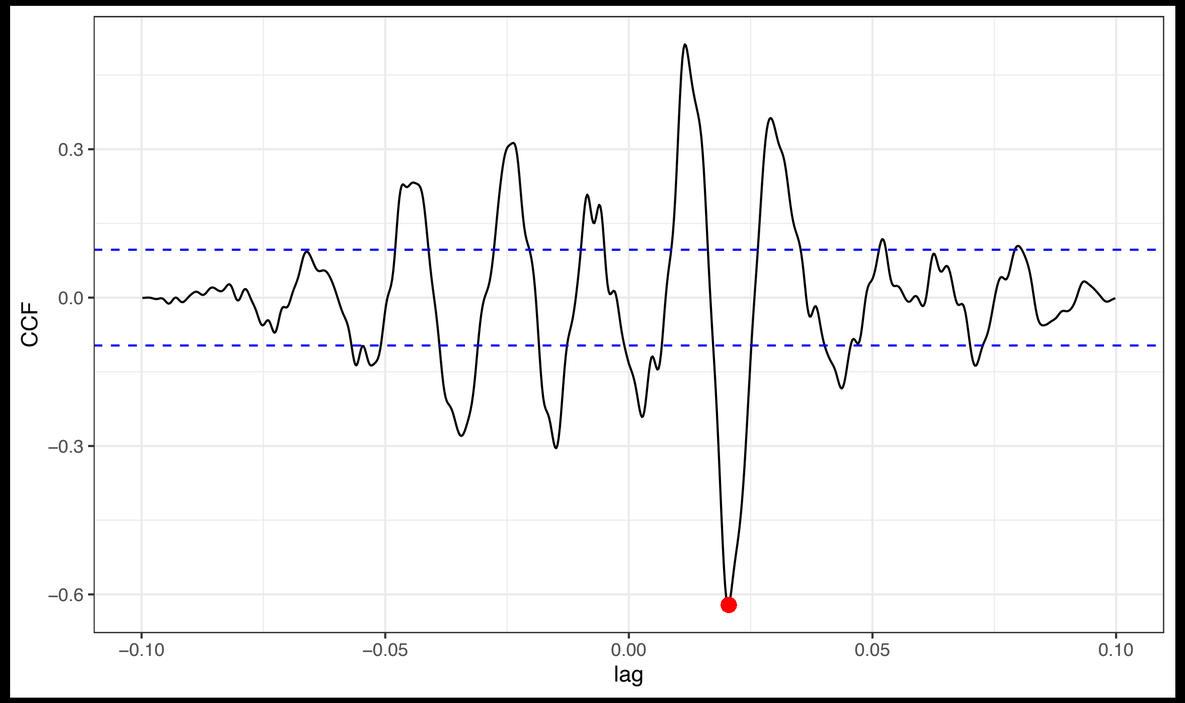


GW170814


Combined
H
L
Zoom-in
500
400
300
200
100
50
40
Frequency (Hz)
5
0
−5
h (1e−22)
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t_0 (s)
H
L (shift)
combined

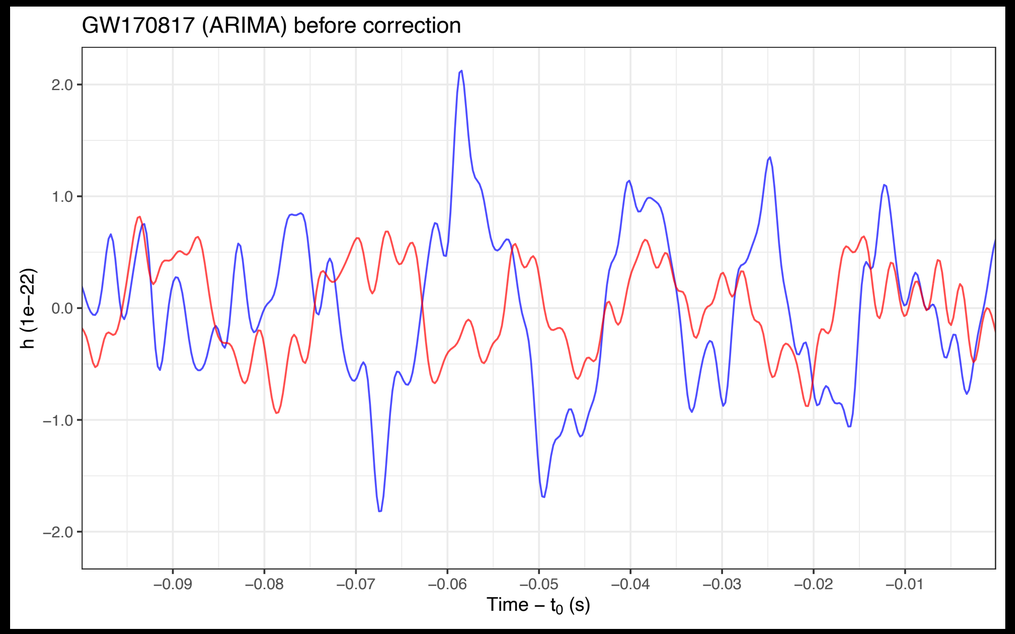
GW170817

CCF

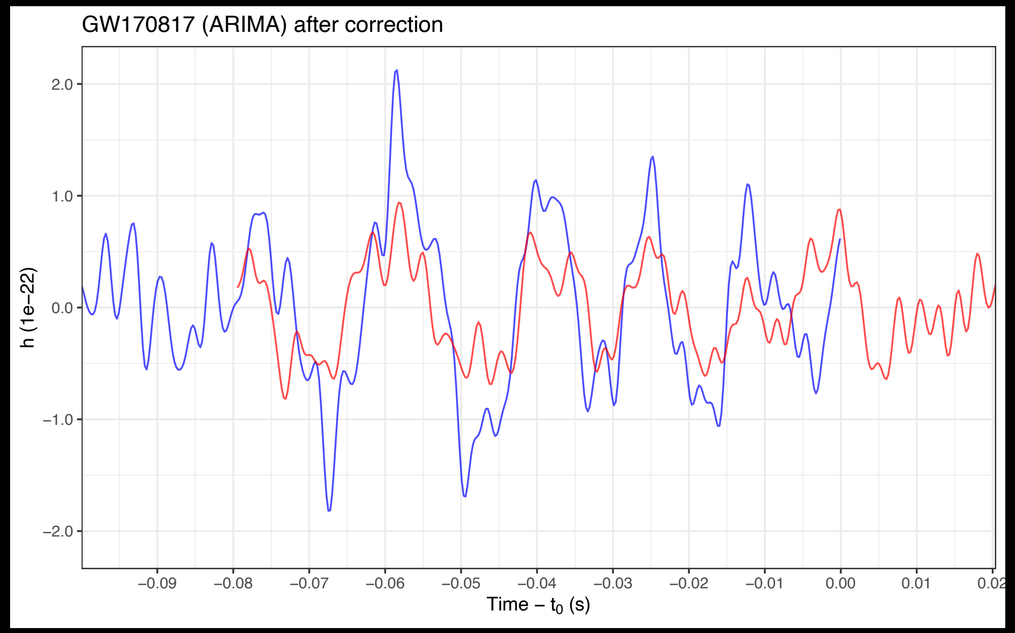


Window = (−0.1, 0) s
C = −0.621
Delay = +20.5 ms
First = Livingston

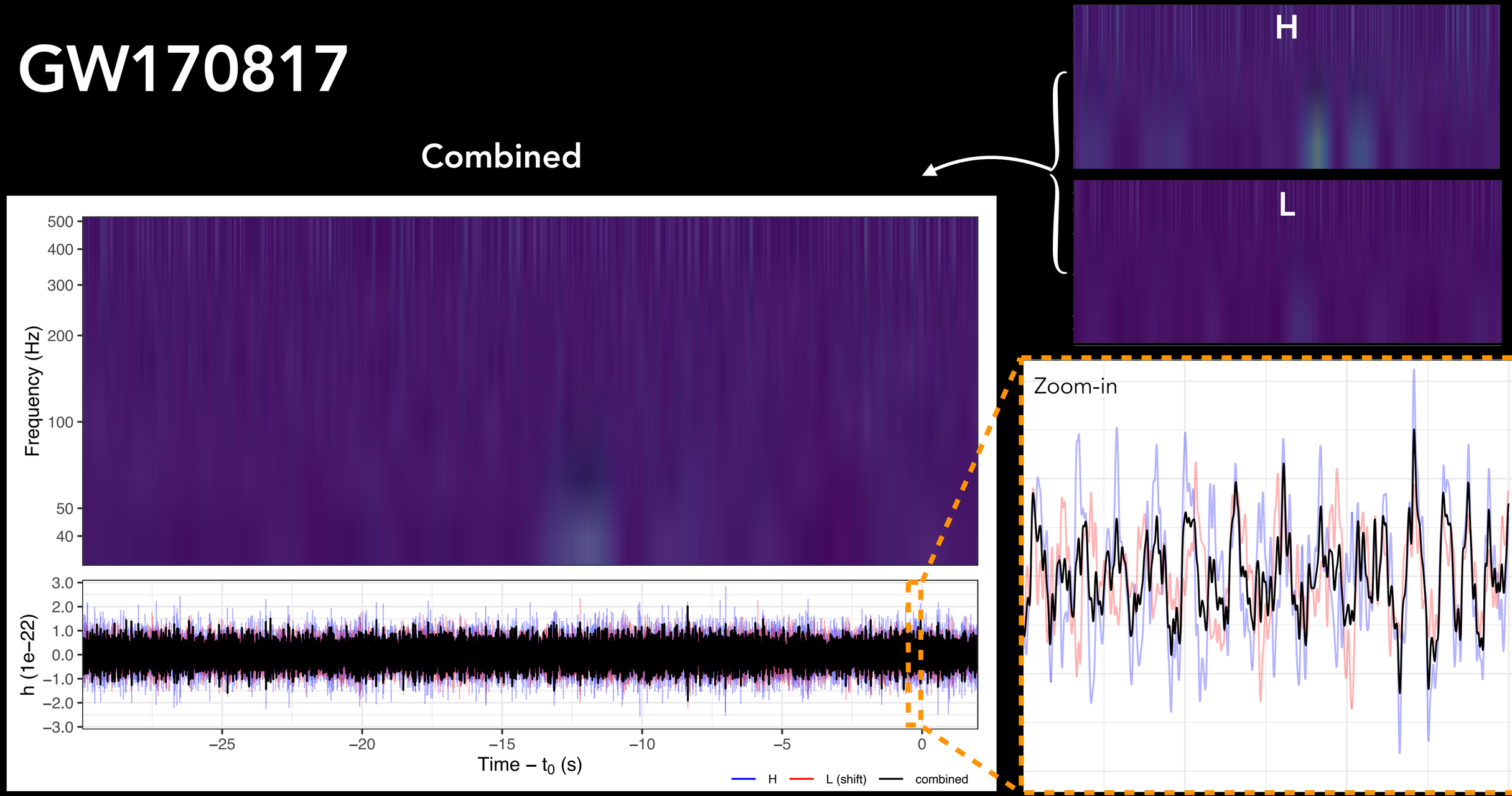
Before



After

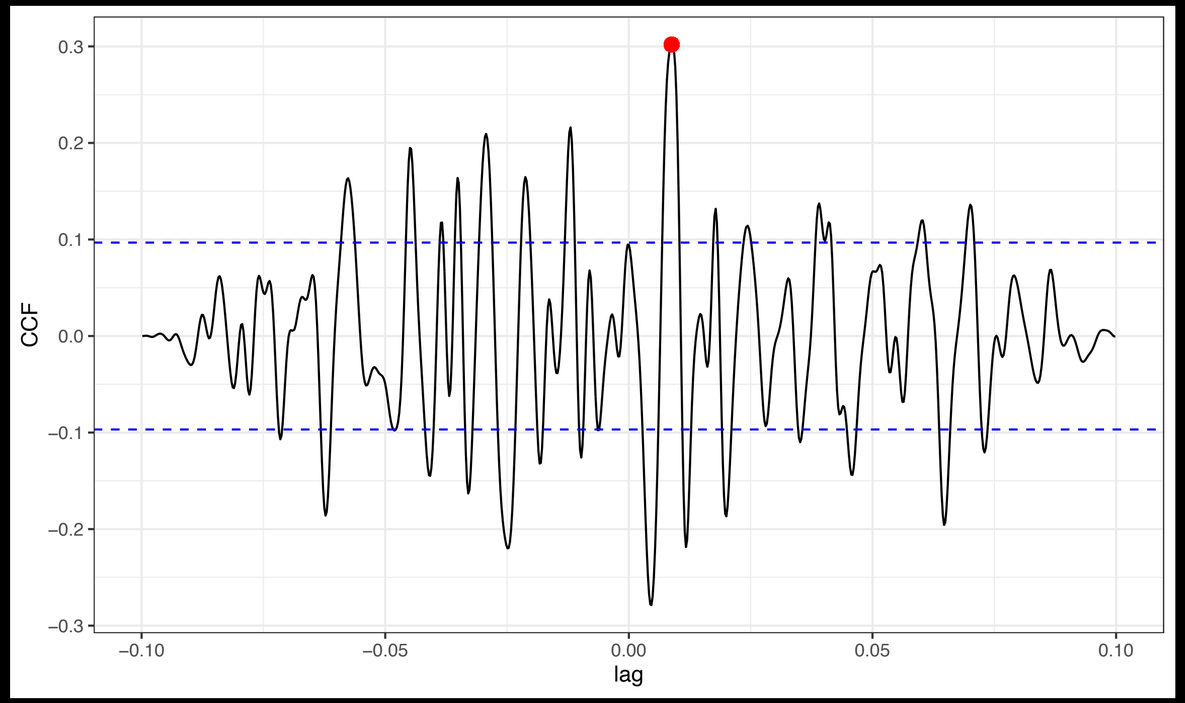


GW170817


Combined
H
L
Zoom-in
Frequency (Hz)
500
400
300
200
100
50
40
h (1e−22)
3.0
2.0
1.0
0.0
−1.0
−2.0
−3.0
−25
−20
−15
−10
−5
0
Time − t_0 (s)
H
L (shift)
combined

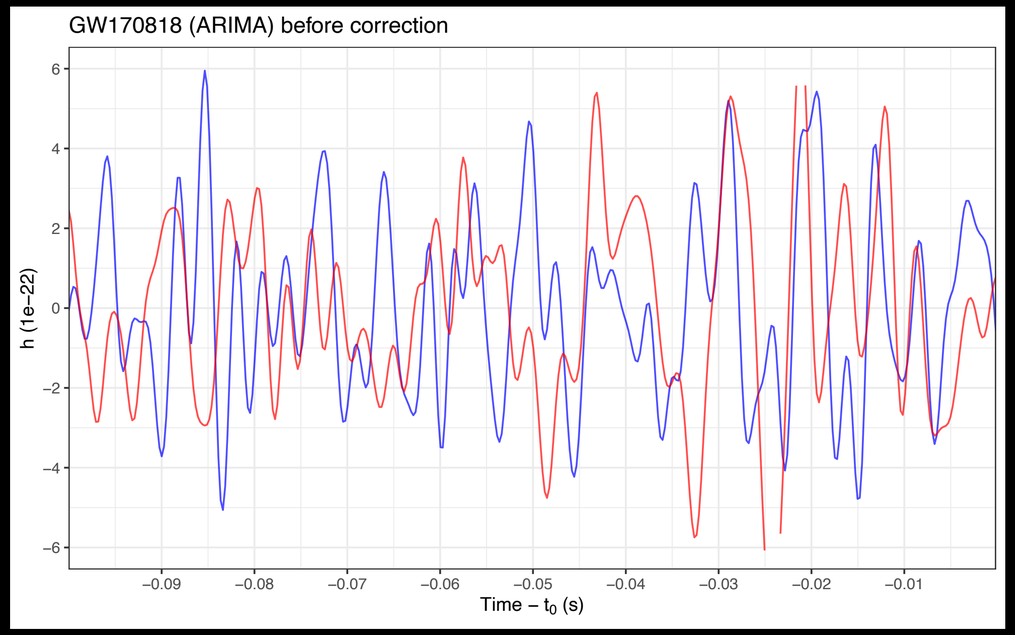
GW170818

CCF

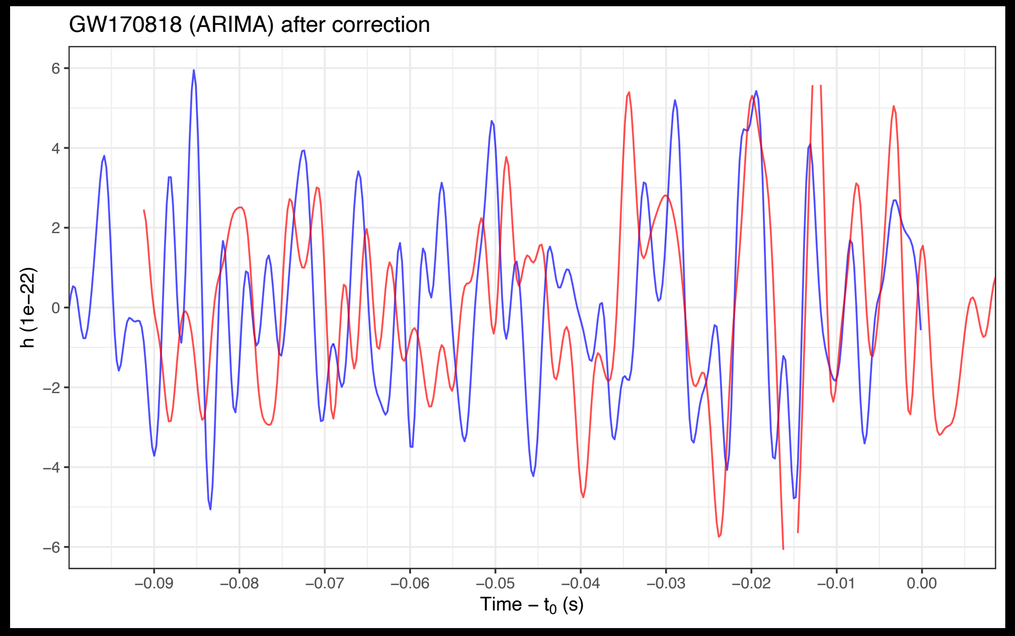


Window = ± 0.1 s
C = 0.302
Delay = +8.79 ms
First = Livingston

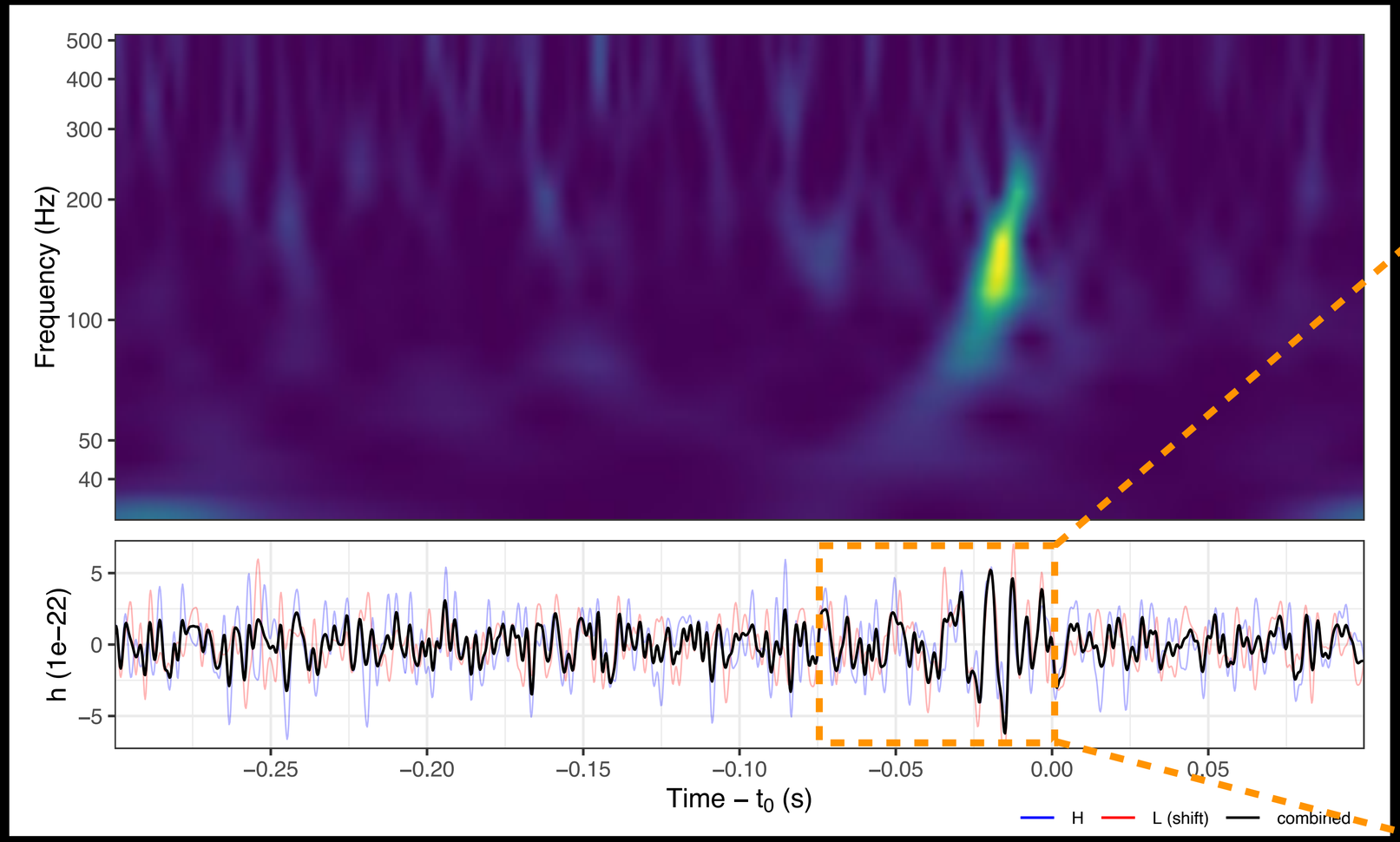
Before

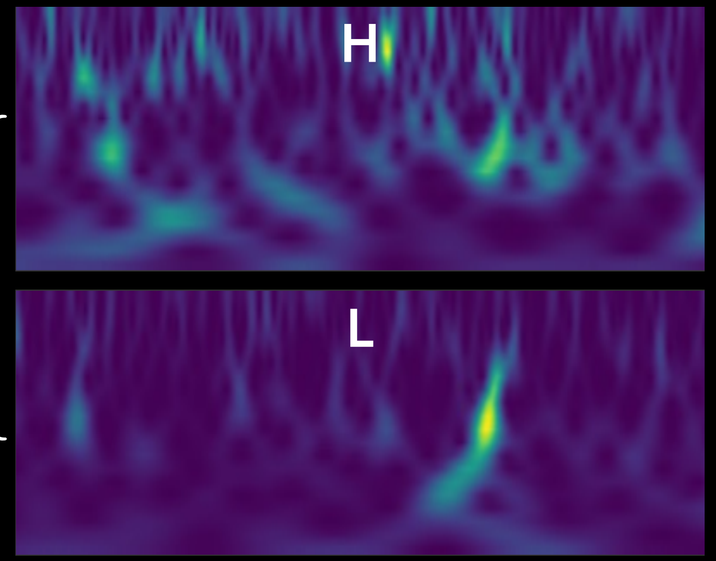


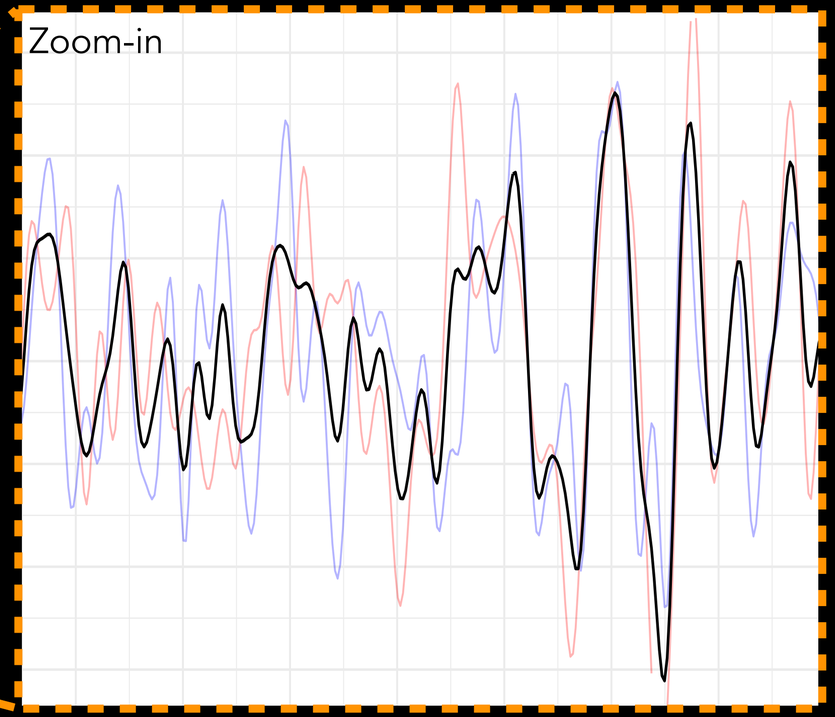
After



GW170818

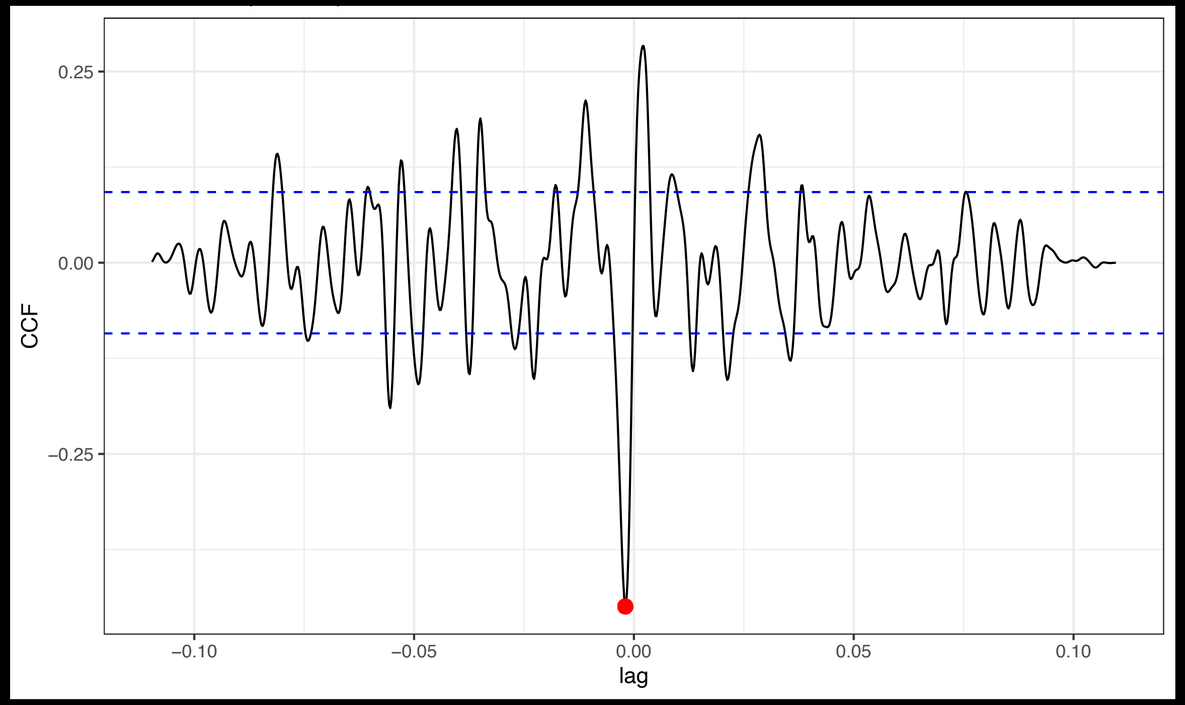

Combined
500
400
300
200
100
50
40
Frequency (Hz)
5
0
−5
h (1e−22)
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t0 (s)
H
L (shift)
combined


H
L


Zoom-in

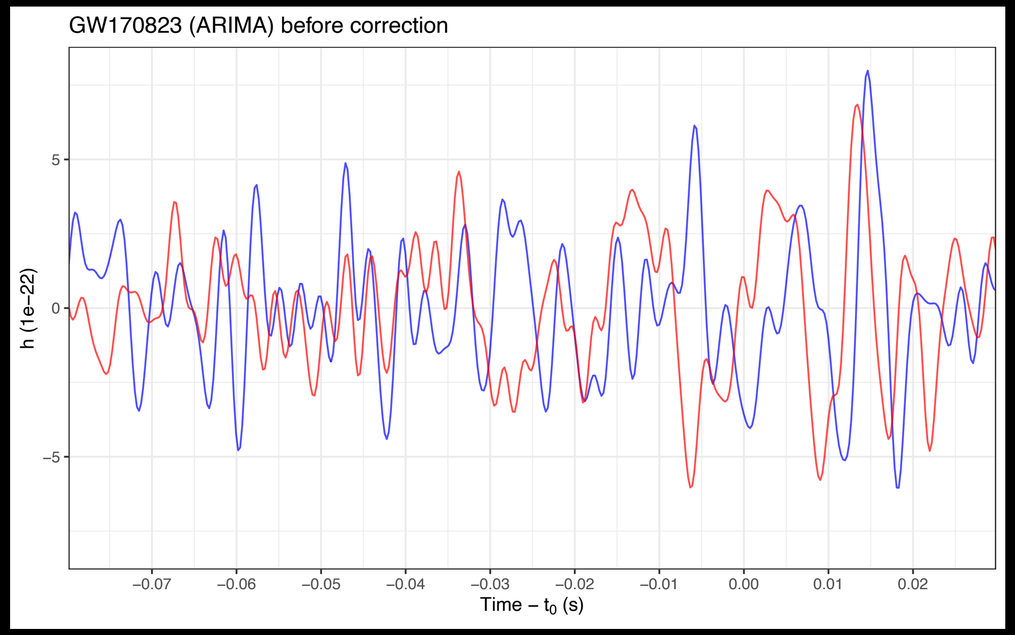
GW170823

CCF

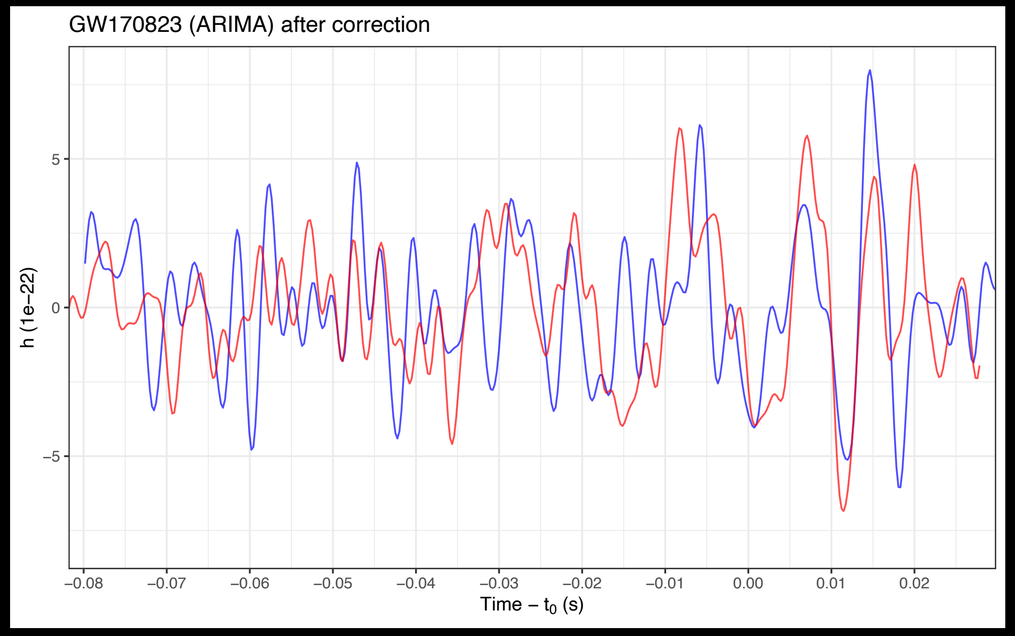


Window = (−0.08, 0.03) s
C = −0.499
Delay = −1.95 ms
First = Hanford

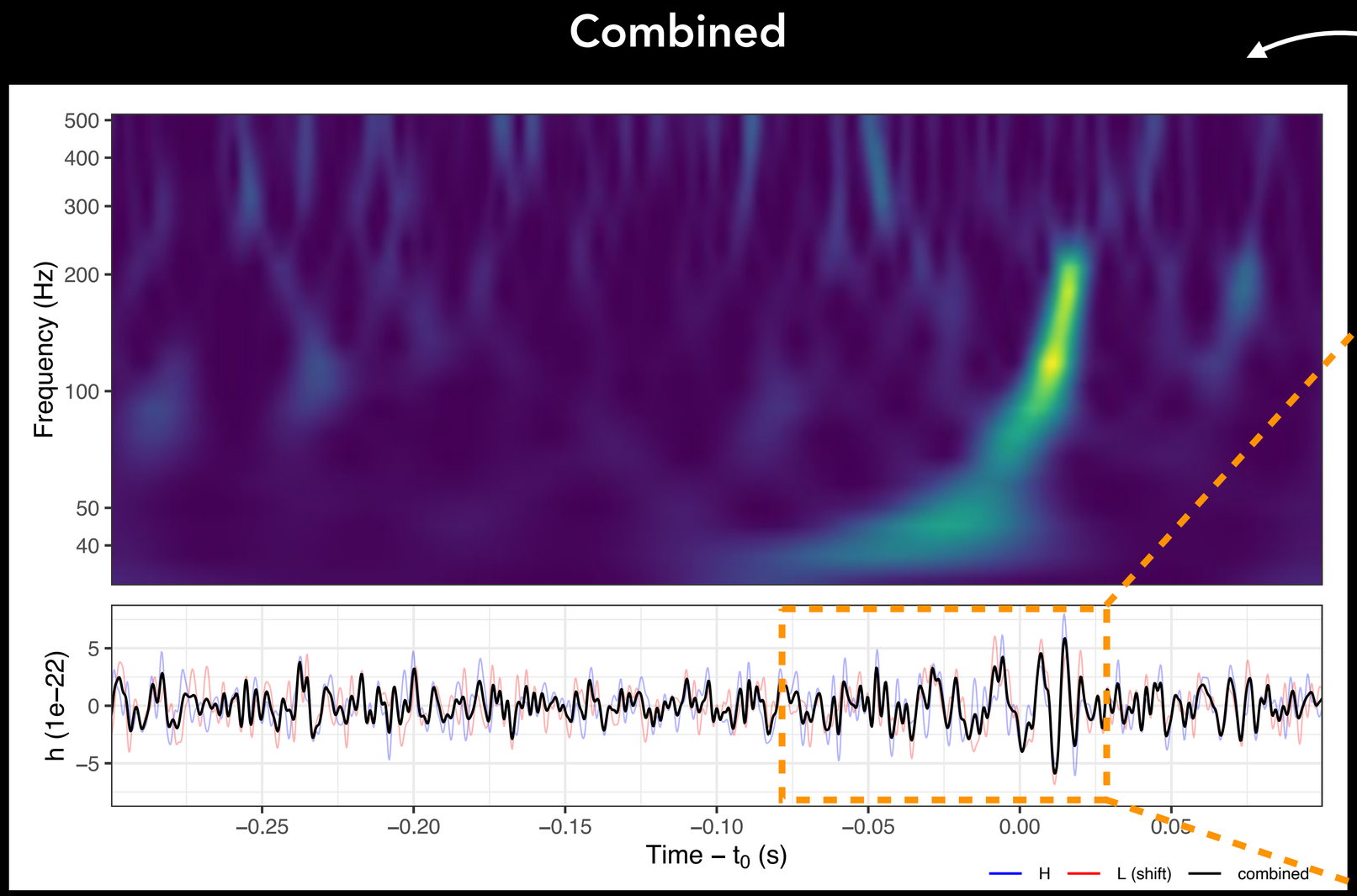
Before

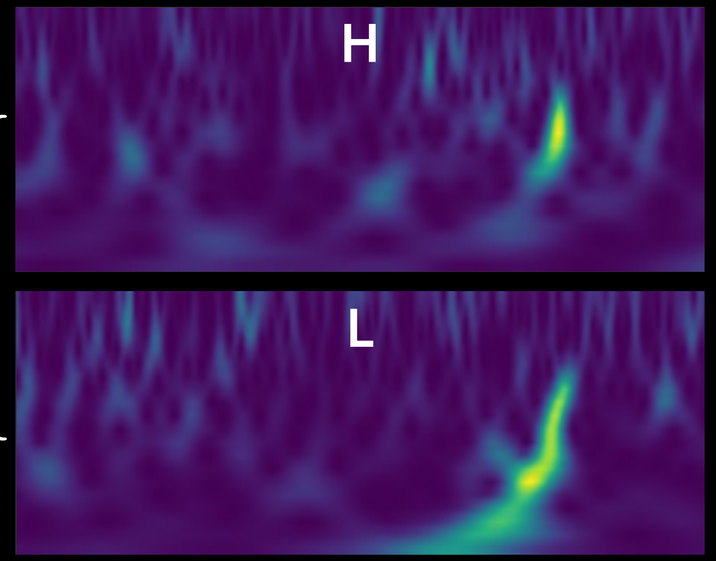


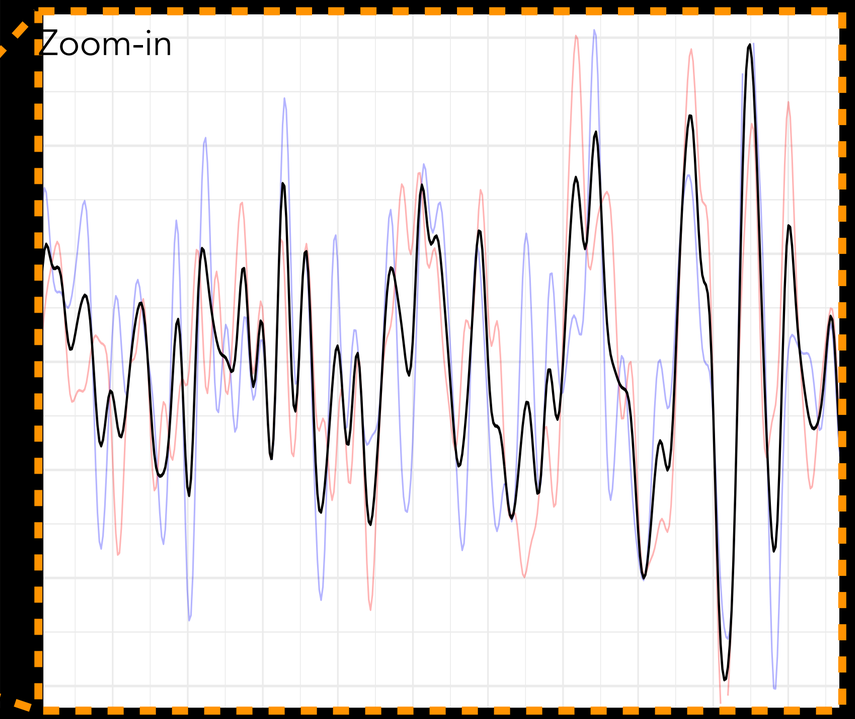
After



GW170823


Combined
500
400
300
200
100
50
40
Frequency (Hz)
5
0
−5
h (1e−22)
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
Time − t0 (s)
H
L (shift)
combined


H
L

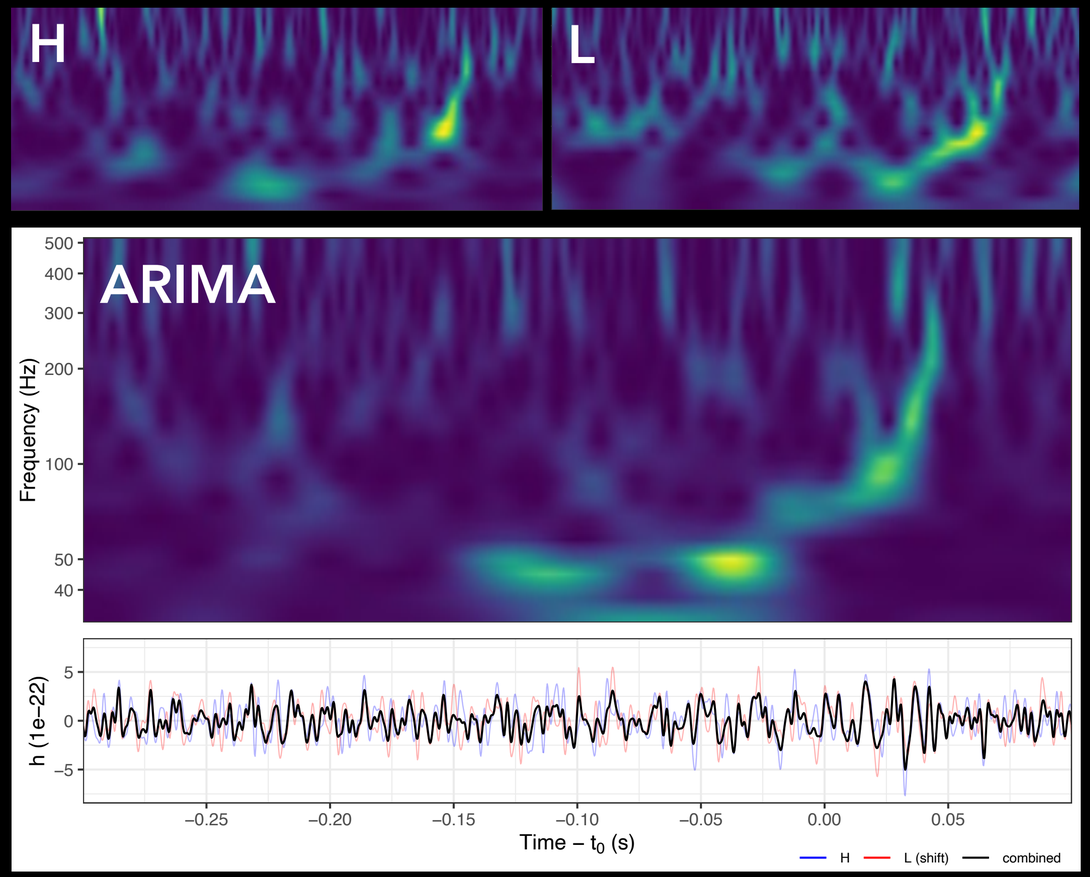

Zoom-in

4) Cross-correlation comparison

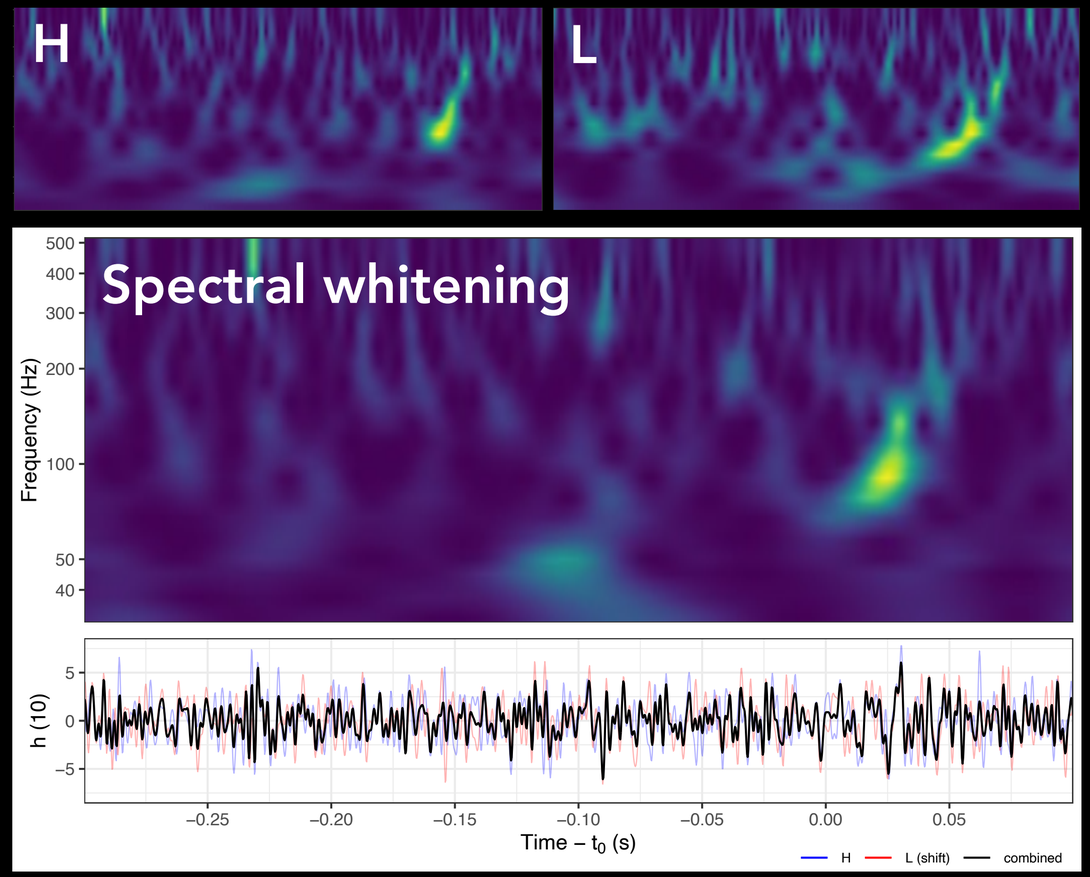
Cross-correlation comparison table

No	source name		C_{whiten}	τ_{whiten} (ms)	C_{ARIMA}	τ_{ARIMA} (ms)	τ_{Akhshi} (ms)	τ_{LIGO} (ms)
1	GW150914	strongest	0.716	7.32 L-first	0.816	7.57 L-first	$7.3^{+0.3}_{-0.5}$ L-first	$6.9^{+0.5}_{-0.4}$ L-first
2	GW151012	weakest	0.356	11.0 L-first	0.430	0.24 **H-first**	$0.5^{+0.5}_{-0.3}$ L-first	n/a
3	GW151226		0.279	1.22 L-first	0.334	1.46 L-first	$1.2^{+0.7}_{-0.5}$ L-first	$1.1^{+0.3}_{-0.3}$ L-first
4	GW170104		0.525	3.17 H-first	0.628	3.42 H-first	$3.2^{+0.5}_{-0.2}$ H-first	$3.0^{+0.4}_{-0.5}$ H-first
5	GW170608		0.398	6.35 H-first	0.464	6.84 H-first	$6.8^{+0.2}_{-0.5}$ H-first	$7\,^{n/a}_{n/a}$ H-first
6	GW170729		0.431	1.46 L-first	0.469	0.98 L-first	$1.8^{+1.0}_{-0.9}$ L-first	n/a
7	GW170809		0.423	9.28 L-first	0.510	9.03 L-first	$9.5^{+0.5}_{-0.5}$ L-first	n/a
8	GW170814		0.486	7.57 L-first	0.542	7.32 L-first	$7.8^{+0.8}_{-0.5}$ L-first	$8\,^{n/a}_{n/a}$ L-first
9	GW170817	BNS	0.335	9.28 L-first	0.621	20.5 L-first	n/a	n/a
10	GW170818		0.188	64.2 H-first	0.302	3.42 H-first	$4.8^{+0.5}_{-0.8}$ L-first	n/a
11	GW170823		0.311	0.98 H-first	0.499	3.42 H-first	$1.5^{+0.7}_{-0.5}$ H-first	n/a

For weakest GW151012,



Combined **ARIMA** cleaned data
with
ARIMA lag = 0.244 ms (C = -0.43)
(**Hanford** first)



Combined **spectral whitened** data
with
Spectral whitening lag = 11 ms (C = -0.356)
(**Livingston** first)

Summary & Future Works

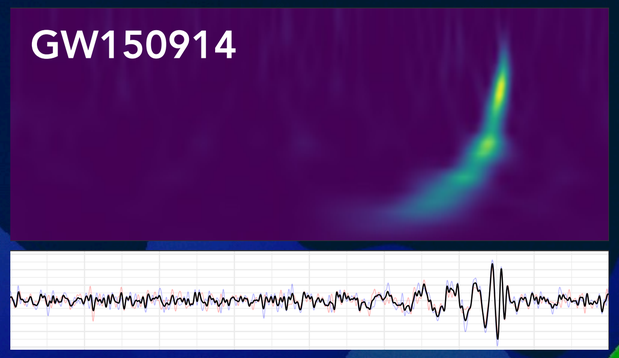
Summary

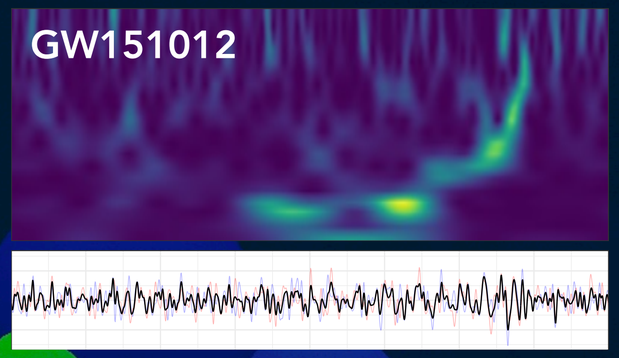
- **ARIMA model** is adopted for noise reduction process
- We have achieved higher CC coefficients than spectral whitening.
- Time delay and combined signals were suggested by CC
- ARIMA cleaned spectrogram and H+L combined spectrogram

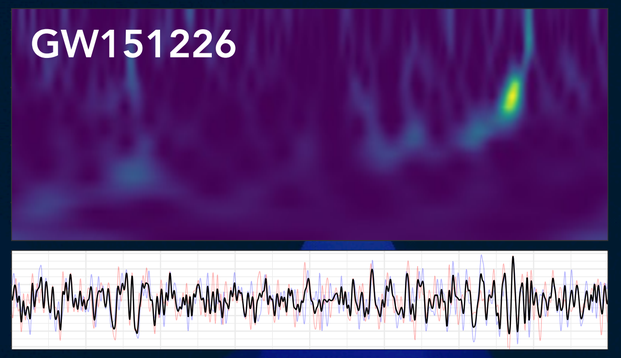
Future Works

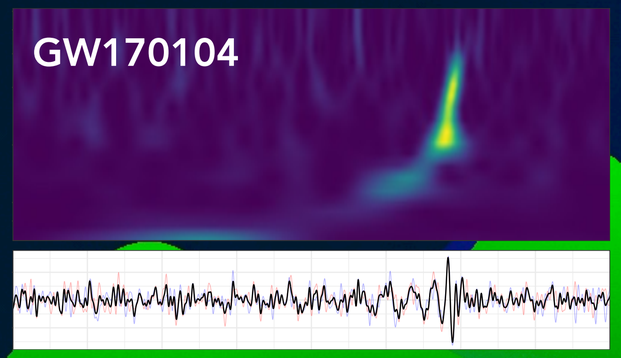
1. Optimal metric for noise reduction performance
2. Automatic parameter search for q & d
3. Additional filters
4. Low-latency pipeline construction
5. Low-uncertainty parameter estimation
6. Noise reduction in CCSN signal

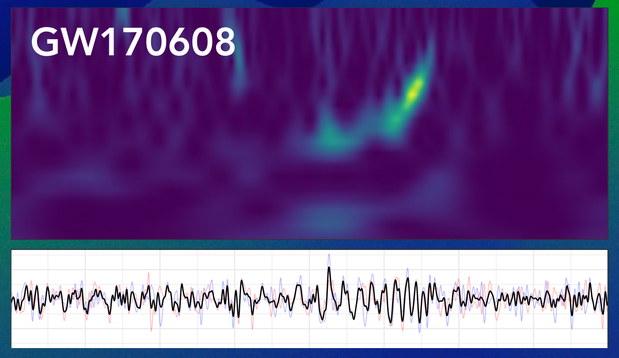
Recovered Gravitational-Wave Transient Catalog-1

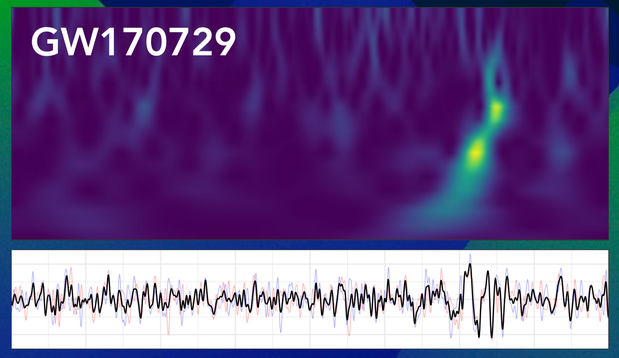


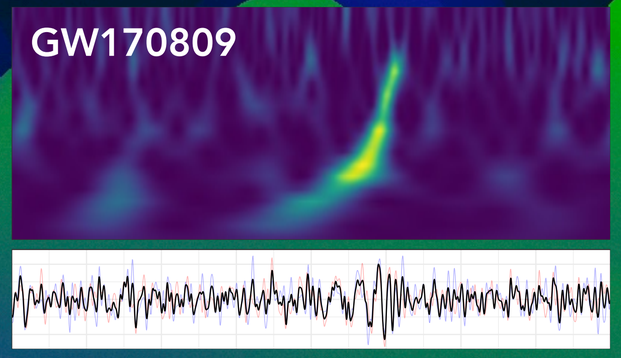


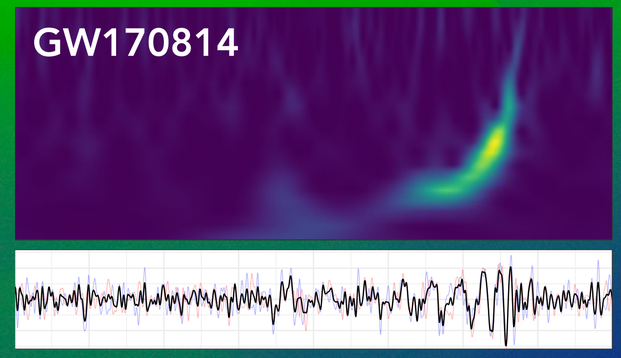


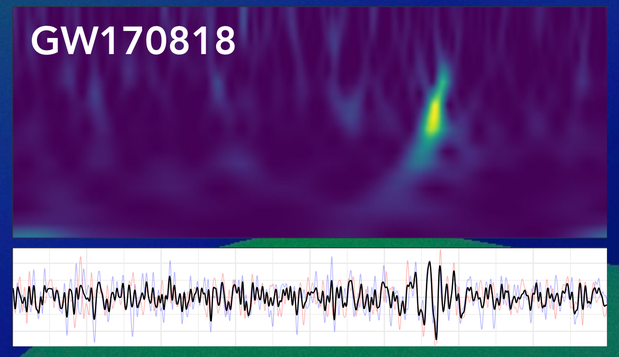


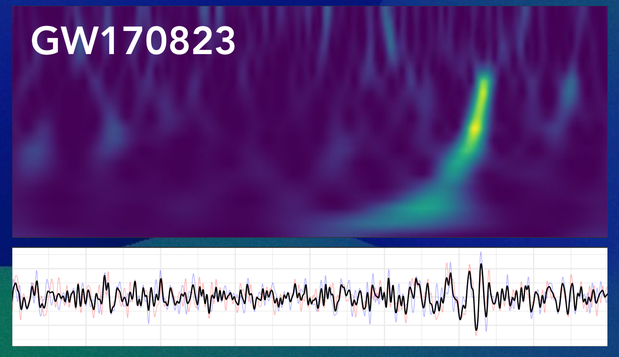


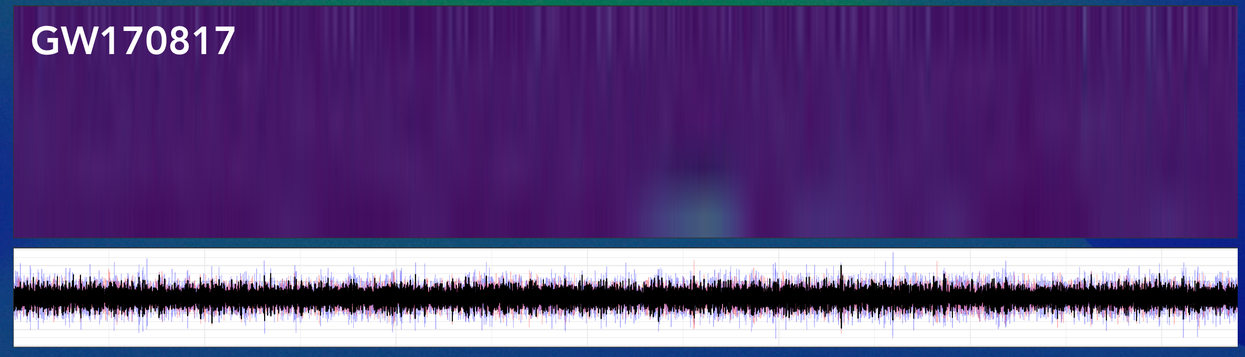






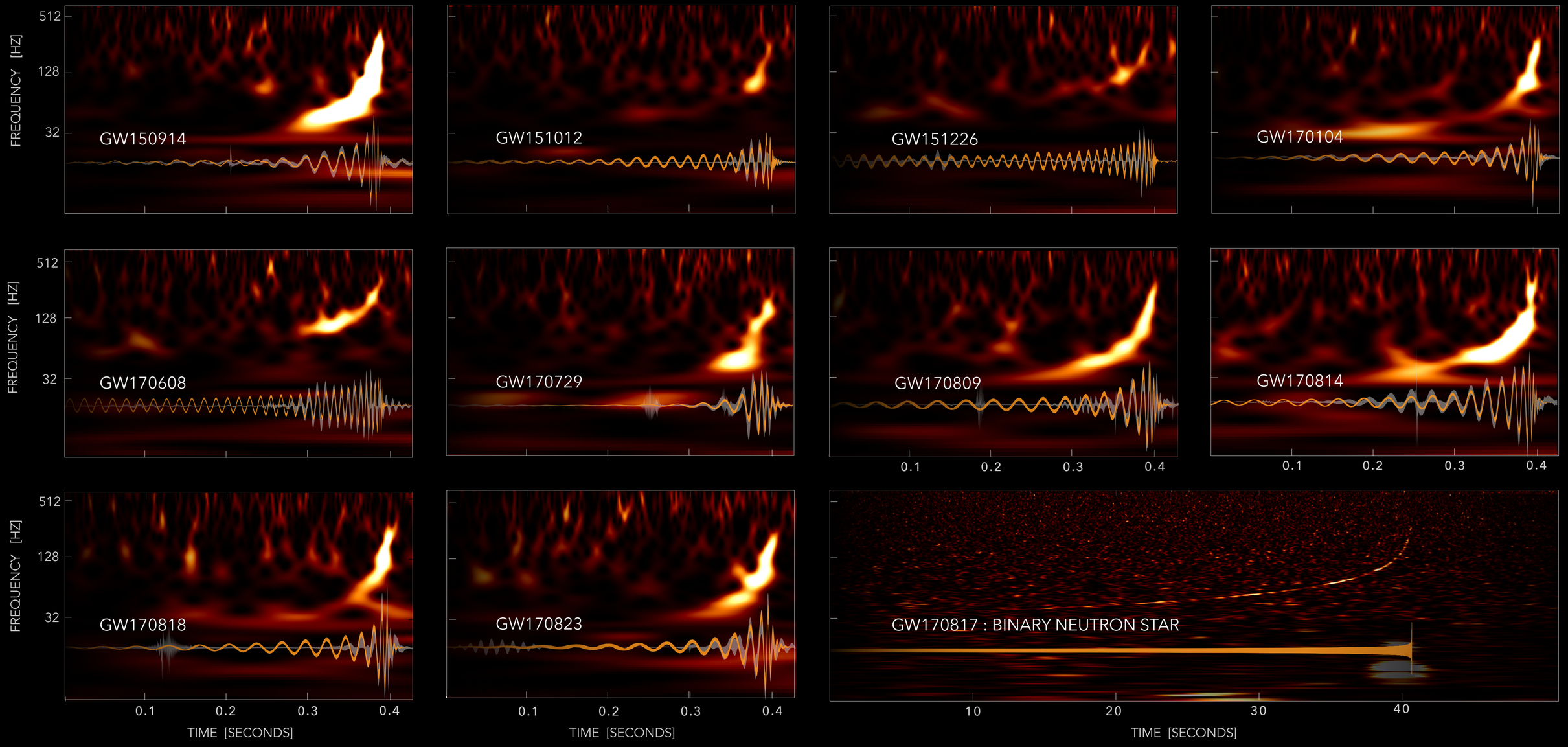






Thank you for your attention!

GRAVITATIONAL-WAVE TRANSIENT CATALOG-1



LIGO-VIRGO DATA: HTTPS://DOI.ORG/10.7935/82H3-HH23

WAVELET (UNMODELED) EINSTEIN'S THEORY

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