2023.03.15.The 68th Workshop on Gravitational Waves and Numerical Relativity

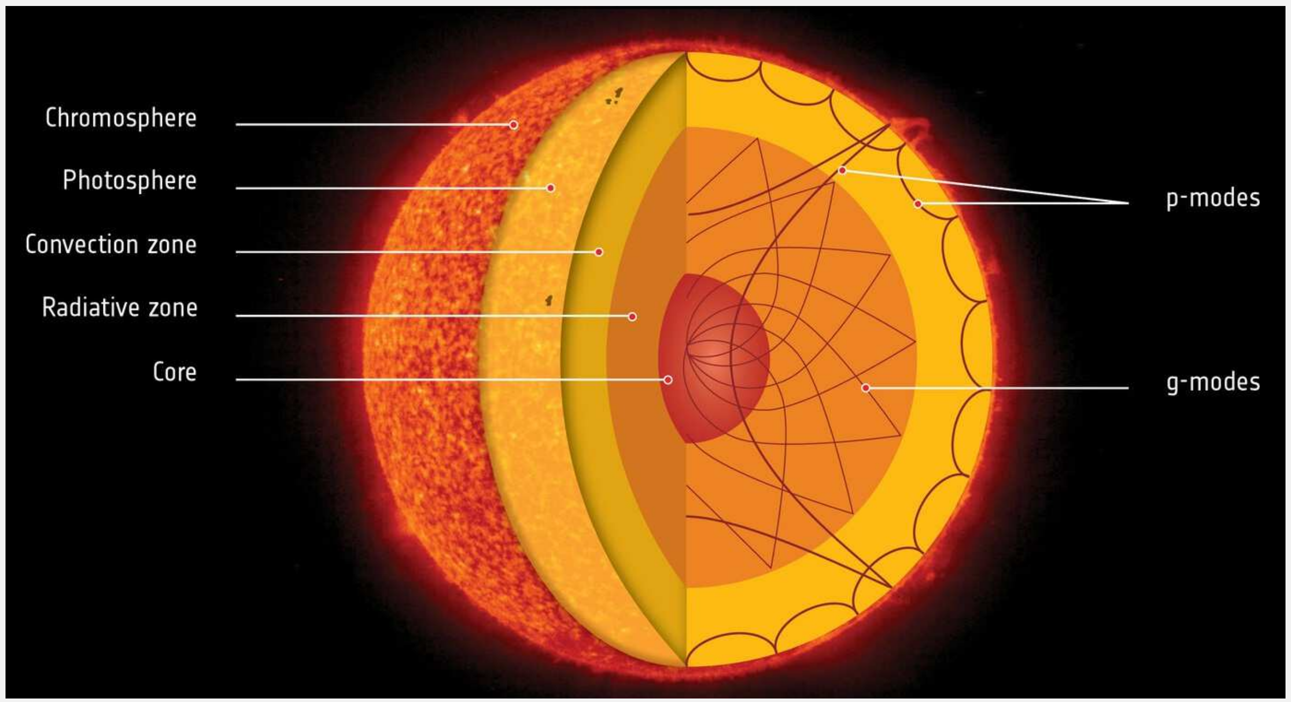
Influence of cluster properties on the oscillation parameters

Yun-A Jo
Kyungpook National University

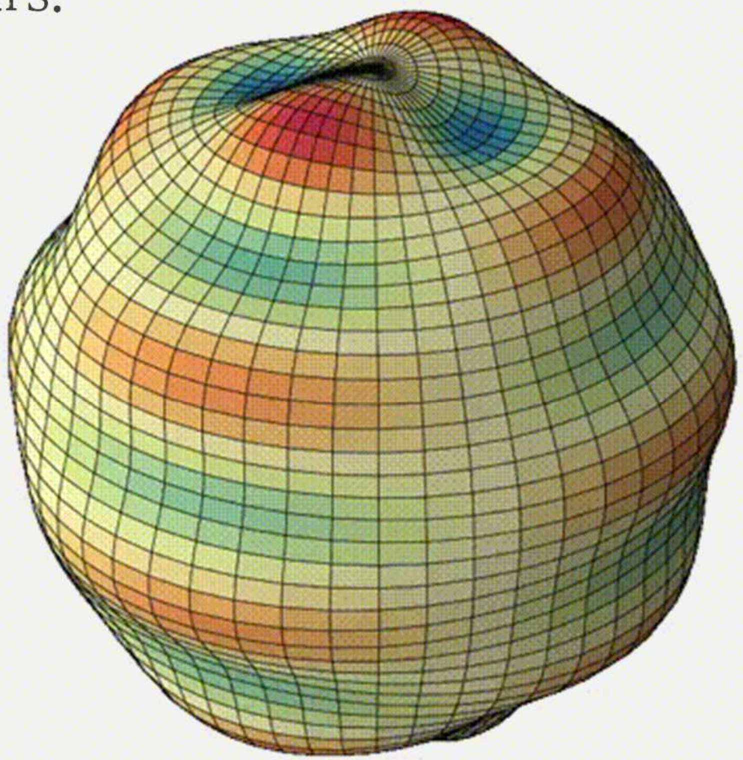
1. Introduction

Asteroseismology

A study of the stellar interiors using the oscillations of stars.



Credit : SOHO (ESA & NASA).

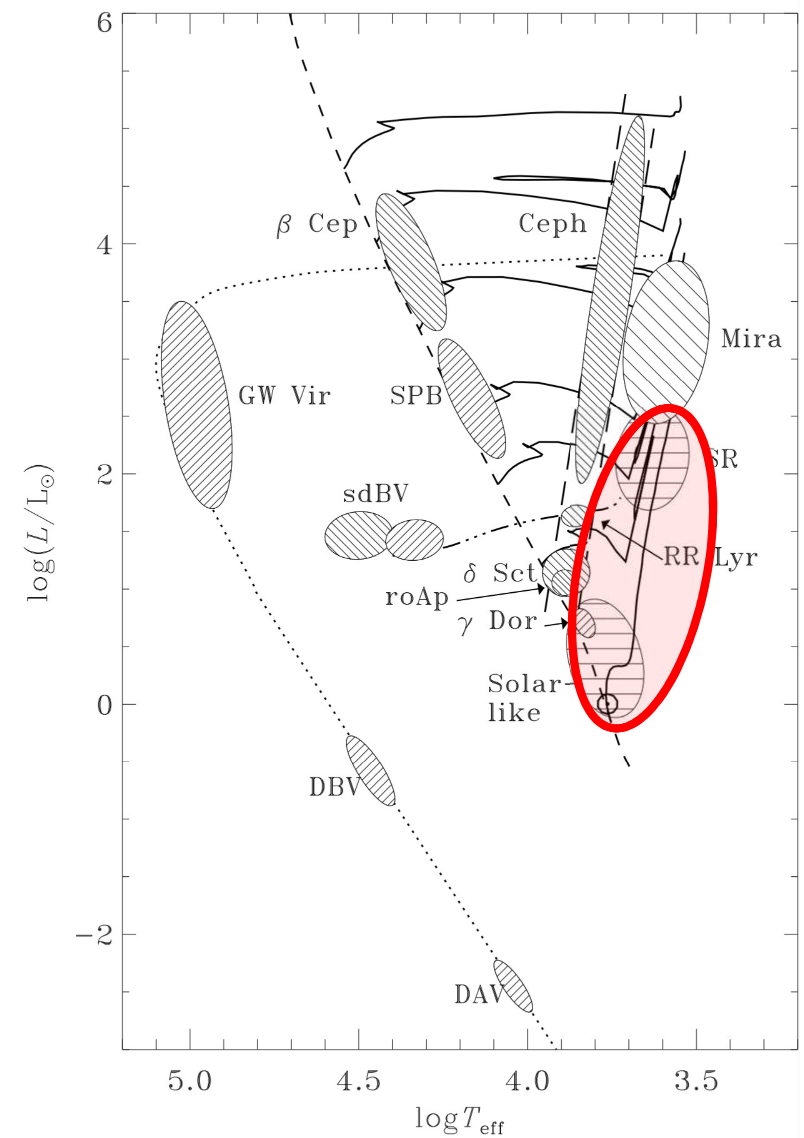


Credit : https://www.thomassteindl.com/physics/

1. Introduction

Pulsating stars

- classical pulsators
  - >> Ceph, RR Lyrae, Mira, etc.
  - >> κ-mechanism (opacity mechanism)
  - >> only a few radial mode
- solar-like star
  - >> MS, SG, RG
  - >> stochastic driving by convective motions
  - >> large number of radial & non-radial modes



The location of pulsating stars in the HR diagram.
(Christensen-Dalsgaard 2008)

1. Introduction

Types of sound waves

	p-mode	g-mode
Frequency	higher (short period)	lower (long period)
Restore force	Pressure	Buoyancy
Propagation region	Outer (convection zone)	Inner (radiative zone)
Asymptotic behavior	Equally spaced in frequency	Equally spaced in period
Examples	Main-sequence stars, giant stars	Neutron star, white dwarfs

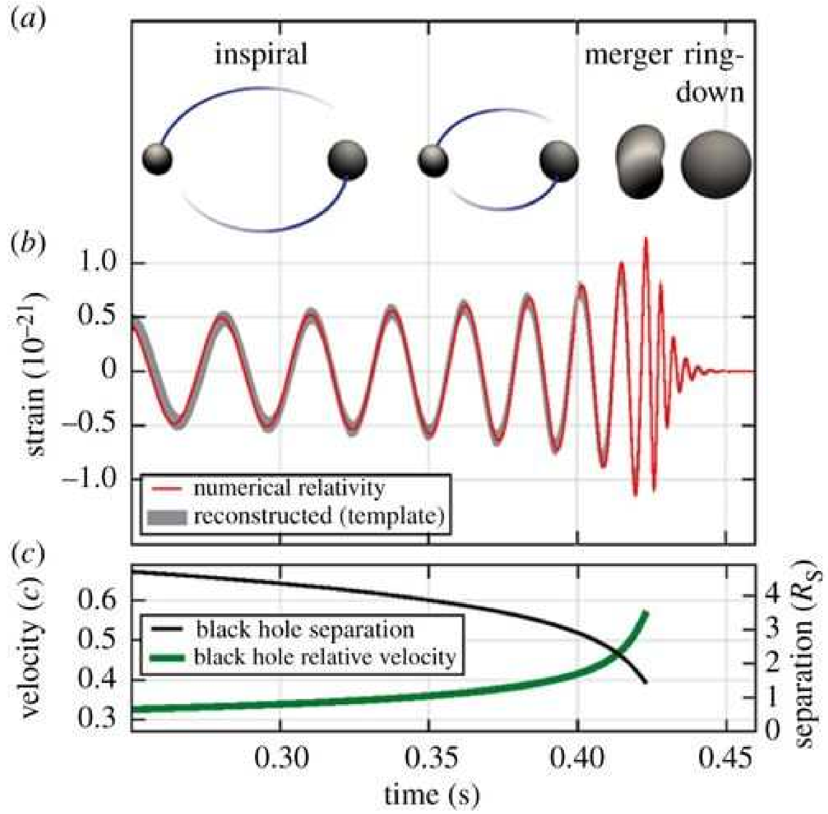
1. Introduction

Neutron stars

- type of oscillations >> f-mode (~ 1 kHz)
  >> p-mode (> 1 kHz)
  >> g-mode (< a few hundred Hz)
  >> r-mode (a few hundred Hz)

- oscillation mode excitation >> stellar rotation
  >> binary coalescence
  >> accretion disk

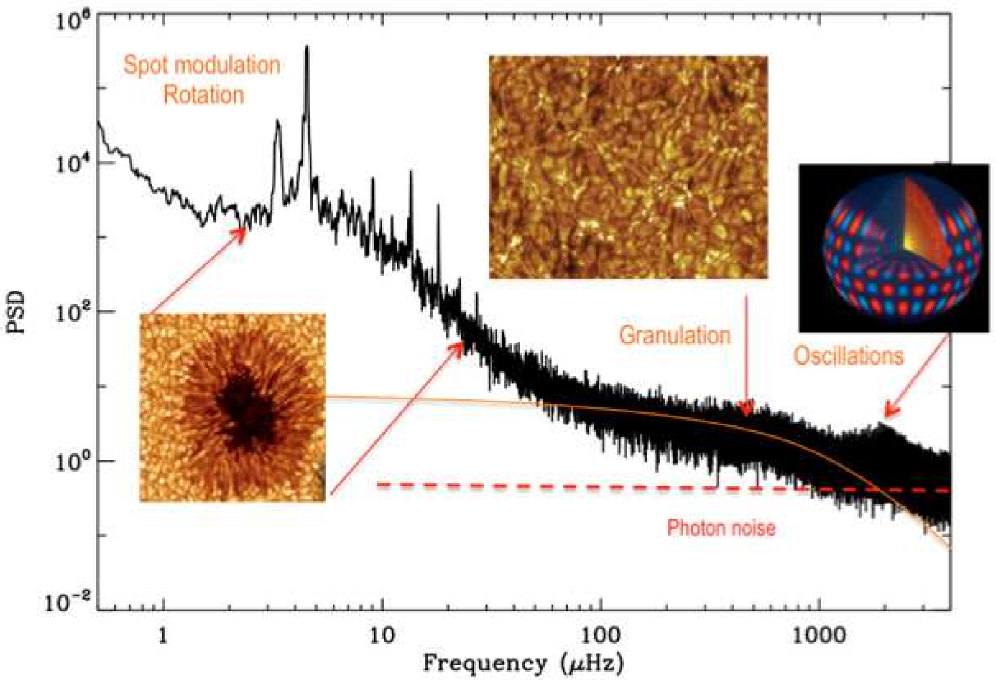
- observed gravitational wave frequency >> a few tens Hz
  ~ several hundreds Hz



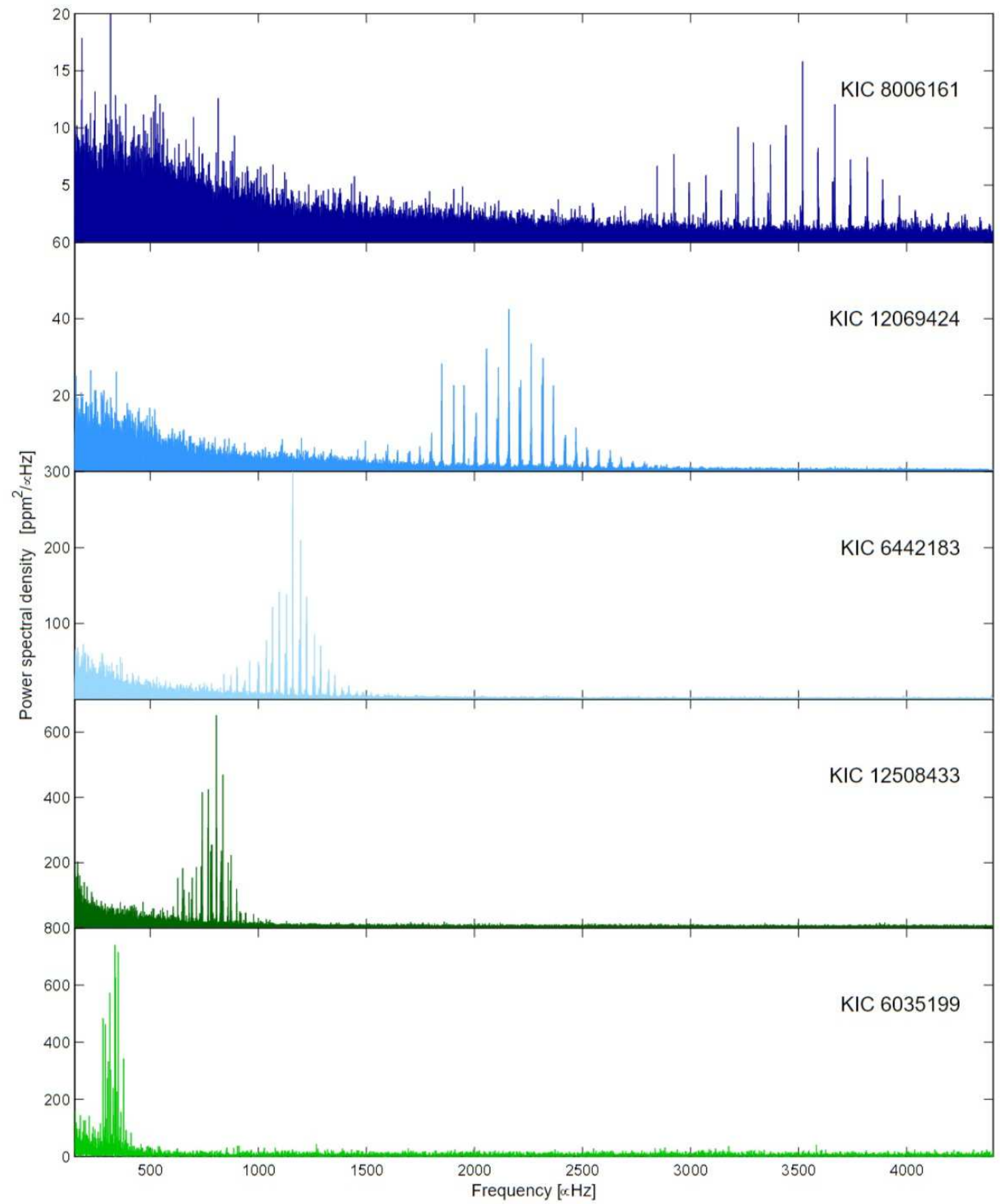
Estimated gravitational wave strain amplitude from GW150914. (Abbott et al. 2016)

1. Introduction

Power spectrum



Power spectrum of KIC 3733735. (Garcia 2015)



main-sequence star

main-sequence star

sub-giant star

sub-giant star

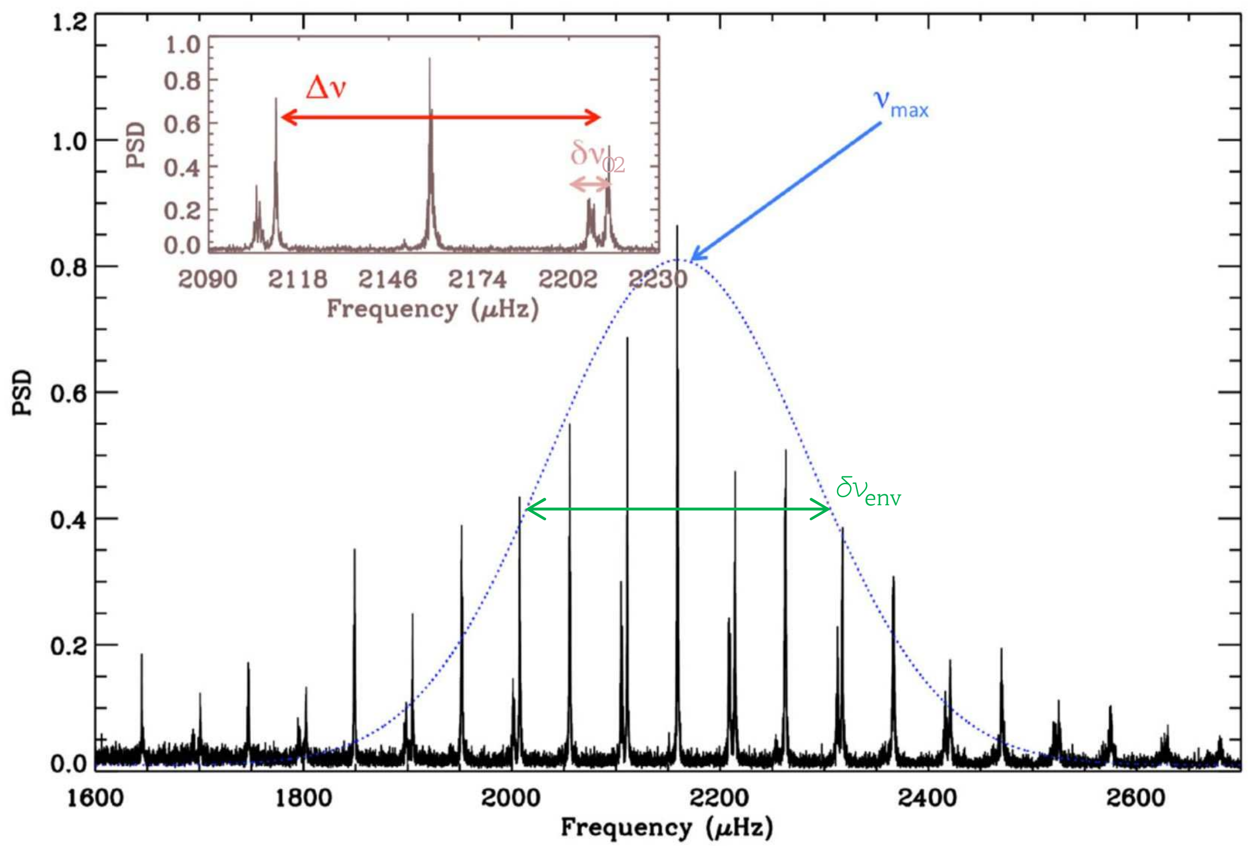
red-giant star

Power spectra of five stars. (Chaplin and Miglio 2013)

1. Introduction

Asteroseismic parameters

- ν_{max} >> The frequency at maximum power
- $\Delta\nu$ >> The large frequency separation (same l, different n)
- $\delta\nu_{02}$ >> The small frequency separation (same n, different l)
- $\delta\nu_{env}$ >> The full-width at half-maximum of the mode envelope



Power spectrum of 16 Cyg A. (Garcia 2015)

1. Introduction

Asteroseismic parameters

"ν_{max} would scale with the acoustic cutoff frequency and hence be related to stellar fundamental properties …"
(Brown et al. 1991; Kjeldsen&Bedding 1995)

→ $\nu_{max} \propto \nu_c \propto g T_{eff}^{-1/2} \propto M R^{-2} T_{eff}^{-1/2}$

→ $\frac{\nu_{max}}{\nu_{max,\odot}} \simeq \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-2}\left(\frac{T_{eff}}{T_{eff,\odot}}\right)^{-1/2}$

"$\Delta\nu$, which probes the sound speed profile and is proportional to the square root of the mean stellar density …"
(Ulrich 1986)

→ $\Delta\nu = \left[2\int_0^R \frac{dr}{c}\right]^{-1}$

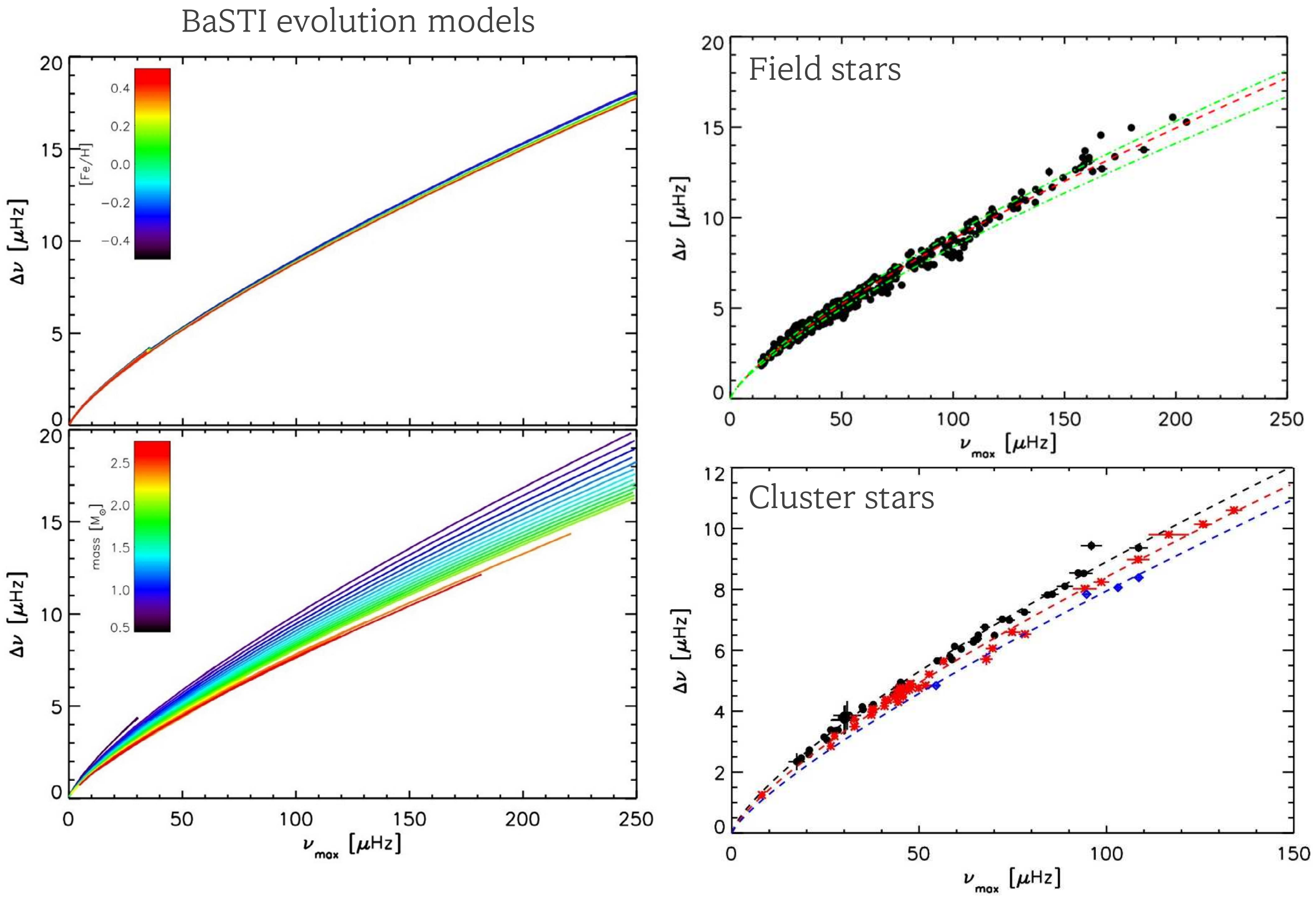
→ $\frac{\Delta\nu}{\Delta\nu} \simeq \left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{R}{R_\odot}\right)^{-3/2}$

1. Introduction

Scaling relations

$$\frac{M}{M_\odot} \simeq \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right)^3 \left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-4} \left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{3/2}$$

$$\frac{R}{R_\odot} \simeq \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right) \left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-2} \left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{1/2}$$



Mass dependency in $\Delta\nu$ vs. ν_{max} correlations. (Hekker et al. 2011)

1. Introduction

Aims

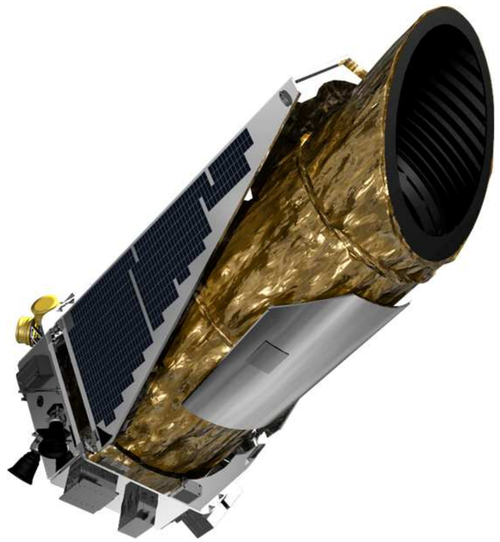
- Properties of global asteroseismic parameters

- Improvement of scaling relations for red giant stars
  → $\nu_{max} - \Delta\nu$
  → $\nu_{max} - \delta\nu_{env}$
  → $\Delta\nu - \delta\nu_{02}$

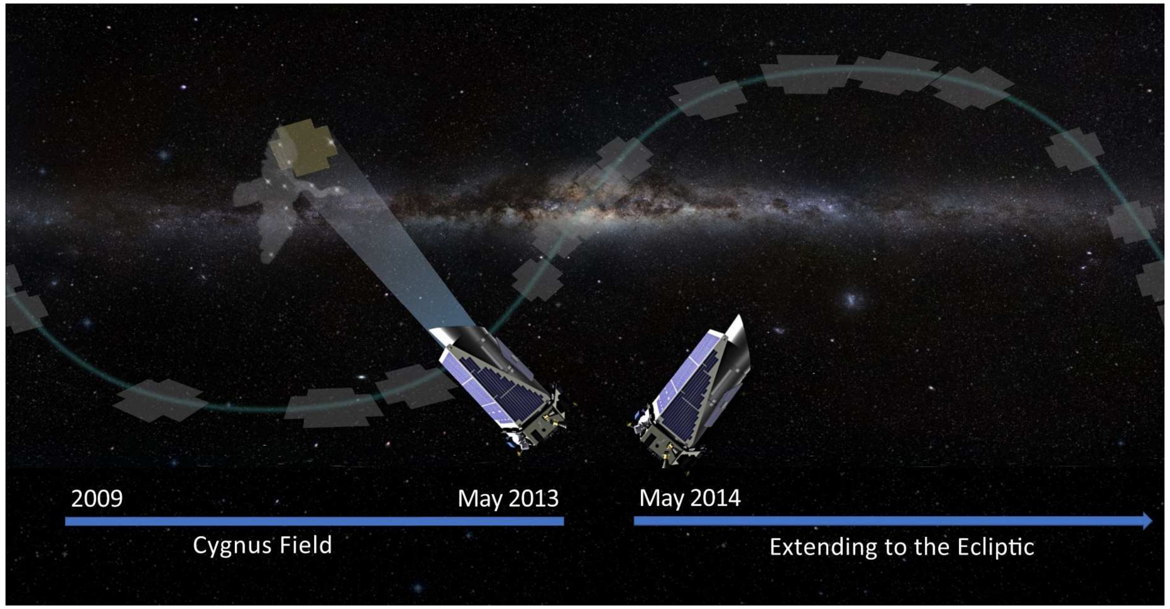
- Influence of cluster properties
  → global asteroseismic parameters vs. cluster properties

2. Data and Data analysis

Kepler/K2 space telescope



- mission purpose >> exploring exoplanets
- mission duration >> *Kepler* : 2009.03. ~ 2013.05.
  K2 : 2014.05. ~ 2018.10.
- wavelength >> 430 – 890 nm
- field of view >> 105 square degrees
  Kepler : Cygnus
  K2 : Ecliptic



Credit : NASA.

2. Data and Data analysis

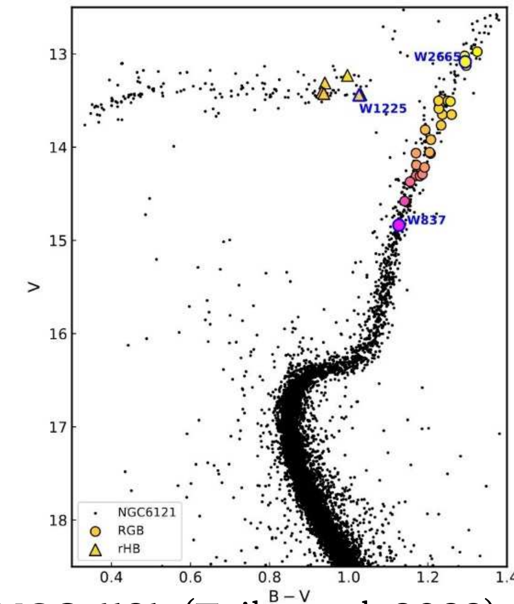
Data sample

- observations >> *Kepler* quarters 0 to 17 (~ 4 years)
  K2 campaigns 5, 16, and 18 (~ 2.5 months)
- clusters >> NGC 1817, 2682, 6791, 6811, 6819 (open)
  NGC 6121 (globular; Tailo et al. 2022)
- sample selection >> $P_{\mathrm{mem}} \geq 70$
  $30\ \mu\mathrm{Hz} < \nu_{\mathrm{max}} < 220\ \mu\mathrm{Hz}$
  (red giants; Aguirre et al. 2020)
- cluster properties >> metallicity, age, mass, *Z* position, radius

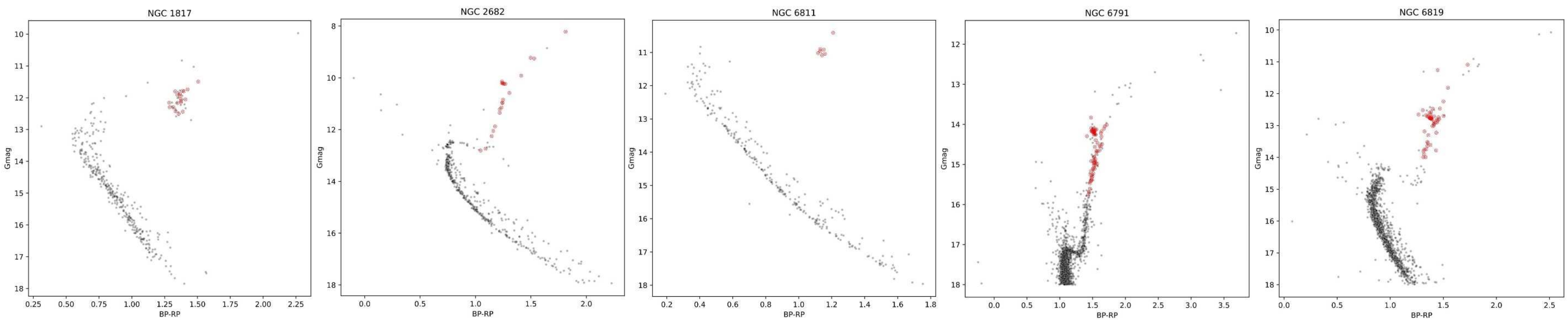
2. Data and Data analysis

Data sample

NGC	[Fe/H]	τ	M	$Z_{\rm pos}$	$R_{\rm core}$	$R_{\rm tidal}$
		log(yr)	log($M_\odot$)	pc	pc	pc
2682	0.072	9.575	3.45	454.3	1.62	17.98
6791	0.399	9.859	4.74	857.3	3.91	29.54
6811	0.032	9.003	3.11	231.3	1.81	6.78
6819	0.093	9.42	4.18	383.4	1.76	13.32
1817	-0.100	9.092	3.41	-388.5	2.42	10.57
6121	-1.4	10.1	4.97	1500	3.70	63.08



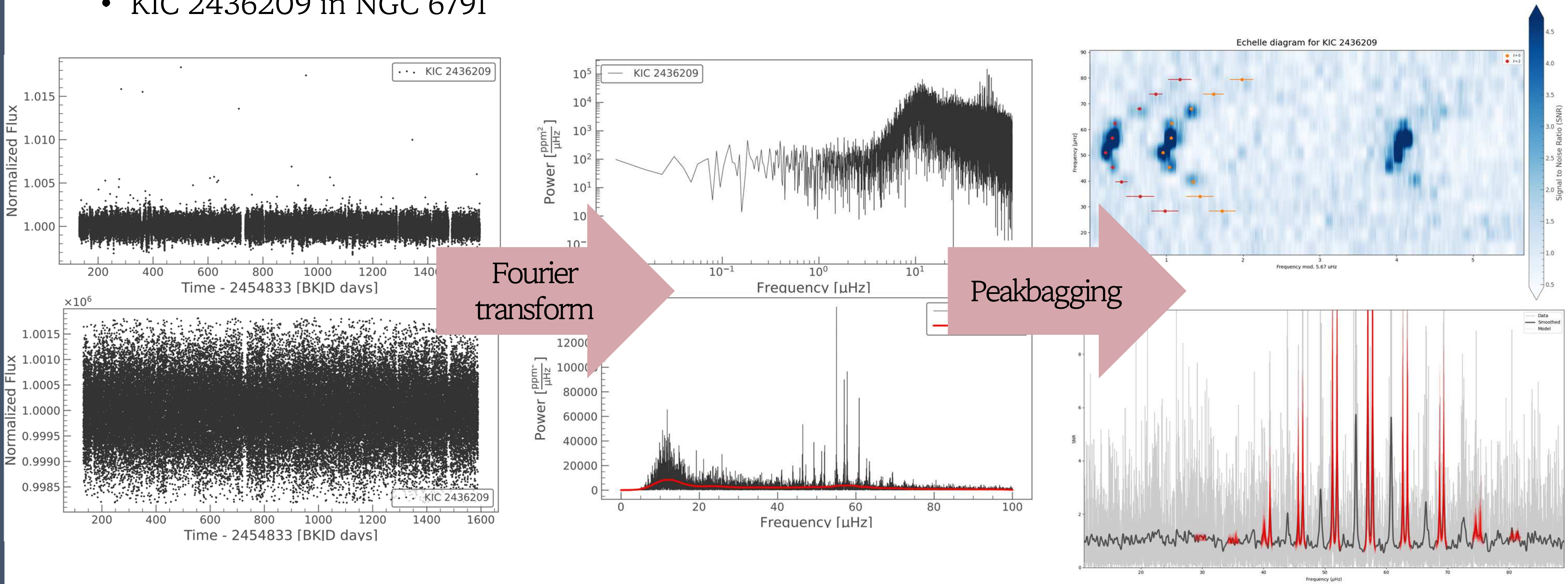
NGC 6121. (Tailo et al. 2022)



2. Data and Data analysis

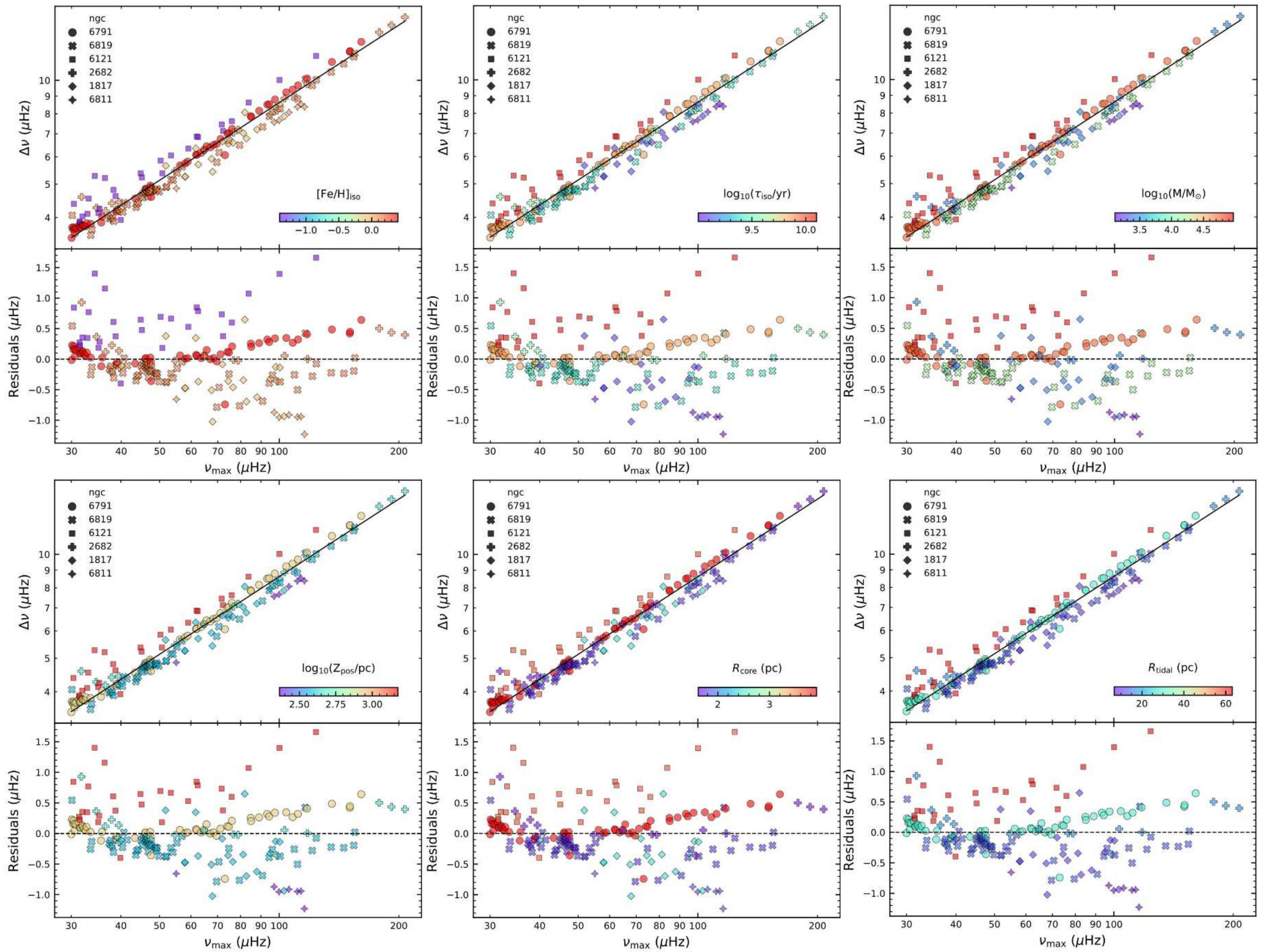
Data reduction

- KIC 2436209 in NGC 6791



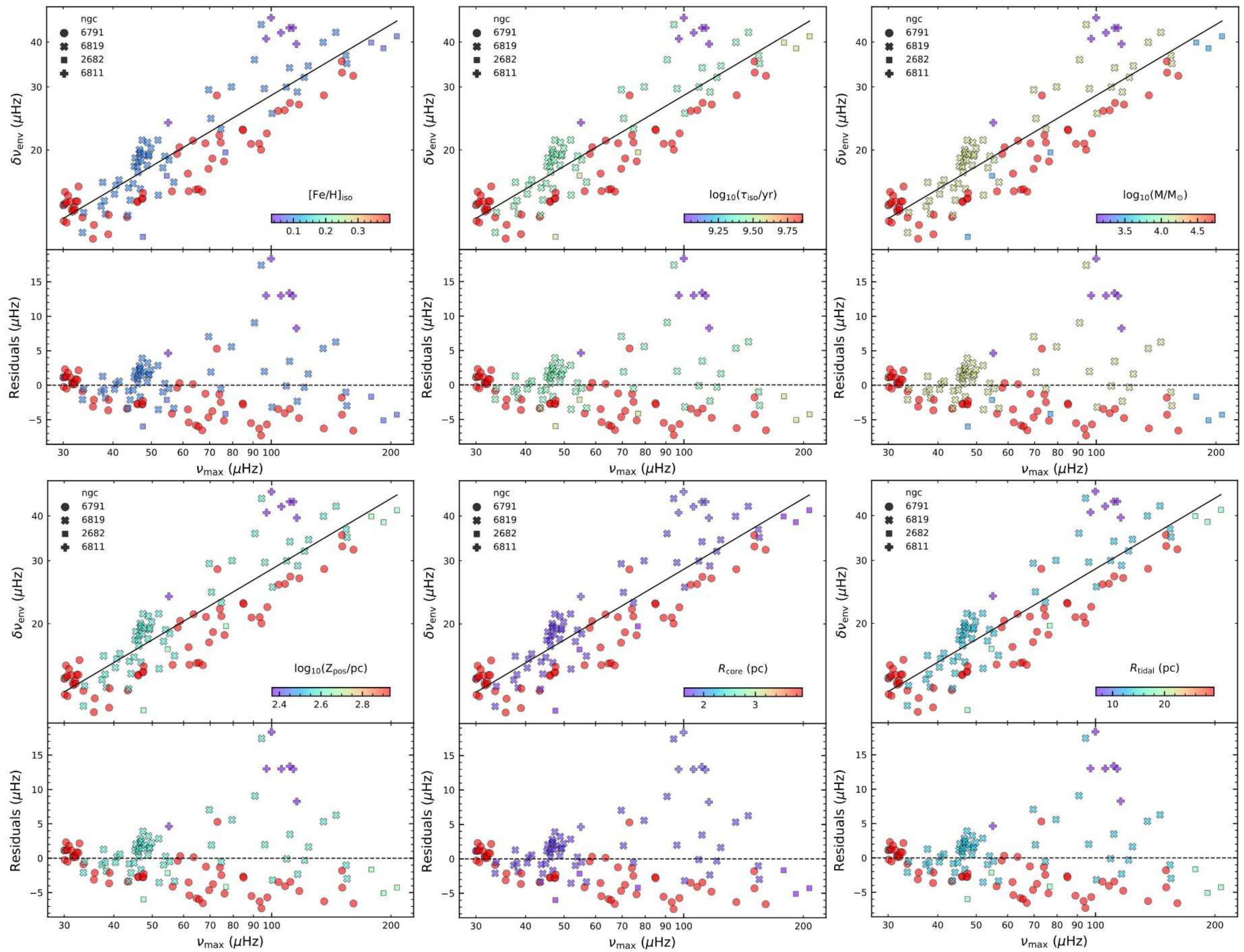
3. Results and Discussions

$$\nu_{\max} - \Delta\nu$$



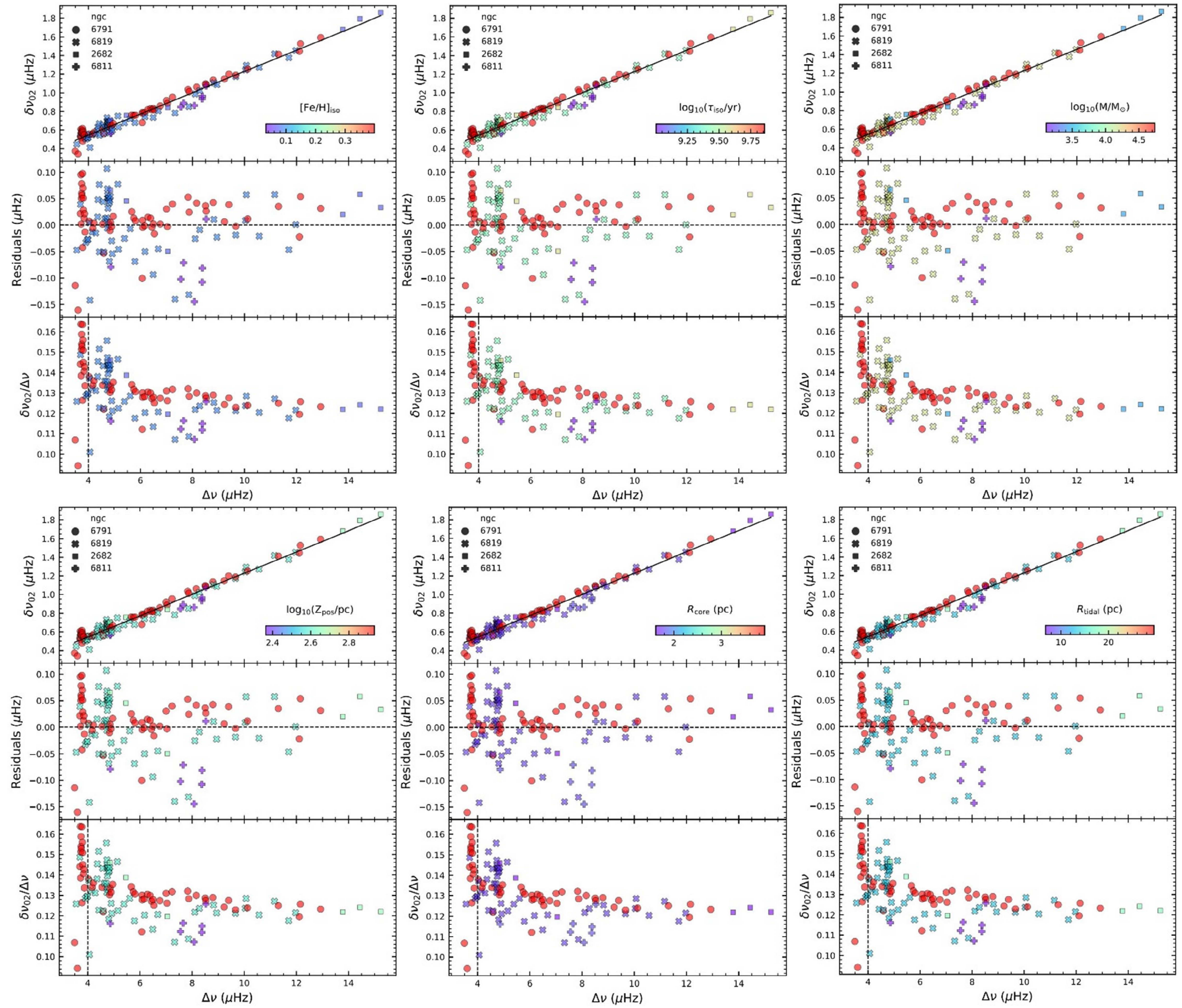
3. Results and Discussions

$\nu_{\max} - \delta\nu_{\mathrm{env}}$



3. Results and Discussions

$$\Delta\nu - \delta\nu_{02}$$

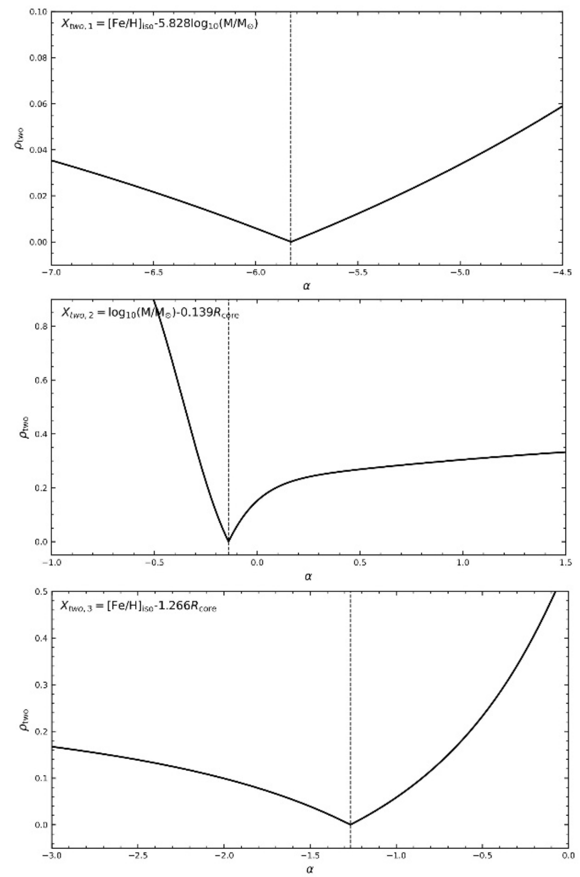


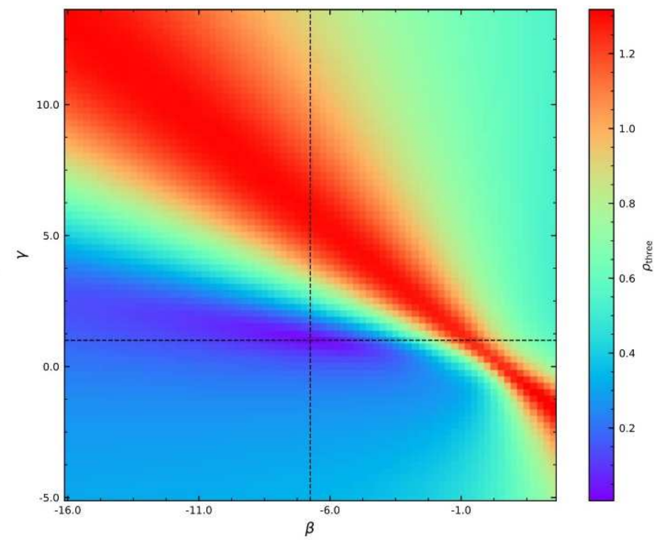
3. Results and Discussions

Determination of primary parameter

ν_{max} >> [Fe/H], M, R_{core}

	[Fe/H]	τ	M	Z_{pos}	R_{core}	R_{tidal}
No correction	0.496	-0.818	-0.905	-0.868	-0.677	-0.766
$\nu_{max,[Fe/H]}$	0.000	-0.661	-0.790	-0.644	-0.563	-0.420
$\nu_{max,\tau}$	0.163	0.000	-0.296	-0.161	-0.140	-0.061
$\nu_{max,M}$	0.227	-0.015	0.000	-0.125	0.152	-0.040
$\nu_{max,Z_{pos}}$	-0.089	0.008	-0.248	0.000	0.125	0.115
$\nu_{max,R_{core}}$	0.325	-0.441	-0.475	-0.396	0.000	-0.344
$\nu_{max,R_{tidal}}$	-0.192	-0.209	-0.421	-0.220	-0.150	0.000
$\nu_{max,X_{two,1}}$	0.000	0.012	0.000	-0.064	0.162	0.088
$\nu_{max,X_{two,2}}$	0.245	-0.012	0.000	-0.184	0.000	-0.079
$\nu_{max,X_{two,3}}$	0.000	-0.384	-0.455	-0.302	0.000	-0.156
$\nu_{max,X_{three}}$	0.002	0.018	0.001	-0.121	0.005	0.057



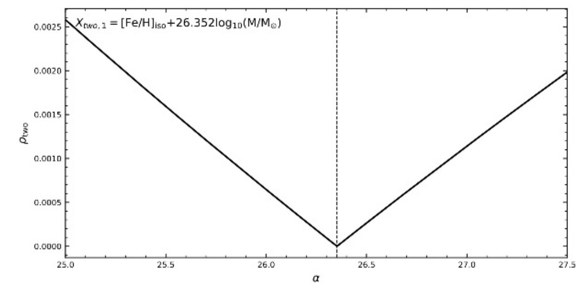


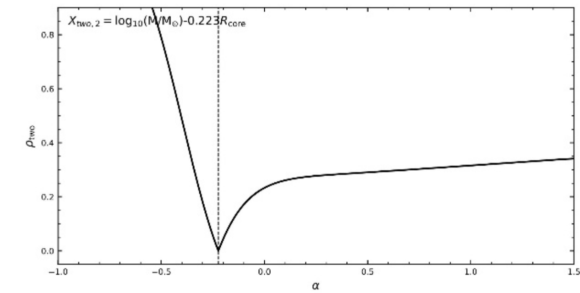
3. Results and Discussions

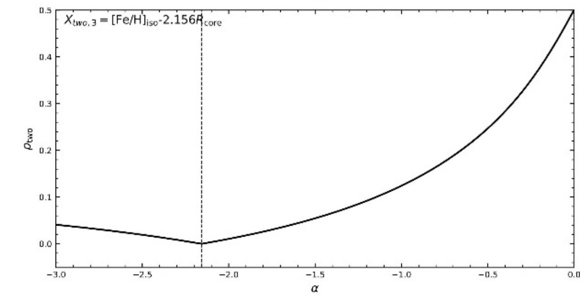
Determination of primary parameter

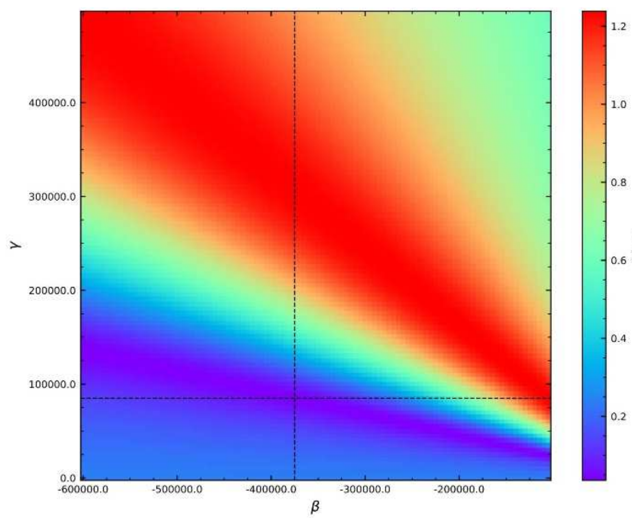
$\Delta\nu$ >> [Fe/H], M

	[Fe/H]	τ	M	Z_{pos}	R_{core}	R_{tidal}
No correction	0.364	-0.677	-0.876	-0.724	-0.605	-0.613
$\Delta\nu_{[Fe/H]}$	0.000	-0.535	-0.768	-0.534	-0.501	-0.342
$\Delta\nu_{\tau}$	0.042	0.000	-0.364	-0.113	-0.152	-0.011
$\Delta\nu_{M}$	-0.046	0.225	0.000	0.133	0.233	0.232
$\Delta\nu_{Z_{pos}}$	-0.126	0.013	-0.324	0.000	0.016	0.107
$\Delta\nu_{R_{core}}$	0.169	-0.296	-0.477	-0.263	0.000	-0.194
$\Delta\nu_{R_{tidal}}$	-0.167	-0.164	-0.466	-0.181	-0.178	0.000
$\Delta\nu_{X_{two,1}}$	0.000	0.220	0.000	0.120	0.233	0.206
$\Delta\nu_{X_{two,2}}$	-0.037	0.252	0.000	0.057	0.000	0.192
$\Delta\nu_{X_{two,3}}$	0.000	-0.258	-0.459	-0.207	0.000	-0.093
$\Delta\nu_{X_{three}}$	-0.036	0.250	-0.002	0.054	-0.006	0.190









3. Results and Discussions

Determination of primary parameter

$\delta\nu_{env}$, $\delta\nu_{02}$

	[Fe/H]	τ	M	Z_{pos}	R_{core}	R_{tidal}
No correction	-0.726	-0.896	-0.930	-0.873	-0.588	-0.815
$\delta\nu_{env,[Fe/H]}$	0.000	-0.445	-0.435	-0.315	0.182	-0.214
$\delta\nu_{env,\tau}$	0.001	0.000	-0.463	0.016	0.071	0.113
$\delta\nu_{env,M}$	0.219	-0.392	0.000	-0.217	0.393	-0.161
$\delta\nu_{env,Z_{pos}}$	0.130	-0.076	-0.379	0.000	0.249	0.108
$\delta\nu_{env,R_{core}}$	-0.185	-0.604	-0.576	-0.489	0.000	-0.395
$\delta\nu_{env,R_{tidal}}$	0.040	-0.186	-0.461	-0.109	0.171	0.000

	[Fe/H]	τ	M	Z_{pos}	R_{core}	R_{tidal}
No correction	-0.361	0.074	-0.522	-0.048	-0.419	-0.018
$\delta\nu_{02,[Fe/H]}$	0.000	0.394	-0.223	0.299	-0.069	0.337
$\delta\nu_{02,\tau}$	-0.422	0.000	-0.583	-0.121	-0.471	-0.090
$\delta\nu_{02,M}$	0.108	0.581	0.000	0.466	-0.009	0.476
$\delta\nu_{02,Z_{pos}}$	-0.318	0.121	-0.482	0.000	-0.381	0.030
$\delta\nu_{02,R_{core}}$	0.055	0.401	-0.211	0.322	0.000	0.369
$\delta\nu_{02,R_{tidal}}$	-0.345	0.091	-0.507	-0.030	-0.404	0.000

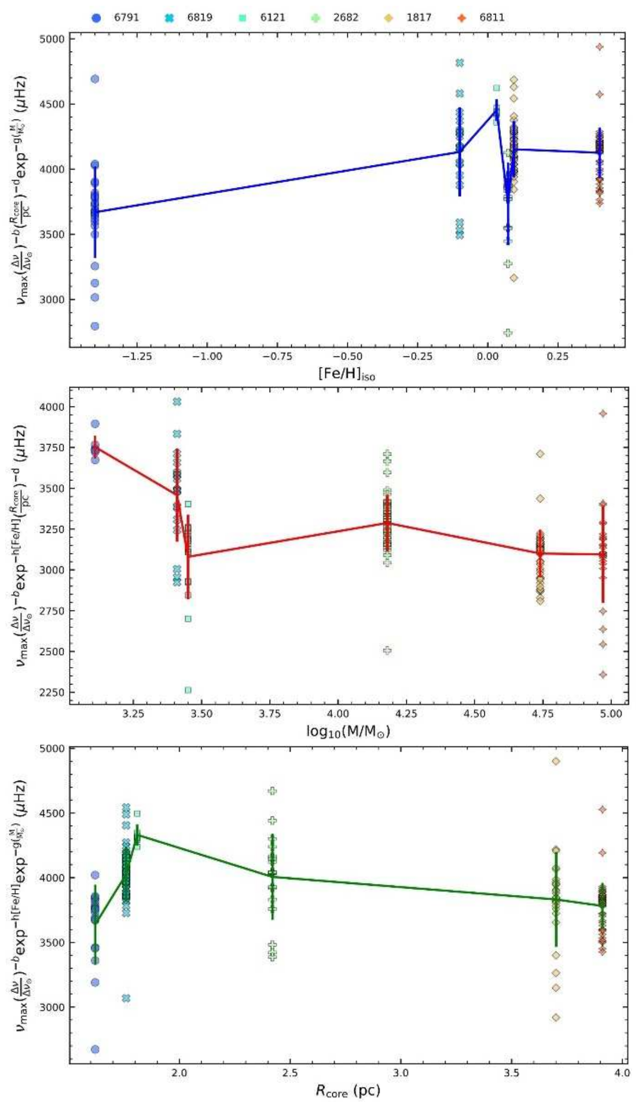
3. Results and Discussions

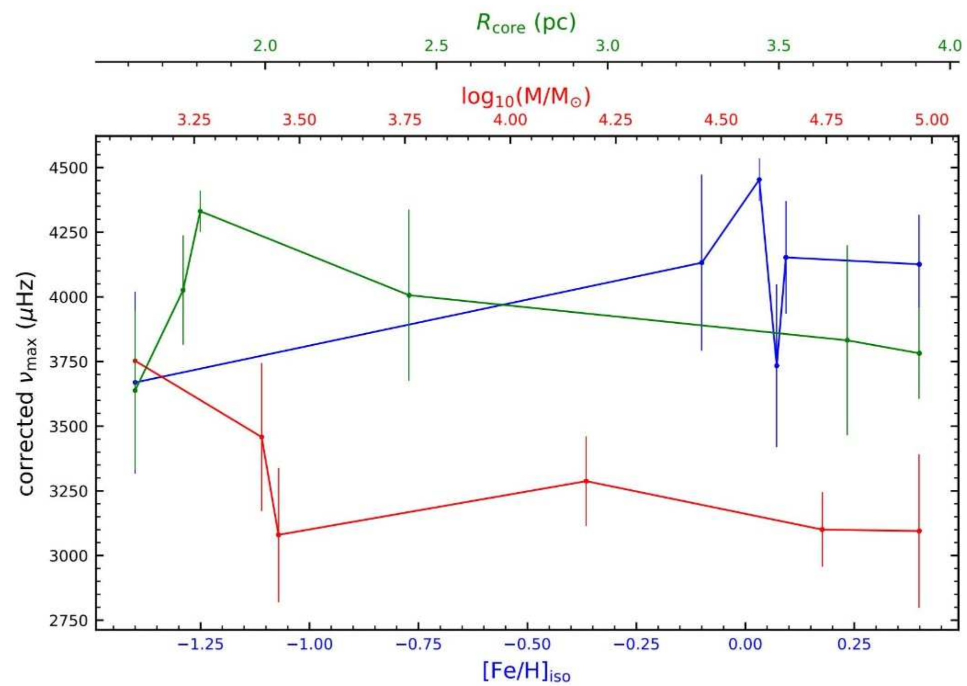
Dependency of cluster properties

ν_{max} >> [Fe/H], M, R_{core}

Scaling relation

$$\nu_{max} = \alpha \, \Delta\nu^{\beta} \, R_{core}{}^{\gamma} \, e^{\delta \, [Fe/H] + \varepsilon M}$$





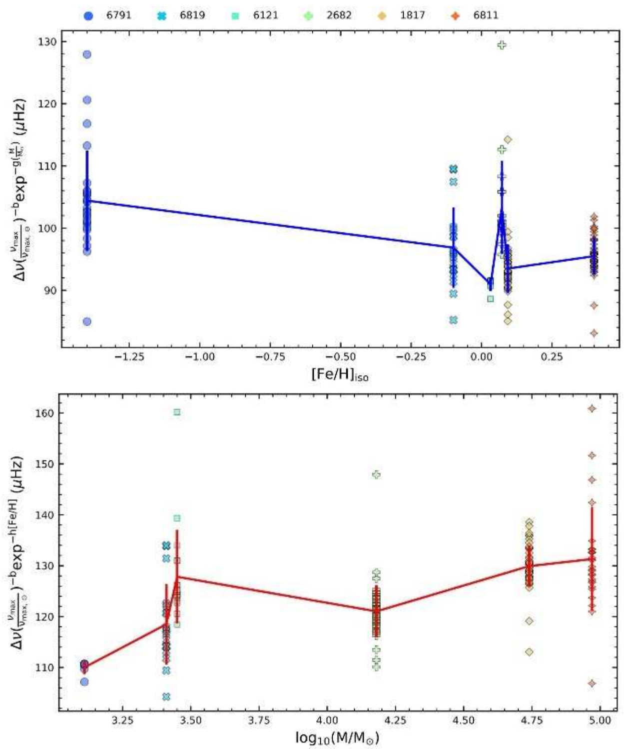
3. Results and Discussions

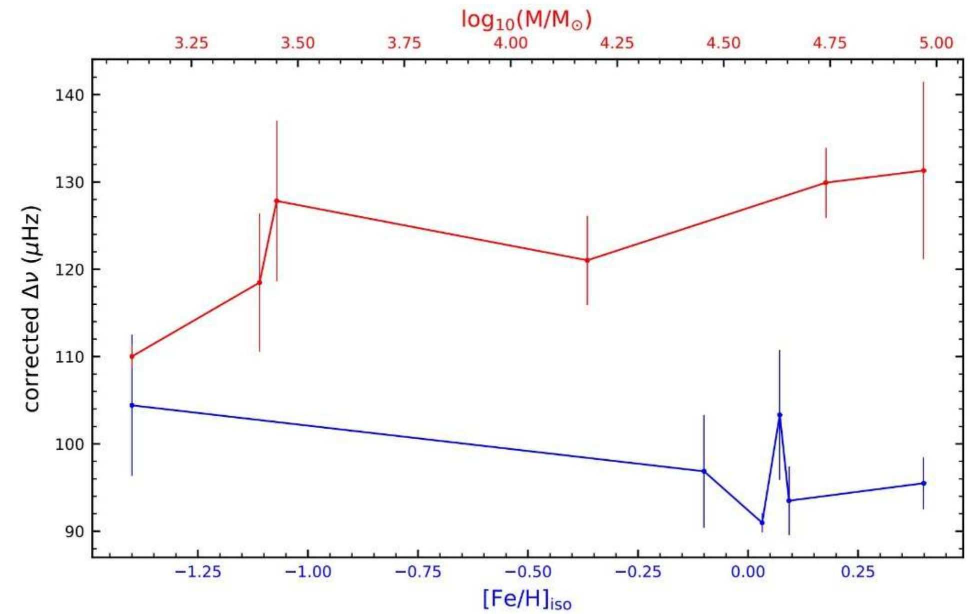
Dependency of cluster properties

$\Delta\nu$ >> [Fe/H], M

Scaling relation

$\Delta\nu = \alpha\, \nu_{max}^{\beta}\, e^{\gamma[Fe/H] + \delta M}$





3. Results and Discussions

Summary

- A direct measure of the cluster properties dependence for the oscillation parameters is derived for the first time.
- [Fe/H], M, and R_{core} are the primary parameters for the oscillation parameters.
- In older clusters, heavy stars have already evolved more than giants, so there are only light stars in the giant stage.

	[Fe/H]	τ	M	Z_{pos}	R_{core}	R_{tidal}	M_{seis}
ν_{max}	+	-	-	-	-	-	+
$\Delta\nu$	-	+	+	+	+	+	-
$\delta\nu_{\mathrm{env}}$	-	-	-	-	-	-	+
$\delta\nu_{02}$	+	+	+	+	+	+	-