Numerical Simulation of Primordial Black Hole Formation

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Introduction

Introduction:Primordial BHs [Zeldovich and Novikov(1967), Hawking(1971)]

◎Remnant of primordial non-linear inhomogeneity

◎Trace the inhomogeneity in the early universe

◎May provide a fraction of dark matter and BH binaries

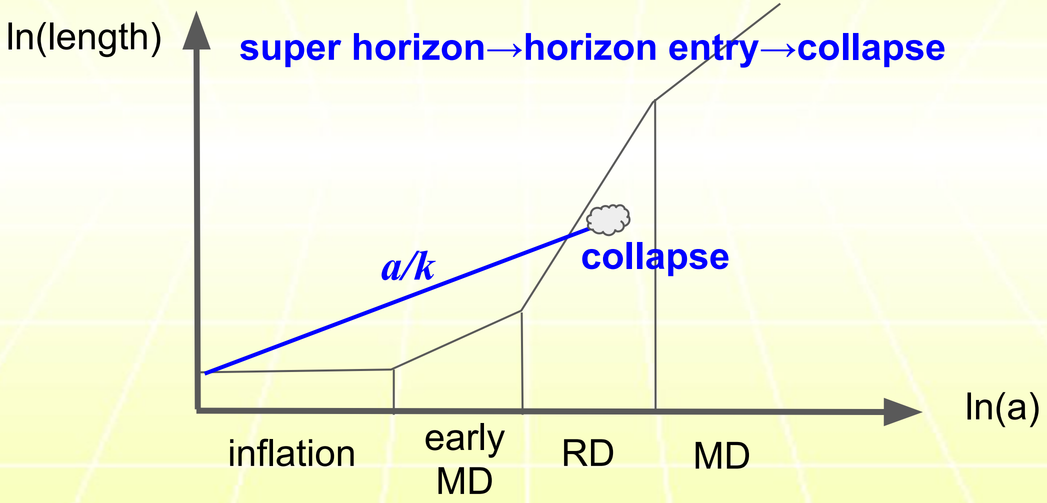
◎Several aspects

- Inflationary models which provide a number of PBHs
- Theoretical estimation and observational constraints on PBH abundance
- Threshold of PBH formation
- Mass and spin distribution of PBHs

Introduction:PBH formation

◎Focus on PBH formation in **radiation dominated era**

◎Comoving scale of an inhomogeneity ~ $1/k$



◎GR simulation starting from a **super-horizon non-linear initial data**

Chulmoon Yoo

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Brief Review of Numerical Relativity

Geometrical Quantities(1)

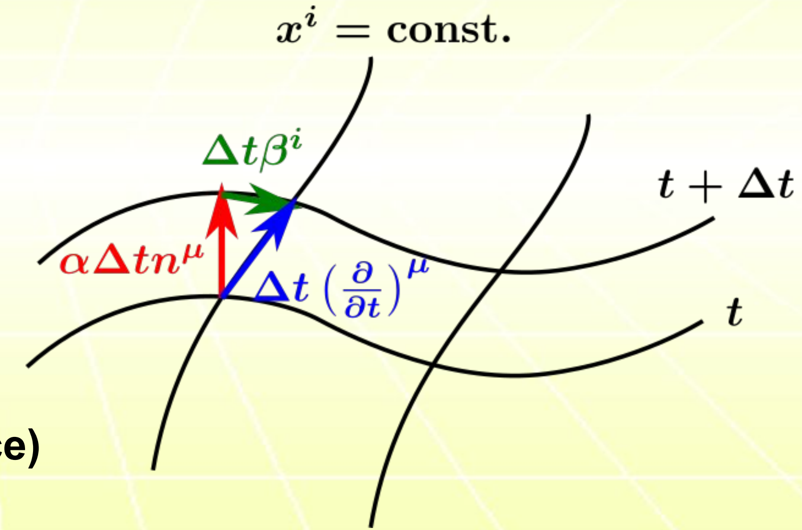
◎**Line elements**

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

α : lapse function(1 component)

β^i : shift vector(3 components)

γ_{ij} : spatial metric(6 components)



◎**Unit normal form to t=const hyper-surface**

$$n_\mu := -N\partial_\mu t$$

◎**Projection tensor(tangent to t=const hyper-surface)**

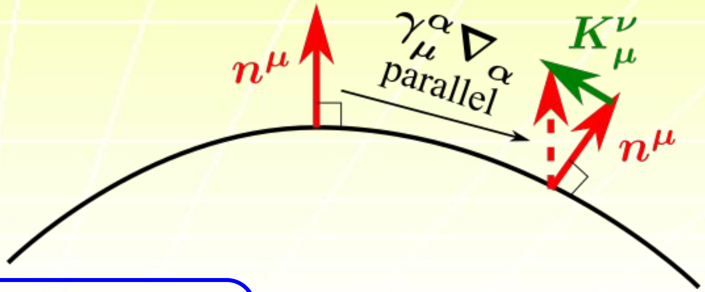
$$\gamma_{\mu\nu} := g_{\mu\nu} + n_\mu n_\nu \qquad n^\mu\gamma_{\mu\nu} = 0$$

3+1 Decomposition

◎Extrinsic curvature (~ "time derivative of $\gamma_{\mu\nu}$")

$$K_{\mu\nu} := -\gamma_\mu^\alpha \nabla_\alpha n_\nu = -\frac{1}{2}\mathcal{L}_{\vec{n}}\gamma_{\mu\nu}$$

$\mathcal{L}_{\vec{n}}$: Lie derivative assoc. with n^μ



◎Decomposition of Einstein eqs.

$$G_{\mu\nu}n^\mu n^\nu = 8\pi T_{\mu\nu}n^\mu n^\nu \Leftrightarrow R + K^2 - K_{ij}K^{ij} = 16\pi E$$ Hamiltonian constraint

$$G_{\mu\nu}n^\mu \gamma_i^\nu = 8\pi T_{\mu\nu}n^\mu \gamma_i^\nu \Leftrightarrow D_j K_i^j - D_i K = 8\pi p_i$$ Momentum constraint

There is no $\partial_t K_{ij}$ term⇒constraint eqs.

$$G_{\mu\nu}\gamma_\alpha^\mu \gamma_\beta^\nu = 8\pi T_{\mu\nu}\gamma_\alpha^\mu \gamma_\beta^\nu \Leftrightarrow (\partial_t - \mathcal{L}_\beta) K_{ij} = -D_i D_j N + N\left\{R_{ij} + KK_{ij} - 2 - 2K_{ik}K^k_j + 4\pi\left[(S-E)\gamma_{ij} - 2S_{ij}\right]\right\}$$

Evolution eqs. for K_{ij}

Further decomposition is needed for numerical simulation

Conformal 3+1 Decomposition

◎Spatial metric

$$\gamma_{ij} = e^{4\psi}\tilde{\gamma}_{ij}$$ where $\psi = \ln(\det\gamma)$, and $\det\tilde{\gamma} = \det f$ with f_{ij} being the flat metric

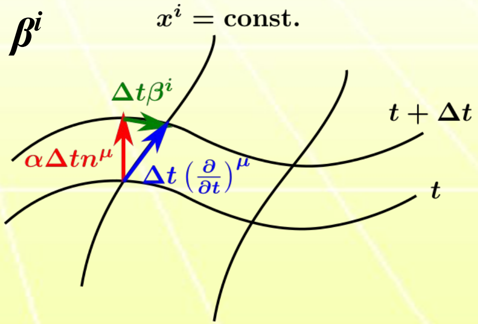
◎Extrinsic curvature

$$K_{ij} = e^{4\psi}\tilde{A}_{ij} + \frac{1}{3}\mathrm{tr}K\gamma_{ij}$$ where $\mathrm{tr}\tilde{A} = 0$

$\tilde{\gamma},\ \psi,\ \mathrm{tr}K,\ \tilde{A}$ are used as independent variables

◎Gauge degrees of freedom(coord. choice) to choose α, β^i

⇒ need to be fixed by hand



Baumgarte-Shapiro-Shibata-Nakamura formalism

Shibata, Nakamura(1995)
Baumgarte, Shapiro(1999)

◎Spatial metric

$$\left(\partial_t - \beta^i \partial_i\right) \psi = \frac{1}{6}\psi\left(\partial_i \beta^i - \alpha K\right)$$

$$\left(\partial_t - \beta^k \partial_k\right) \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} + \tilde{\gamma}_{ik}\partial_j \beta^k + \tilde{\gamma}_{jk}\partial_i \beta^k - \frac{2}{3}\partial_k \beta^k \tilde{\gamma}_{ij}$$

◎Extrinsic curvature

$$\left(\partial_t - \beta^k \partial_k\right) \mathrm{tr}K = \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{2}{3}\mathrm{tr}K^2\right) - \triangle\alpha$$

$$\left(\partial_t - \beta^k \partial_k\right) \tilde{A}_{ij} = \text{functions of } [\psi, \tilde{\gamma}, \mathrm{tr}K, \tilde{A}, \tilde{\Gamma}, \alpha, \beta, \partial\psi, \triangle\psi, \cdots]$$

◎Auxiliary variable for numerical stability $\tilde{\Gamma}^i := -\mathcal{D}_j \tilde{\gamma}^{ij}$

$$\left(\partial_t - \beta^k \partial_k\right) \tilde{\Gamma}^i = \text{functions of } [\psi, \tilde{\gamma}, \mathrm{tr}K, \tilde{A}, \tilde{\Gamma}, \alpha, \beta, \partial\psi, \triangle\psi, \cdots]$$

17 evolution eqs.

◎Eqs. for gauge fixing

Dynamical Gauge Conditions

◎Time slicing condition(modified version of the “1+log slice”)

$$\left(\partial_t - \beta^i \partial_i\right) \alpha = -2\alpha(\mathrm{tr}K + 3H_b)$$

◎Spatial coordinates(∼“Hyperbolic Gamma driver”)

$$\left(\partial_t - \beta^k \partial_k\right) \beta^i = \frac{3}{4} B^i$$

$$\left(\partial_t - \beta^k \partial_k\right) B^i = \partial_t \tilde{\Gamma}^i - 3H_b B^i$$

◎17 + 1 + 6 = 24 variables for geometry

Relativistic Hydro-dynamics

◎Energy momentum tensor

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}$$

◎Lorentz factor for n^μ

$$\Gamma = -u^\mu n_\mu$$

◎Velocity U^μ relative to n^μ

$$u^\mu = \Gamma(n^\mu + U^\mu)$$

◎Rest mass density(ρ_0), specific int. ene.(ε)

$$\rho = \rho_0(1 + \varepsilon)$$

◎Rest mass density measured by n^μ

$$D = \rho_0 \Gamma$$

◎"Dynamical" variables

$$\rho_* := \sqrt{\gamma} D,\ S_0 := \sqrt{\gamma} E,\ S_i := \sqrt{\gamma} p_i$$

◎Fluid equations

$$\partial_t \rho_* + \partial_i f^i_{\rho_*} = 0$$
$$\partial_t S_0 + \partial_i f^i_{S_0} = -S^i \partial_i \alpha + \alpha \sqrt{\gamma} S_{ij} K^{ij}$$
$$\partial_t S_i + \partial_j f^i_{S_j} = -S_0 \partial_i \alpha + S_j \partial_i \beta^j - \frac{1}{2} \alpha \sqrt{\gamma} S_{jk} \partial_i \gamma^{jk}$$

with

$$f^i_{\rho_*} = \rho_* V^i = \rho_*(\alpha U^i - \beta^i)$$
$$f^i_{S_0} = S_0 V^i + \sqrt{\gamma} P (V^i + \beta^i)$$
$$f^i_{S_j} = S_j V^i + \alpha \sqrt{\gamma} \delta^i_j P$$

◎"Primitive" variables

$$\rho,\ V^i := u^i / u^0,\ \varepsilon$$

Barotropic EoS case

◎Relation between the variables

$$\rho_* = \sqrt{\gamma}\Gamma\frac{\rho}{1+\varepsilon}$$

$$S_0 = \sqrt{\gamma}[\Gamma^2(\rho+P)-P]$$

$$S_i = \sqrt{\gamma}(E+P)U_i = \frac{1}{\alpha}(S_0+\sqrt{\gamma}P)\gamma_{ij}(V^i+\beta^i)$$

$$p^\mu p_\mu - E^2 - (P-\rho)E + \rho P = 0$$

dynamical var

EoS $P=(\rho,\varepsilon)$

primitive var

◎Barotropic EoS $P=P(\rho)$

$$\rho = \rho(\text{dynamical variables})$$

$$V^i = \alpha U^i - \beta^i = \alpha\frac{\gamma^{ij}S_j}{S_0+\sqrt{\gamma}P} - \beta^i$$

Equations are closed without ρ_* (or equivalently ε)
⇒we don't need to solve the continuity eq.

◎$P=w\rho$

$$\rho = \frac{1}{2w}\left[-(1-w)E+\sqrt{E^2(1-w)^2+4w(E^2-p^\mu p_\mu)}\right]$$

Equations for fluid

◎4(+1) equations

$$\partial_t S_0 + \partial_i f^i_{S_0} = -S^i \partial_i \alpha + \alpha \sqrt{\gamma} S_{ij} K^{ij}$$

$$\partial_t S_i + \partial_j f^i_{S_j} = -S_0 \partial_i \alpha + S_j \partial_i \beta^j - \frac{1}{2} \alpha \sqrt{\gamma} S_{jk} \partial_i \gamma^{jk}$$

$$(\partial_t \rho_* + \partial_i f^i_{\rho_*} = 0)$$

◎Scheme for the flux calculation

A central scheme with MUSCL(Mono Upstream-centered Scheme for Conservation Laws)

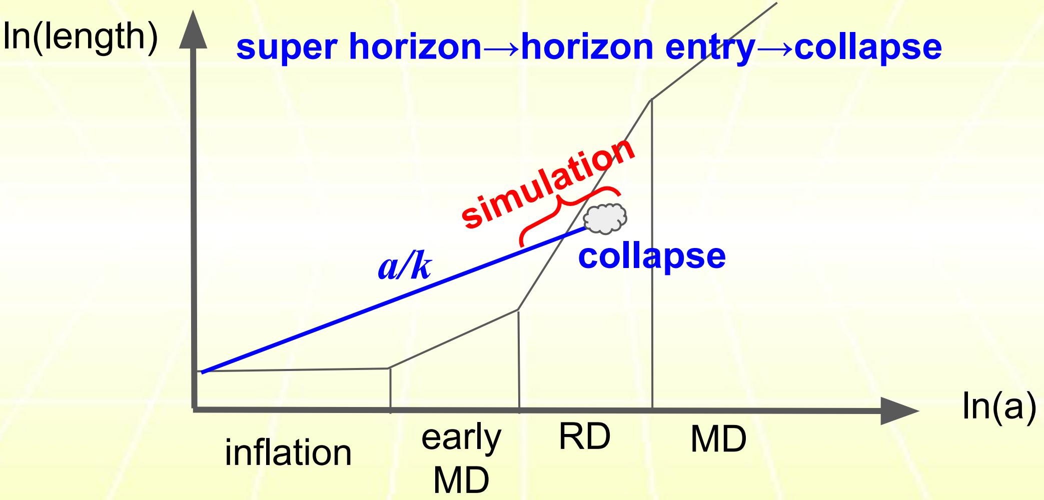
Kurganov, Tadmor(2000)
Shibata, Font(2005)

◎Totally 24 equations for geometry and 4+1 equations for fluid = 28 +1 equations

Cosmological long-wavelength perturbation

Introduction:PBH formation

◎Focus on PBH formation in radiation dominated era



◎GR simulation starting from a super-horizon non-spherical initial data

◎We approximately solve the equations in the long-wavelength approximation

Cosmological conformal 3+1 decomposition

◎Decomposition of metric variables

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- spatial metric

$$\gamma_{ij} = \psi^4 a(t)^2 \tilde{\gamma}_{ij}$$

- reference flat spatial metric f_{ij}

$$\tilde{\gamma} := \det(\tilde{\gamma}_{ij}) = f := \det(f_{ij})$$

- extrinsic curvature

$$K_{ij} = \psi^4 a^2 \tilde{A}_{ij} + \frac{\gamma_{ij}}{3} K$$

◎Equations from the definition of the extrinsic curvature

$$(\partial_t - \mathcal{L}_\beta)\psi = -\frac{2\dot{a}}{a}\psi + \frac{\psi}{6}(-\alpha K + \mathcal{D}_k \beta^k)$$

$$(\partial_t - \mathcal{L}_\beta)\tilde{\gamma}_{ij} = -2\tilde{A}_{ij} - \frac{2}{3}\tilde{\gamma}_{ij}\mathcal{D}_k\beta^k$$

$\mathcal{D}$:covariant derivative w.r.t. f_{ij}

Field equations

◎Constraint equations

$$\tilde{\Delta}\psi = \frac{\tilde{R}}{8}\psi - 2\pi\psi^5 a^2 E - \frac{\psi^5 a^2}{8}\left(\tilde{A}_{ij}\tilde{A}^{ij} - \frac{2}{3}K^2\right)$$

⇒E can be calculated from geometrical variables

$$\tilde{D}^j\left(\psi^6\tilde{A}_{ij}\right) - \frac{2}{3}\psi^6\tilde{D}_i K = 8\pi p_i\psi^6$$

⇒p_i can be calculated from geometrical variables

◎Evolution equations

$$(\partial_t - \mathcal{L}_\beta)\tilde{A}_{ij} = \frac{1}{a^2\psi^4}\left[\alpha\left(R_{ij} - \frac{8\pi}{a^2\psi^4}S_{ij}\right) - D_i D_j\alpha\right]_{\rm TL} + \alpha\left(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k_j\right) - \frac{2}{3}\tilde{A}_{ij}\mathcal{D}_k\beta^k$$

$$(\partial_t - \mathcal{L}_\beta)K = \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2\right) - D_k D^k\alpha + 4\pi\alpha\left(E + S\right)$$

◎+ energy conservation and relativistic Euler equations for fluid

Fluid variables

◎Energy momentum tensor

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}$$

◎Four velocity

$$u^\mu = \Gamma(n^\mu + U^\mu)$$

$$\Gamma = \left(1 - U^i U_i\right)^{-1/2}$$

◎Equation of state

$$w := P/\rho = 1/3$$

◎Energy and momentum densities for an Eulerian observer

$$E := T^{\mu\nu} n_\mu n_\nu = \Gamma^2(\rho + P) - P = \rho(4\Gamma^2 - 1)/3$$

$$p_i := -T^{\mu\nu} n_\mu \gamma_{\nu i} = (E + P)U_i = (E + \rho/3)U_i$$

◎ρ and U_i can be calculated from $E(\rho, U_i)$ and $p_i(\rho, U_i)$

→ρ and U_i can be given by geometrical variables through constraint equations

Gradient expansion(Anti-Newtonian)

[Salopek and Bond(?)(1990)] [Tomita(1972,1975)]

◎Assumption 1

$$\partial_i \sim k \ll aH_b$$ H_b:=background Hubble

⇒small parameter $\epsilon = k/(aH_b)$

◎Assumption 2: flat FLRW for ε→0

$$\alpha - 1 = \mathcal{O}(\epsilon),\ \ \beta^i = \mathcal{O}(\epsilon)$$ and $$\dot{\tilde{\gamma}}_{ij} = \mathcal{O}(\epsilon) \xrightarrow{\text{O(ε) decays}} \dot{\tilde{\gamma}}_{ij} = \mathcal{O}(\epsilon^2)$$

[Lyth,Malik,Sasaki(2005)]

◎Orders of variables from EoM

$$\psi = \mathcal{O}(\epsilon^0),\ U^i = \mathcal{O}(\epsilon),\ \rho = \rho_b(1+\mathcal{O}(\epsilon^2)),\ \tilde{A}_{ij} = \mathcal{O}(\epsilon^2)$$

$$\tilde{\gamma}_{ij} = f_{ij} + \mathcal{O}(\epsilon^2), \alpha = 1 + \mathcal{O}(\epsilon^2),\ K = 3H_b(1+\mathcal{O}(\epsilon^2)$$

Cosmological long-wavelength solution

◎Field variables

$$\psi(t,x^k)=\Psi(x^k)(1+\xi(t,x^k)),\ \delta=\frac{\rho-\rho_b}{\rho_b},\ h_{ij}=\tilde{\gamma}_{ij}-f_{ij},\ \chi=\alpha-1,\ \kappa=\frac{K-K_b}{K_b},\ \tilde{A}_{ij}$$

◎Growing mode solutions in the constant-mean-curvature(uniform Hubble) slicing(κ=0)

$$p_{ij}(x^k):=\frac{1}{\Psi^4}\left[-\frac{2}{\Psi}\left(\bar{\mathcal{D}}_i\bar{\mathcal{D}}_j\Psi-\frac{1}{3}f_{ij}\bar{\triangle}\Psi\right)+\frac{6}{\Psi^2}\left(\bar{\mathcal{D}}_i\Psi\bar{\mathcal{D}}_j\Psi-\frac{1}{3}f_{ij}\bar{\mathcal{D}}^k\Psi\bar{\mathcal{D}}_k\Psi\right)\right]$$

$$\begin{cases} h_{ij}=-\frac{4}{(3w+5)(3w-1)}p_{ij}\left(\frac{1}{aH_b}\right)^2+\mathcal{O}(\epsilon^4) \\ \tilde{A}_{ij}=\frac{2}{3w+5}p_{ij}H_b\left(\frac{1}{aH_b}\right)^2+\mathcal{O}(\epsilon^4) \end{cases}$$

$$f(x^k):=-\frac{4}{3}\frac{\bar{\triangle}\Psi}{\Psi^5}$$

$$\begin{cases} \chi=-\frac{3w+1}{3w+3}f\left(\frac{1}{aH_b}\right)^2+\mathcal{O}(\epsilon^4) \\ \xi=-\frac{1}{6w+6}f\left(\frac{1}{aH_b}\right)^2+\mathcal{O}(\epsilon^4) \\ \delta=f\left(\frac{1}{aH_b}\right)^2+\mathcal{O}(\epsilon^4) \\ U_i=\frac{2}{3(w+1)(3w+5)}\partial_i fa\left(\frac{1}{aH_b}\right)^3+\mathcal{O}(\epsilon^5) \end{cases}$$

$\Psi(x^k)$ fixes everything

Note : $\gamma_{ij}\simeq\Psi^4a(t)^2\tilde{\gamma}_{ij}$

→ **We do not use these expressions (see next slide)** (refers to the δ and U_i expressions)

Fluid variables

◎Constraint equations

$$\tilde{\Delta}\psi = \frac{\tilde{R}}{8}\psi - 2\pi\psi^5 a^2 E - \frac{\psi^5 a^2}{8}\left(\tilde{A}_{ij}\tilde{A}^{ij} - \frac{2}{3}K^2\right)$$

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$$p_i := -T^{\mu\nu}n_\mu\gamma_{\nu i} = (E + P)U_i = (E + \rho/3)U_i$$

◎ρ and U_i can be calculated from E and p_i

→ρ and U_i can be given by geometrical variables through constraint equations

→We set ρ and U_i s.t. constraint equations are exactly satisfied initially

About the numerical code

◎Originally provided by Hirotada Okawa (for E-eqs and real scalar field w/ periodic BC)

◎COSMOS(秋桜)code by C++ [CY, Hirotada Okawa, Ken-ichi Nakao(1306.1389)]

◎Basically follows the SACRA(桜)code by Fortran

[Tetsuro Yamamoto, Masaru Shibata, Keisuke Taniguchi(arXiv:0806.4007)]

◎Inhomogeneous coordinate system has been implemented

[CY, Taishi Ikeda,Hirotada Okawa(arXiv:1811.00762)]

◎Fluid evolution code has been implemented

[CY, Tomohiro Harada,Hirotada Okawa(arXiv:2004.01042)]

◎1+1 code for spherical systems has been developed based on COSMOS

[CY, Harada, Hirano, Okawa, Sasaki(arXiv:2112.12335)]

◎Recently, a mesh refinement procedure has been implemented

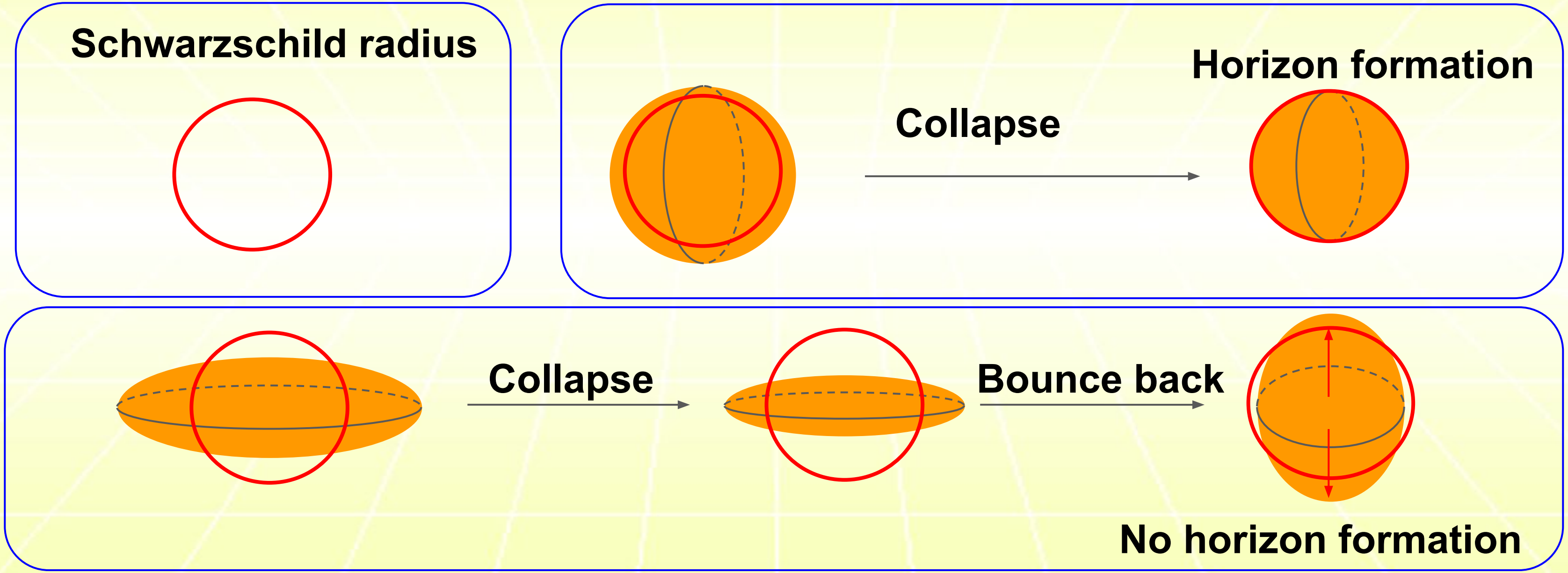
[CY, ongoing]

Simulation of non-spherical PBH formation

[3] 2004.01042 CY, Tomohiro Harada, Hirotada Okawa

Effect of ellipticity

◎Ellipticity makes black hole formation harder



◎How significant is this effect?

Probability for profile

Curvature perturbation

◎Curvature perturbation

$$\zeta(x^k) := -2\ln\Psi(x^k)$$

◎Random Gaussian variable with the power spectrum $\mathcal{P}(k)$

⇒ gradient moments $\sigma_n^2 := \int \frac{dk}{k} k^{2n}\mathcal{P}(k) \ll k_*^{2n}$

◎Taylor expansion around a peak($\partial_i\zeta=0$), by taking a principal direction

$$\zeta(x^k) = \zeta_0 + \frac{1}{2}\left(\lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^3\right) + \mathcal{O}(x^3) \qquad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0$$

◎Useful probability variables

$$\mu = -\zeta_0/\sigma_0 \quad \xi_1 := (\lambda_1 + \lambda_2 + \lambda_3)/\sigma_2 \quad \xi_2 := \frac{1}{2}(\lambda_1 - \lambda_3)/\sigma_2 \quad \xi_3 := \frac{1}{2}(\lambda_1 - 2\lambda_3 + \lambda_3)/\sigma_2$$

◎Ellipticity is characterized by ξ_2 and ξ_3

Peak probability

◎**Peak probability for μ, ξ_1, ξ_2, ξ_3** [Bardeen,Bond,Kaiser,Szalay(1986)]

$$P(\mu,\vec{\xi})d\mu d\vec{\xi} = P_1(\mu,\xi_1)P_2(\xi_2,\xi_3)d\mu d\vec{\xi}$$

$$P_1(\mu,\xi_1)d\mu d\xi_1 = \frac{1}{2\pi}\frac{1}{1-\gamma^2}\exp[-\frac{1}{2}(\mu^2+\frac{(\xi_1-\gamma\mu)^2}{1-\gamma^2})]d\mu d\xi_1$$

$$P_2(\xi_2,\xi_3)d\xi_2 d\xi_3 = \frac{5^{5/2}3^2}{\sqrt{2\pi}}\xi_2(\xi_2^2-\xi_3^2)\exp\left[-\frac{5}{2}\left(3\xi_2^2+\xi_3^2\right)\right]d\xi_2 d\xi_3$$

◎**No correlation between (μ,ξ_1) and (ξ_2, ξ_3) ⇒** $\xi_2 \sim 1 \quad \xi_3 \sim 1$

◎**Dimensionless parameter for the extent of the ellipticity**

$$\chi_1 := \xi_2/\xi_1 \quad \chi_2 := \xi_3/\xi_1$$

◎**For PBH formation(rare events for very high peaks)**

$|\zeta_0| \sim 1$ → $\mu \gg 1$ → $\xi_1 \gg 1$ → $\chi_1 \ll 1,\ \chi_2 \ll 1$

high amplitude

$\sigma_0 \ll 1$

Correlation between μ and ξ_1

Our initial data setting

◎Curvature perturbation profile

$$\zeta = \zeta_0 \exp\left[-\frac{1}{2}\left(k_1^2 x^2 + k_2^2 y^2 + k_3^2 z^2\right)\right] \times [\text{window function to eliminate the tail}]$$

We fix $k^2 = k_1^2 + k_2^2 + k_3^2$

$$\chi_1 = (2k_1^2/k^2 + k_2^2/k^2 - 1)/2$$

$$\chi_2 = (1 - 3k_2^2/k^2)/2$$

$\longrightarrow$

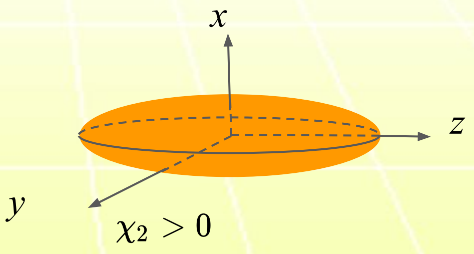
$$k_1^2 = \frac{1}{3}\left(\hat{\xi}_1 + 3\hat{\xi}_2 + \hat{\xi}_3\right) = \frac{k^2}{3}(1 + 3\chi_1 + \chi_2)$$

$$k_2^2 = \frac{1}{3}\left(\hat{\xi}_1 - 2\hat{\xi}_2\right) = \frac{k^2}{3}(1 - 2\chi_2)$$

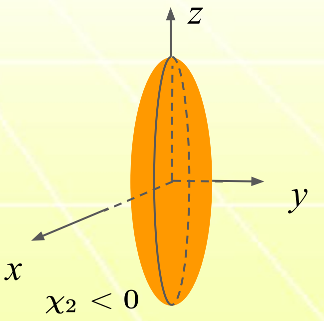
$$k_3^2 = \frac{1}{3}\left(\hat{\xi}_1 - 3\tilde{\xi}_2 + \hat{\xi}_3\right) = \frac{k^2}{3}(1 - 3\chi_1 + \chi_2)$$

◎We focus on spheroidal shape initial data

given by $|\hat{\xi}_3| = \hat{\xi}_2 \Leftrightarrow |\chi_2| = \chi_1$



oblate



prolate

Initial data setting

Parameter settings

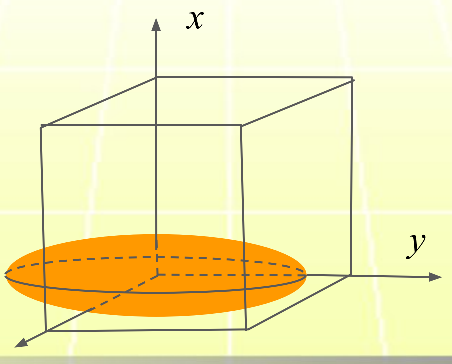
◎Domain for simulation: ⅛ region with reflecting boundary condition

◎Initial scale factor $a_0=1$

◎Unit of length: edge length of the cubic domain $= L$

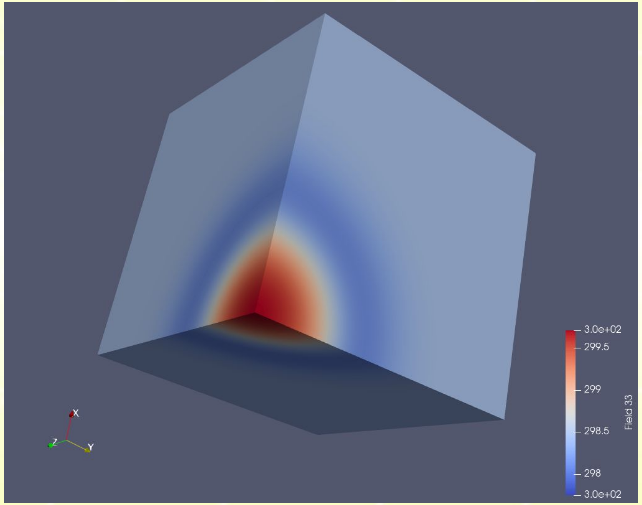
◎Scale of the inhomogeneity $1/k = L/10$

◎Initial Hubble parameter $1/H_0=L/50$



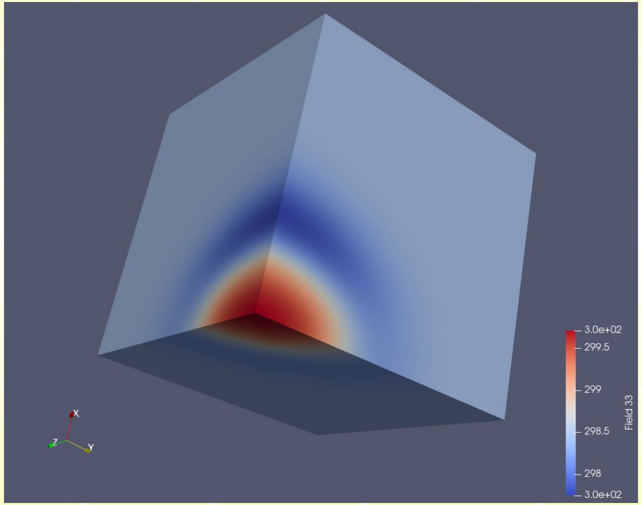
Initial density profile

◎Spherical initial data density profile



$$\mu = 0.8,\ k = 10,\ \chi_1 = \chi_2 = 0$$

◎Oblate initial data density profile



$$\mu = 0.8,\ k = 10,\ \chi_1 = \chi_2 = 0.1$$

Note:both are compensated density profiles

Numerical schemes

Summary of schemes (w/o mesh refinement)

◎Reflection boundary condition

◎for geometry

BSSN ＋ 1+log slice + Gamma driver

◎for fluid evolution

Central scheme with MUSCL method

◎Resolution

・Scale-up reference coordinates x^i related to the Cartesian coord. X^i by

$$X^i = x^i - \frac{S}{1+S}\frac{L}{\pi}\sin\left(\frac{\pi}{L}x^i\right) \text{ with } S = 15$$

・Resolution at the center

$$\Delta X|_{\text{center}} = \frac{1}{1+S}\Delta x = \frac{1}{16}\frac{L}{100} = \frac{L}{1600}$$

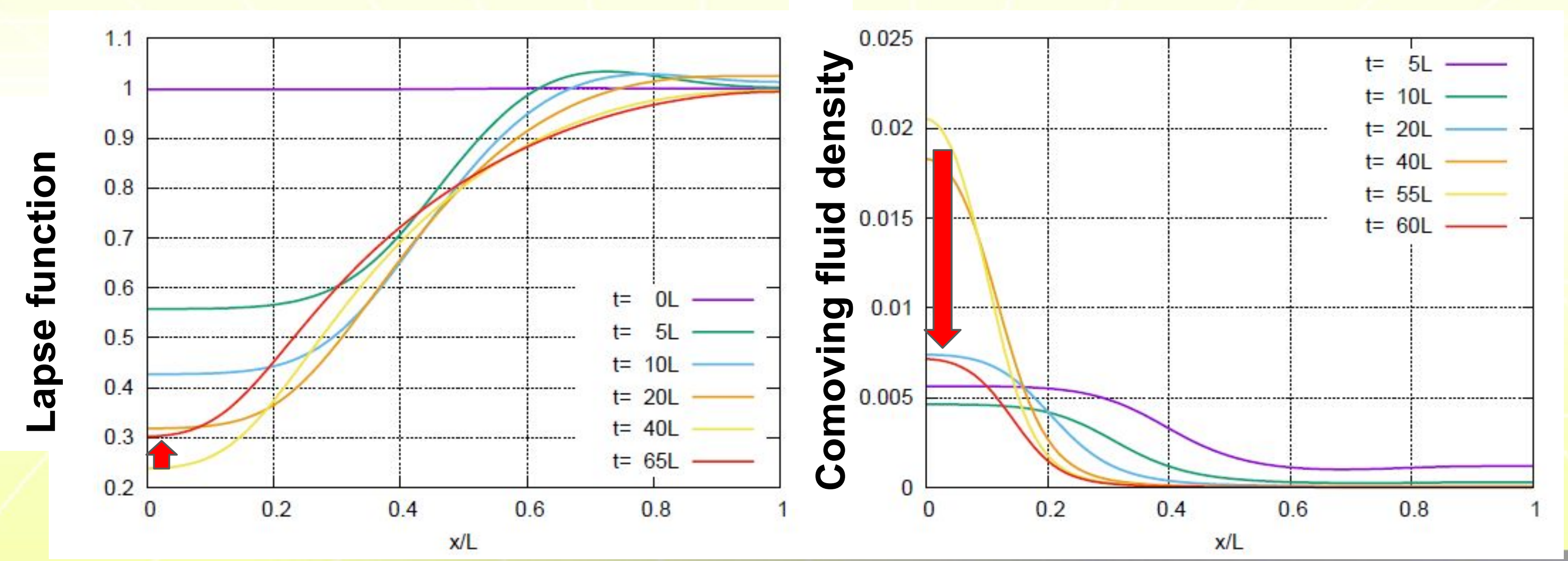
Results:spherical case

Just below the threshold:no horizon formation

◎**Parameters**

$$|\zeta_0| = 0.795,\ \chi_1 = \chi_2 = 0$$

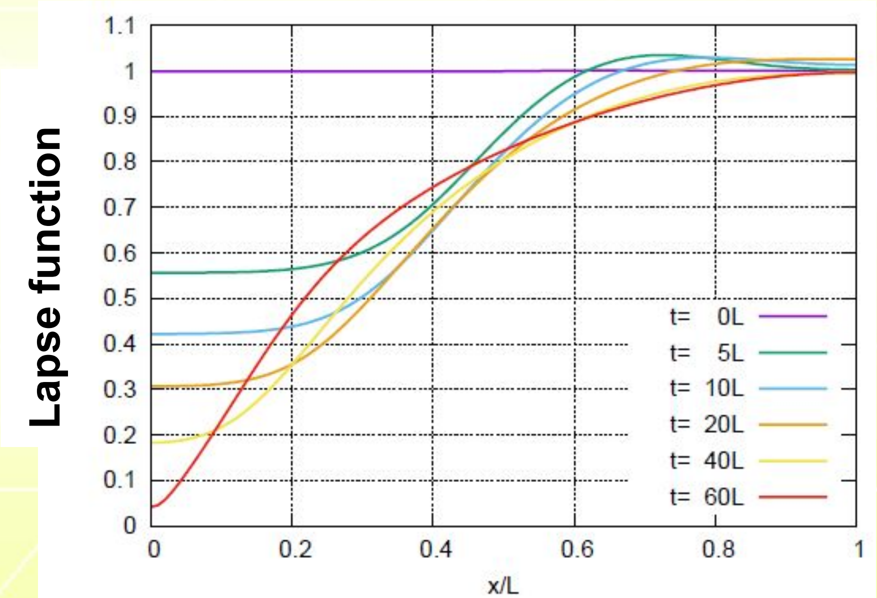
◎**Bouncing back**

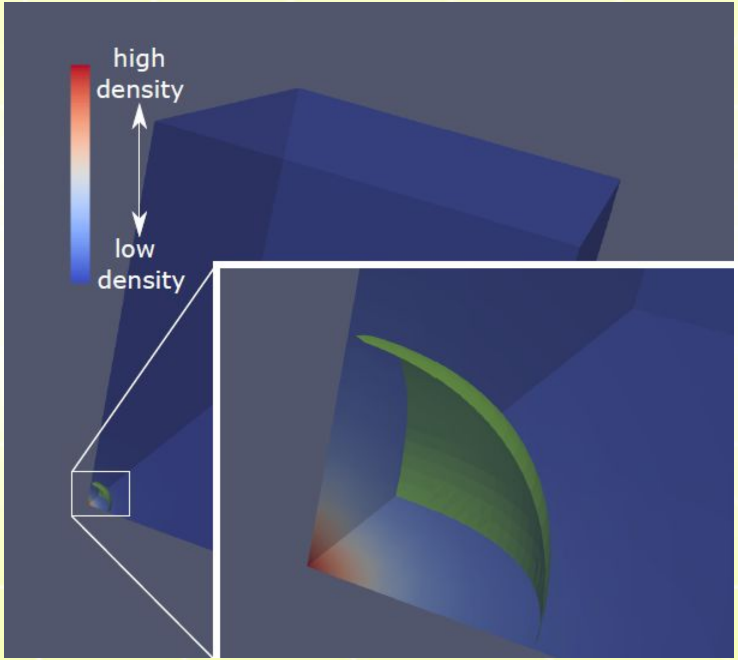


Just above the threshold:horizon formation

◎Parameters

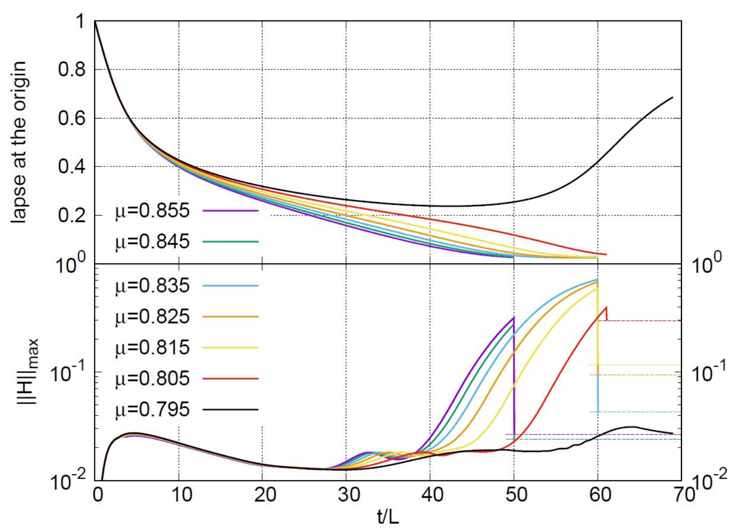
$$|\zeta_0| = 0.805,\ \chi_1 = \chi_2 = 0$$





Constraint violation

◎Unfortunately, constraints are significantly violated near the horizon



◎But, well suppressed for bouncing case and we can read off the threshold: μ~0.8

◎The threshold value is consistent with an accurate spherically symmetric simulation

Results:non-spherical case

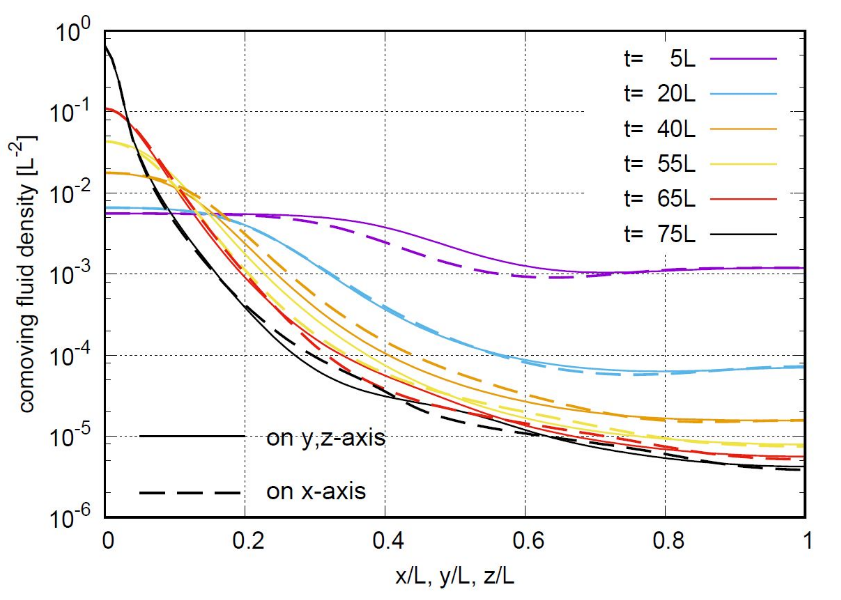
Non spherical spheroidal initial configuration

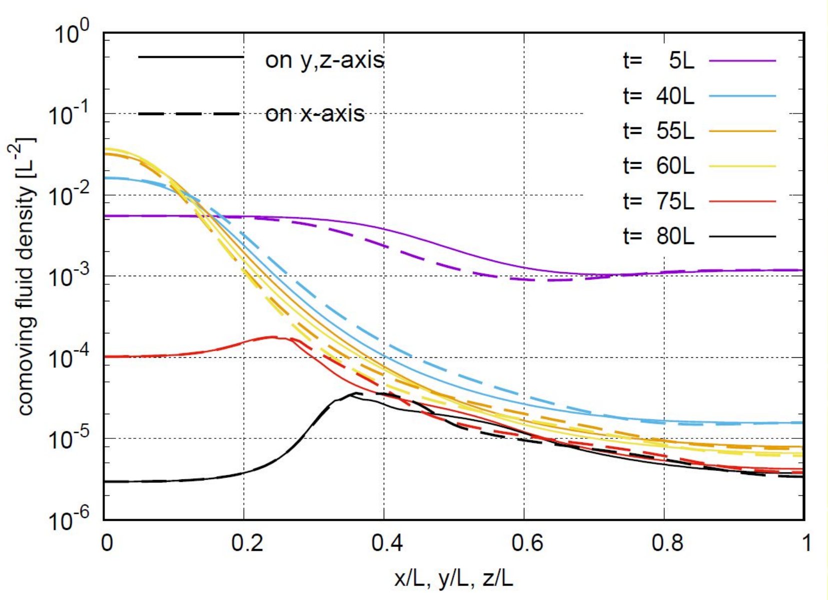
◎ $|\zeta_0| = 0.805$

◎Horizon formation for χ_2=0.08

◎Bounce for χ_2=0.09

Comoving density

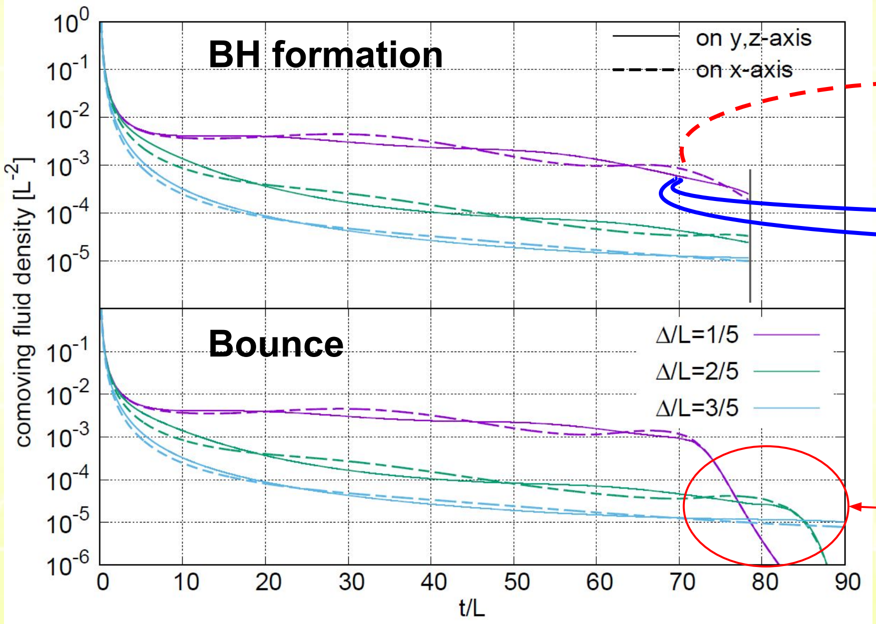


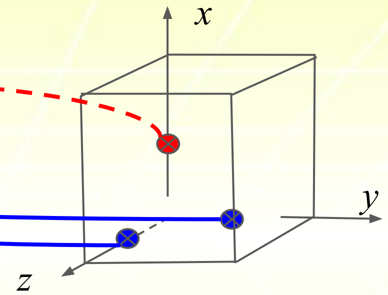


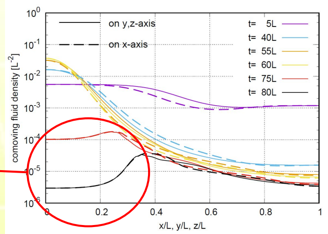
◎Late time configuration near the center is highly spherical in both cases

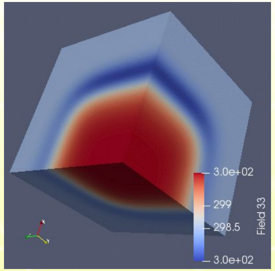
Oscillation

◎Time evolution of the comoving fluid density at fixed spatial points









◎Spherical shape is stable and the oscillation can be found

Conclusion

◎Ellipticity makes the threshold of the amplitude larger, namely, horizon formation harder

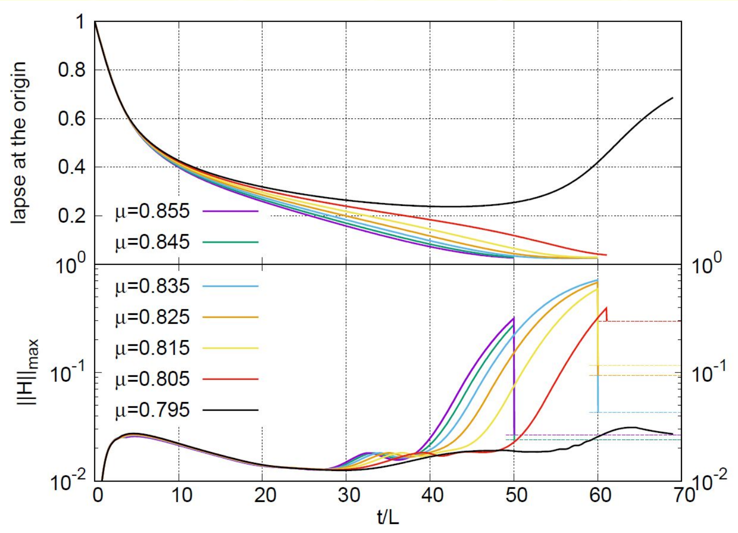
◎But, $\chi_2=|\chi_3|\sim 1$ would be needed for a significant effect

◎Since $\chi_2=\chi_3<<1$ for a realistic situation, the effect would be negligible

◎Significant constraint violation

Constraint violation

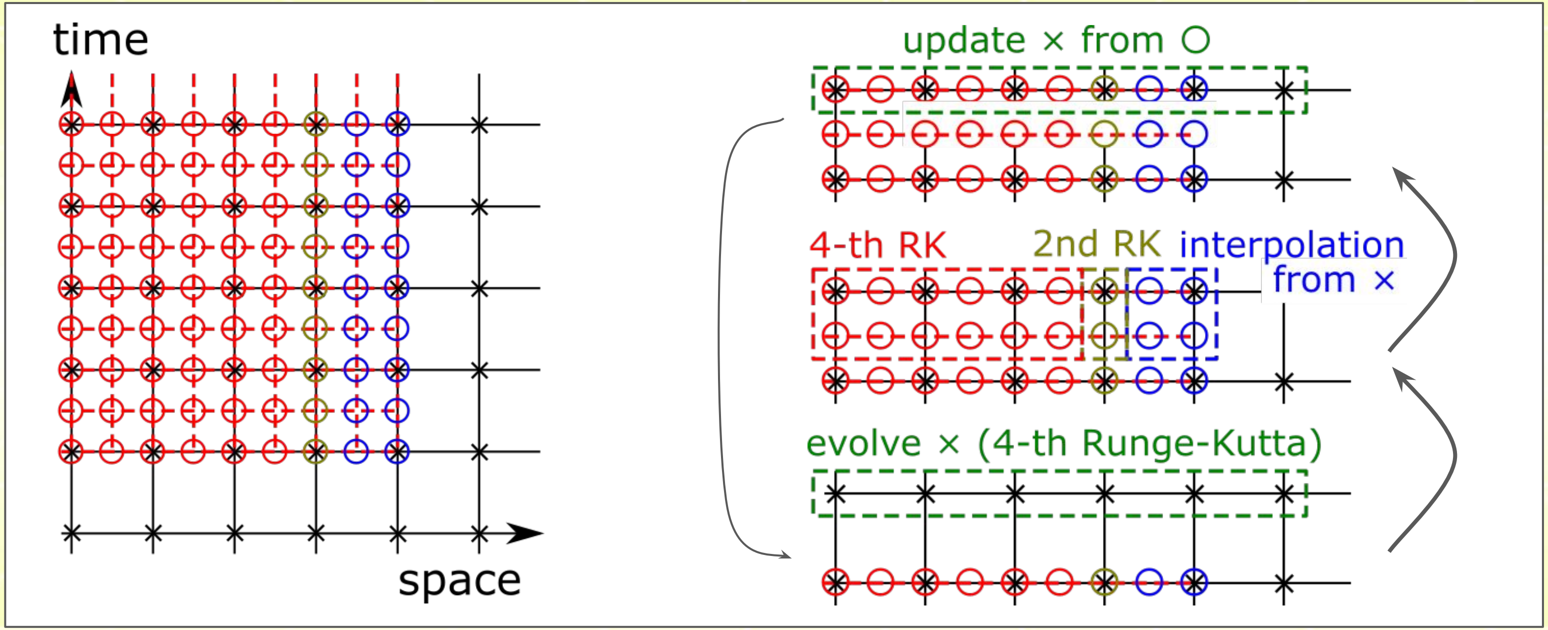
◎Unfortunately, constraints are significantly violated near the horizon



◎Much finer resolution near the center is needed

Mesh refinement

Rough sketch of mesh refinement



[Tetsuro Yamamoto, Masaru Shibata, Keisuke Taniguchi(arXiv:0806.4007)]

◎Twice finer resolution in a local spacetime patch

Summary of resolution difference

◎Resolution in previous simulation

- Scale-up reference coordinates x^i related to the Cartesian coord. X^i by

$$X^i = x^i - \frac{S}{1+S}\frac{L}{\pi}\sin\left(\frac{\pi}{L}x^i\right) \text{ with } S = 15$$

- Resolution at the center ($\Delta x = L/100$)

$$\Delta X|_{\text{center}} = \frac{1}{1+S}\Delta x = \frac{1}{16}\frac{L}{100} = \frac{L}{1600}$$

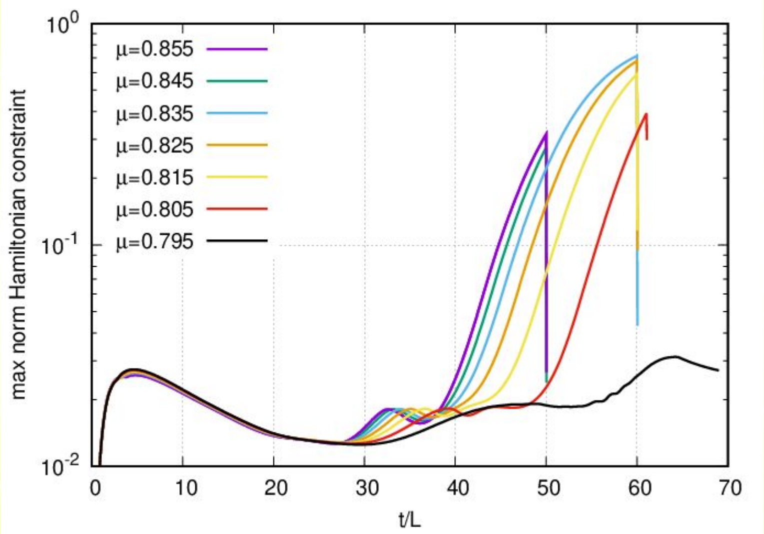
◎New simulation with mesh refinement

- $S = 10, \Delta x = L/60$
- Two additional layers for the mesh refinement

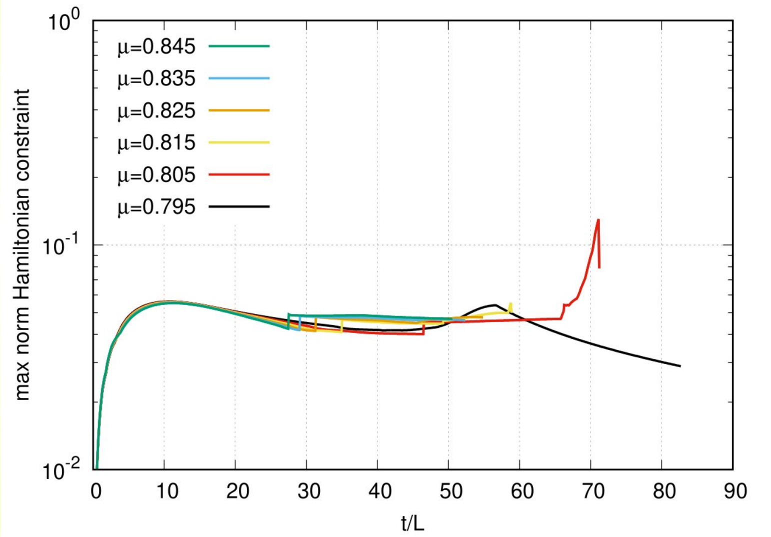
$$\Delta X|_{\text{center}} = \frac{1}{1+S} \times \frac{1}{2^2} \times \Delta x = \frac{1}{44} \times \frac{L}{60} = \frac{L}{2640}$$

Constraint violation w/ mesh refinement

◎Without mesh refinement



◎With mesh refinement



◎The mesh refinement reduces the constraint violation!

Towards the simulation of a spinning PBH

Very Very Preliminary

Initial condition

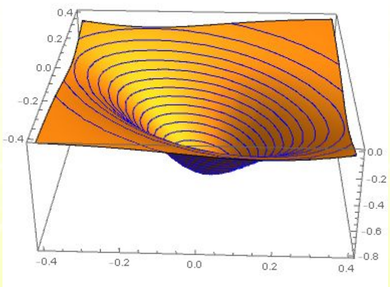
◎Initial curvature perturbation

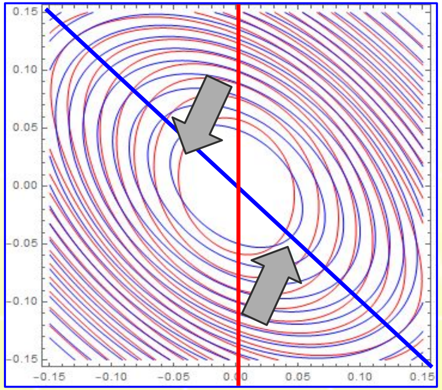
$$\frac{\zeta}{\mu} \simeq -1 + \frac{1}{2}\left(k_1^2(x+y)^2/2 + k_2^2(x-y)^2/2 + k_3^2 z^2\right) + \mathcal{O}(r^4)$$

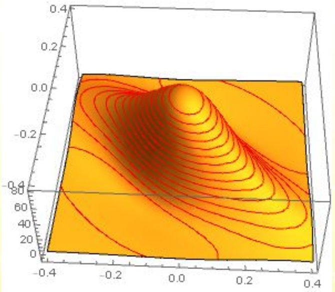
$$\frac{\triangle\zeta}{\mu k^2} \simeq 1 - \frac{1}{2}\left(k_1^2 x^2 + k_2^2 y^2 + k_3^2 z^2\right) + \mathcal{O}(r^4)$$

ζ∼gravitational potential on (x,y) plane

Δζ∼energy density on (x,y) plane



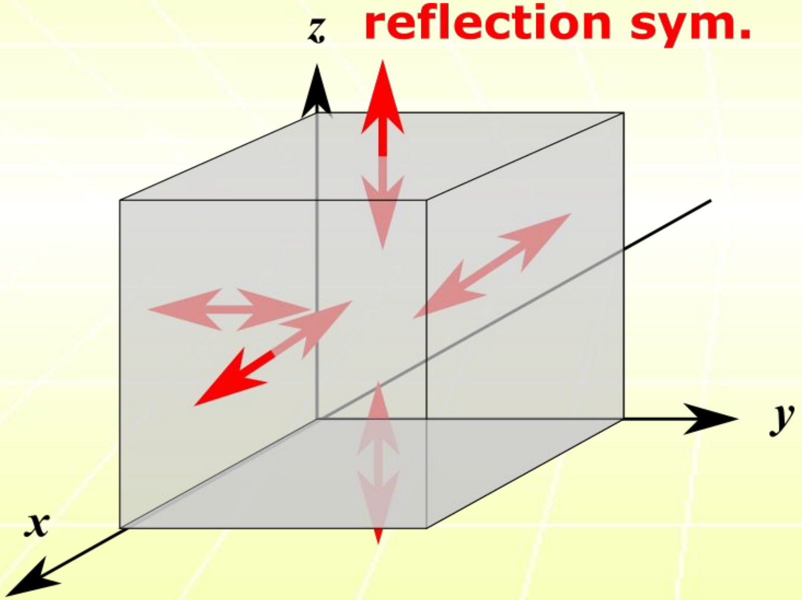


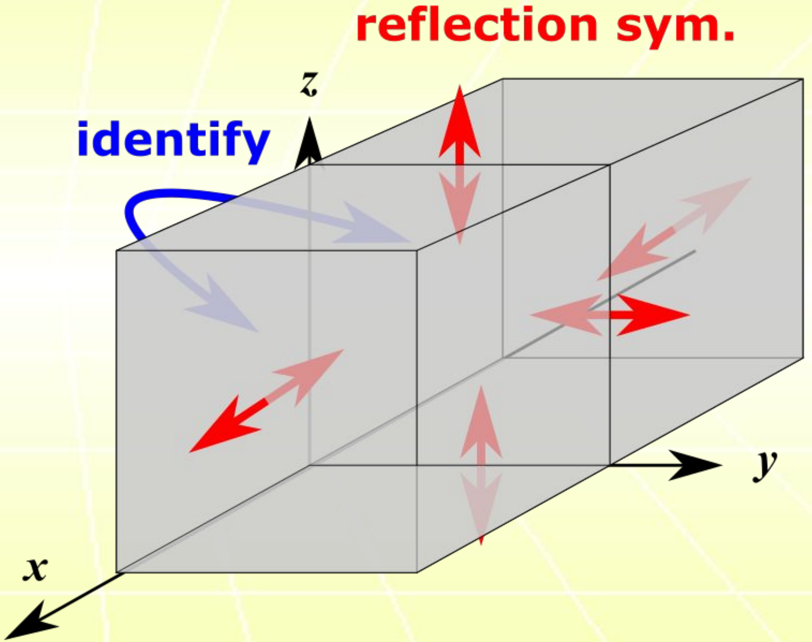


tidal torque ⇒ angular momentum transfer ⇒ spinning PBH

Extension of the domain and boundary cond.

◎1/8 region → 1/4 region





Initial condition

◎Initial curvature perturbation

$$ds^2 \simeq -dt^2 + a(t)^2 e^{-2\zeta(x)} d\vec{x}\cdot d\vec{x}$$

$$\zeta = -\mu\Big[1 + \frac{1}{2}\left(k_1^2(x+y)^2/2 + k_2^2(x-y)^2/2 + k_3^2 z^2\right) + \frac{1}{4}\left(k_1^2(x+y)^2/2 + k_2^2(x-y)^2/2 + k_3^2 z^2\right)^2 + \frac{1}{280}k^2r^2\left(9\kappa_1^2 - \kappa_2^2 - \kappa_3^2\right)x^2 + (\kappa_1^2 - 9\kappa_2^2 + \kappa_3^2)y^2 + (\kappa_1^2 + \kappa_2^2 - 9\kappa_3^2)z^2\Big]^{-1} \exp\left[-\frac{1}{2880}k^6r^6\right]$$

$$\frac{\zeta}{\mu} \simeq -1 + \frac{1}{2}\left(k_1^2(x+y)^2/2 + k_2^2(x-y)^2/2 + k_3^2 z^2\right) + \mathcal{O}(r^4)$$

$$\frac{\triangle\zeta}{\mu k^2} \simeq 1 - \frac{1}{2}\left(\kappa_1^2 x^2 + \kappa_2^2 y^2 + \kappa_3^2 z^2\right) + \mathcal{O}(r^4)$$

Initial condition

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ζ∼gravitational potential on (x,y) plane

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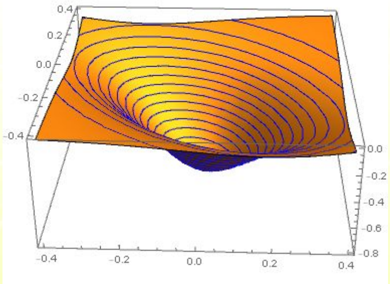
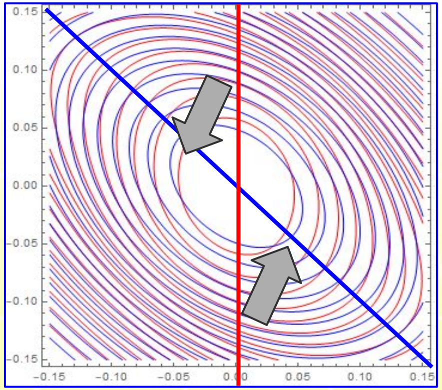
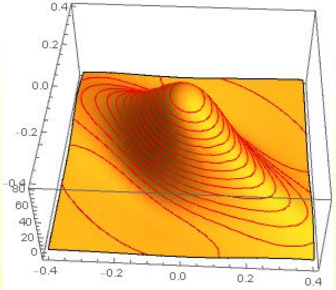


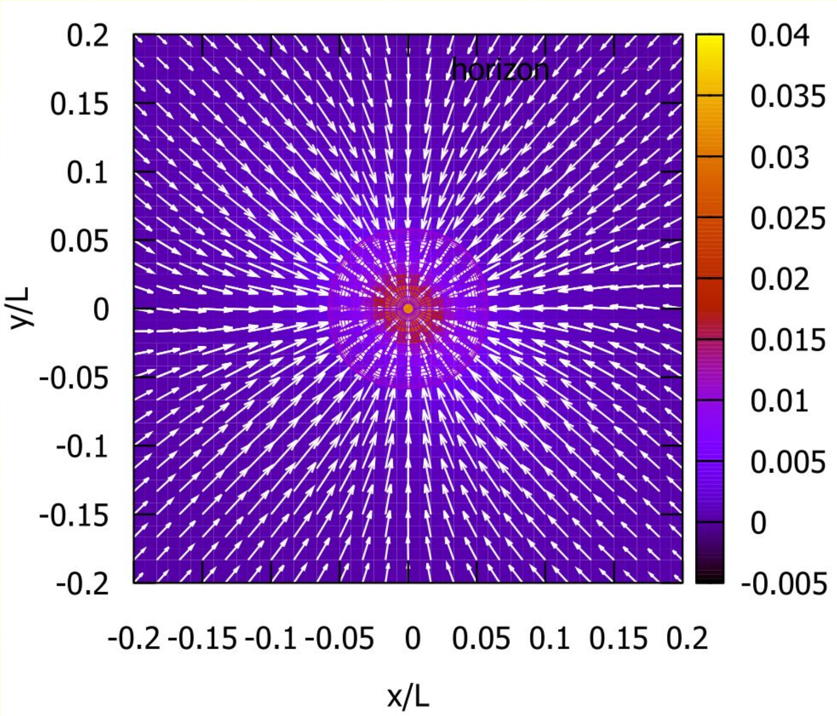




tidal torque ⇒ angular momentum transfer ⇒ spinning PBH

Spinning PBH formation?

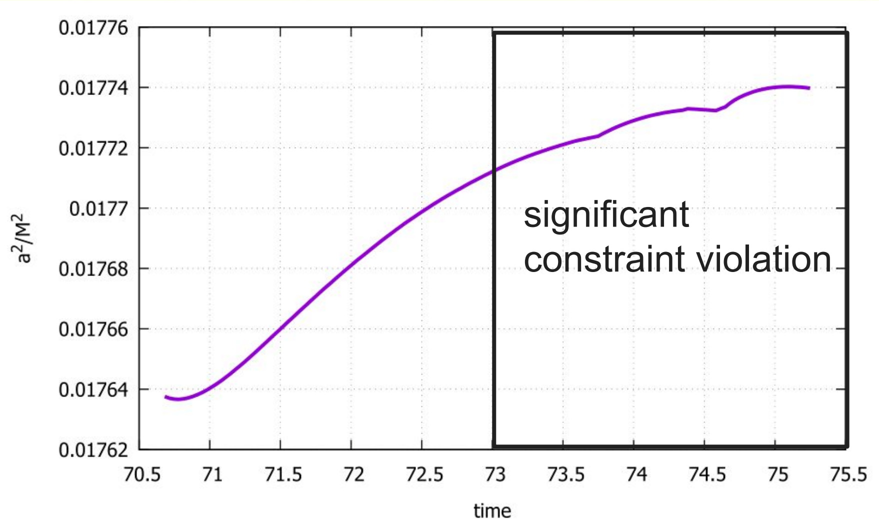
◎Non-zero spin parameter…?



$$A_{\rm Kerr} = 8\pi(M^2 + \sqrt{M^4 - a^2 M^2})$$

$$A_{\rm Kerr} = 8\pi(M^2 + \sqrt{M^4 - a^2 M^2})$$

$$\Rightarrow \left(\frac{a^2}{M^2}\right)_{\rm eff} = \frac{4\pi A(l^2 - \pi A)}{l^4}$$



Better resolution is needed…