Mini-Neutron Star Collision on Laptop

Dae Ik Kim[1]

in collaboration with C.-H. Lee[1], S. Jeon[2], Y. Kim[3] and K. Kim[4]

[1]Department of Physics, Pusan National University, South Korea
[2]Department of Physics, McGill University, Canada
[3]Center of Exotic Nuclear Studies, Institute of Basic Science, South Korea
[4]Institute for Rare Isotope Science, Institute of Basic Science, South Korea

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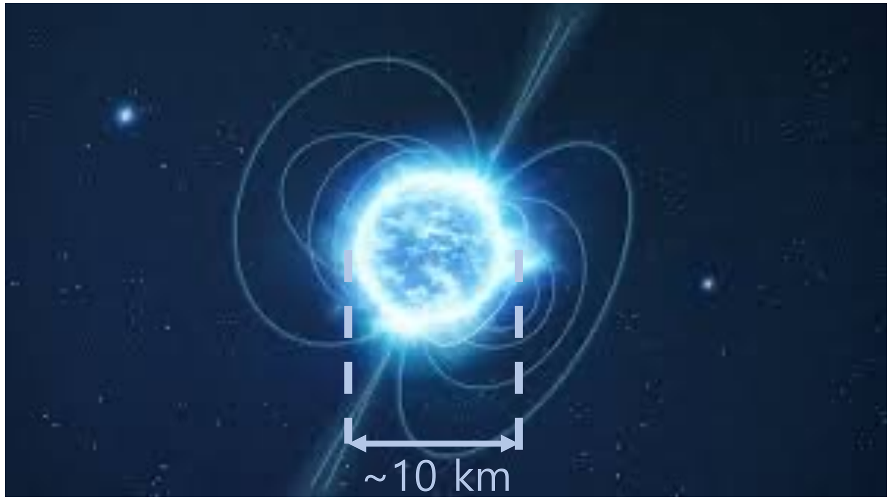
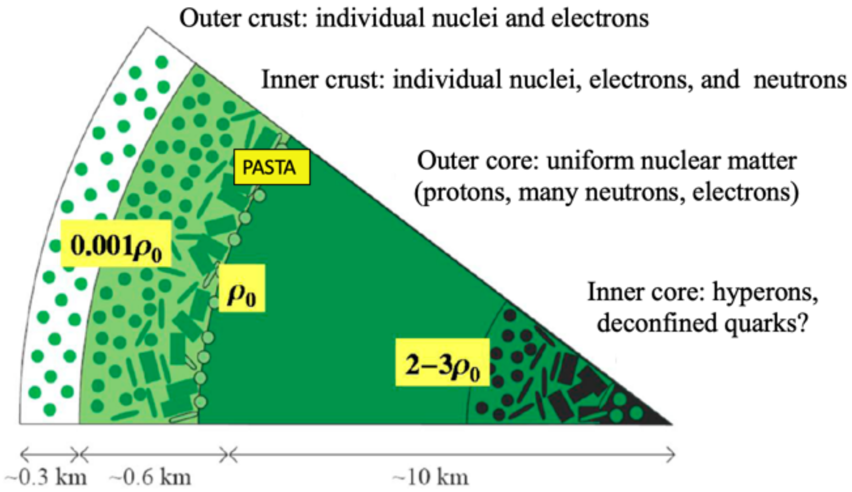
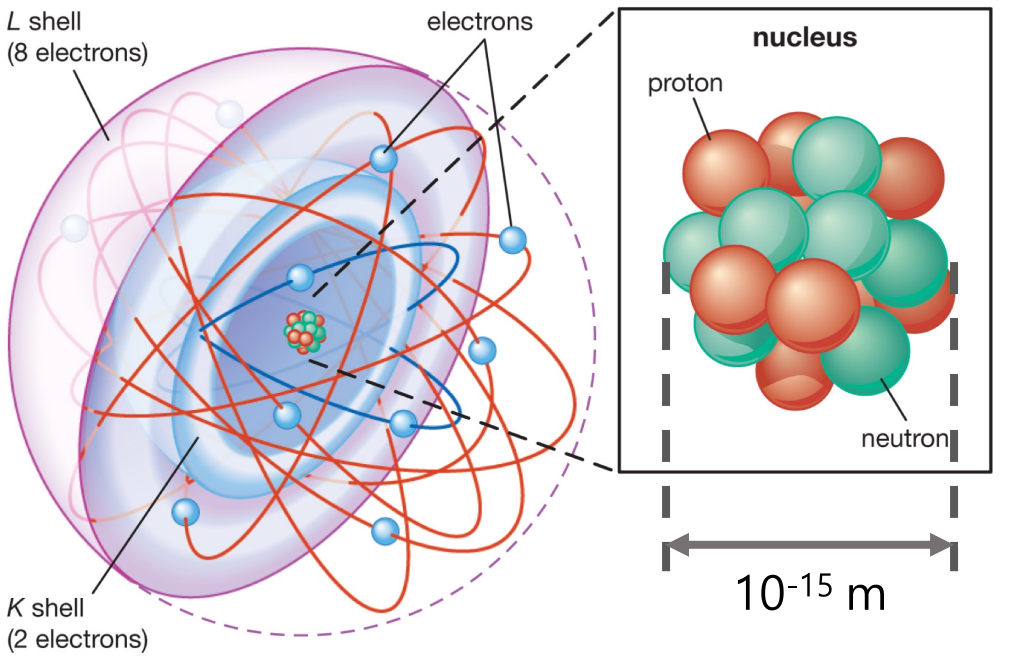
Introduction

Neutron Star, Mini-Neutron Star



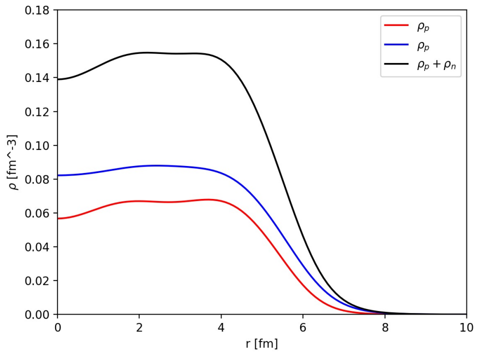


Dr. Veronica Dexhemimer, Kent State Univeristy



Mini-Neutron Star = Nucleus
Quantum many-body system
Saturation density ρ_0

Nuclear Physics ~ Astrophysics



From Nuclei

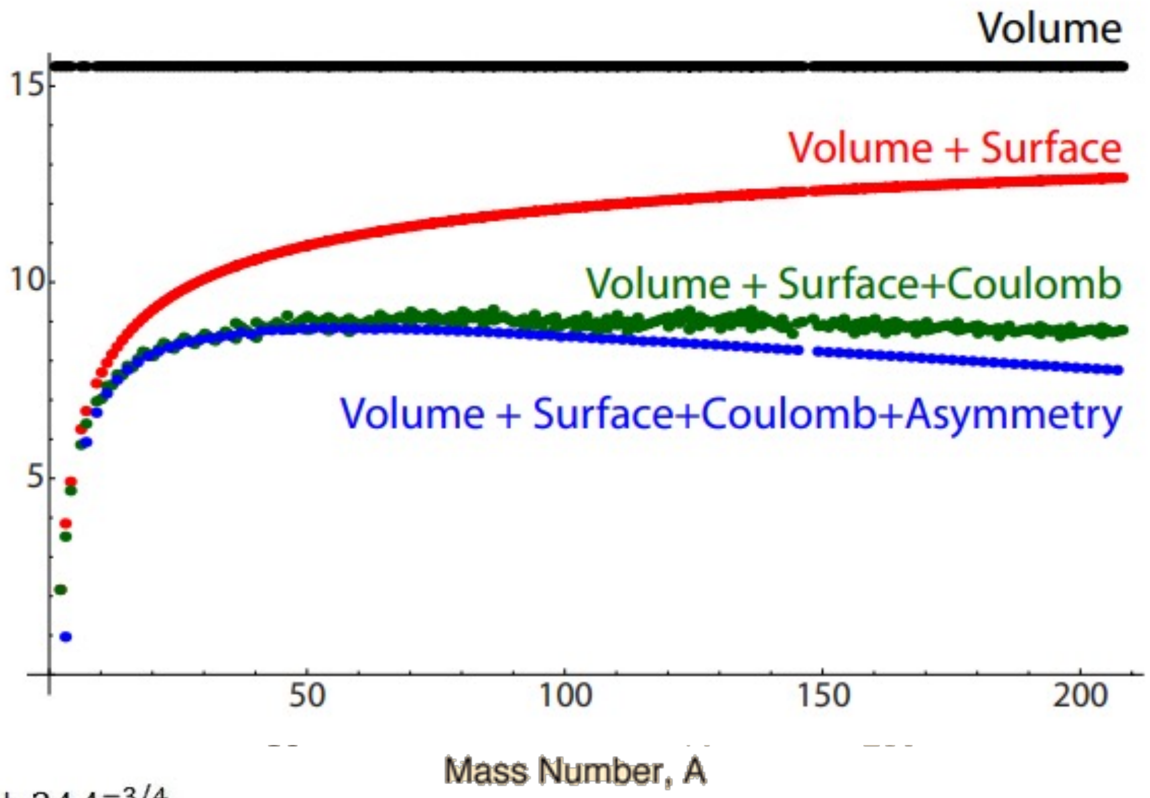
Binding energy of Nuclei

$$m_{\text{nucleus}}c^2 = Zm_pc^2 + Nm_nc^2 - E_b$$

Weizsäcker formula

$$E_b(MeV) = a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z^2}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} \pm \delta(A,Z)$$

$$\delta(A,Z) = \begin{matrix} +\delta_0 \; for \; Z,N \; even \\ 0 \\ -\delta_0 \; for \; Z,N \; odd \end{matrix}$$



$$E_b(MeV) = 15.76A - 17.81A^{2/3} - 0.711\frac{Z^2}{A^{1/3}} - 23.7\frac{(N-Z)^2}{A} \pm 34A^{-3/4}$$

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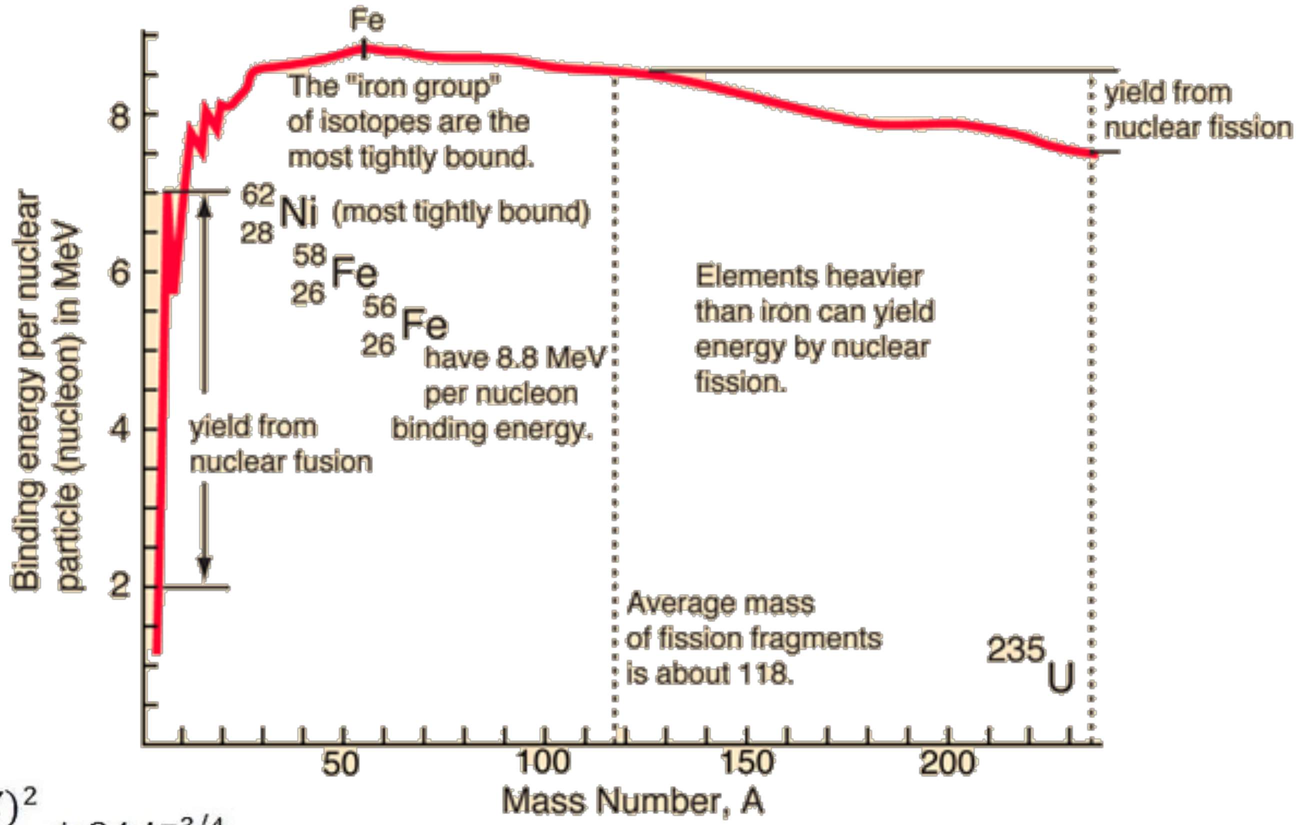


Nuclear matter properties

Binding energy per Nucleon

$$E(\rho,\alpha) = E_0(\rho) + E_{\mathrm{sym}}(\rho)\alpha^2$$

$$= E_0 + \frac{1}{2!}K_0\chi^2 + \left(S_0 + L\chi + \frac{1}{2!}K_{\mathrm{sym}}\chi^2\right)\alpha^2$$

where,

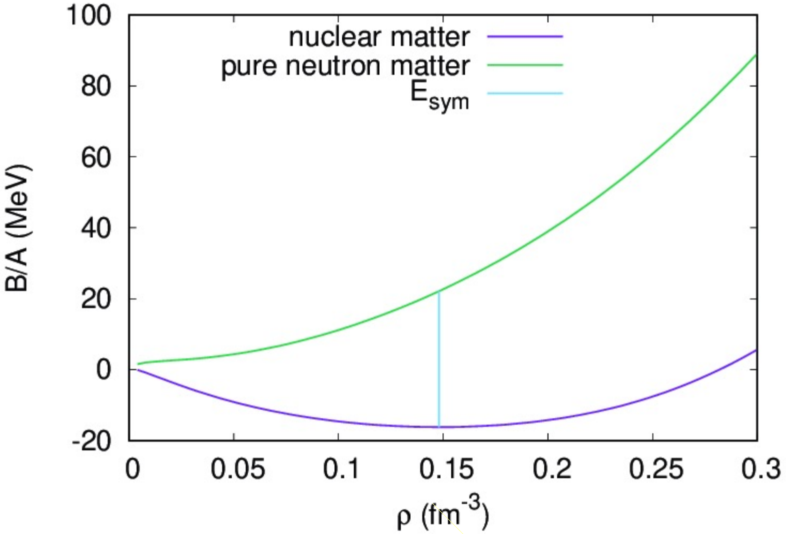
$$\chi = \frac{\rho-\rho_0}{3\rho_0} \quad , \quad \alpha = \frac{\rho_p-\rho_n}{\rho}$$

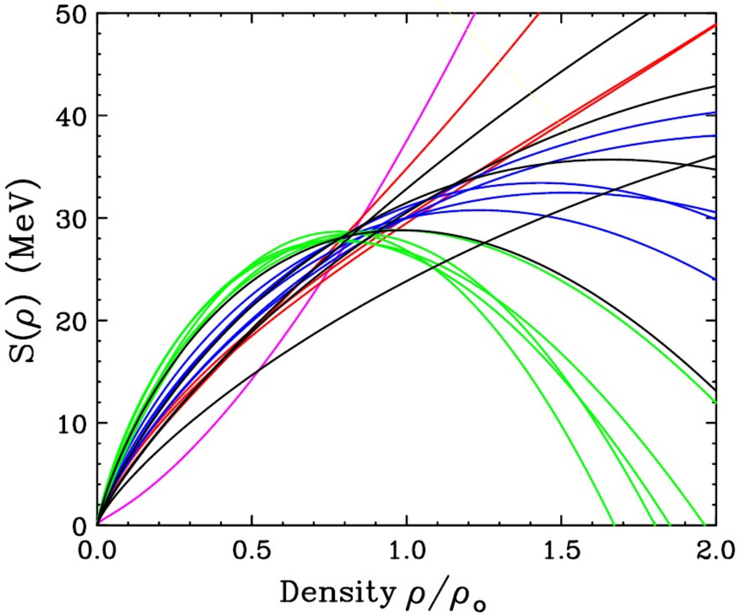
Incompressibility

$$K_0 = 9\rho_0^2 \frac{d^2E_0(\rho)}{d\rho^2}\bigg|_{\rho=\rho_o} = 240\ (\pm)\ \mathrm{MeV}$$

Slope parameter

$$L = 3\rho_0 \frac{dE_{\mathrm{sym}}(\rho)}{d\rho}\bigg|_{\rho=\rho_o} = 50\ (\pm)\ \mathrm{MeV}$$





Brown, Phys. Rev. Lett. 85, 5296 (2001)

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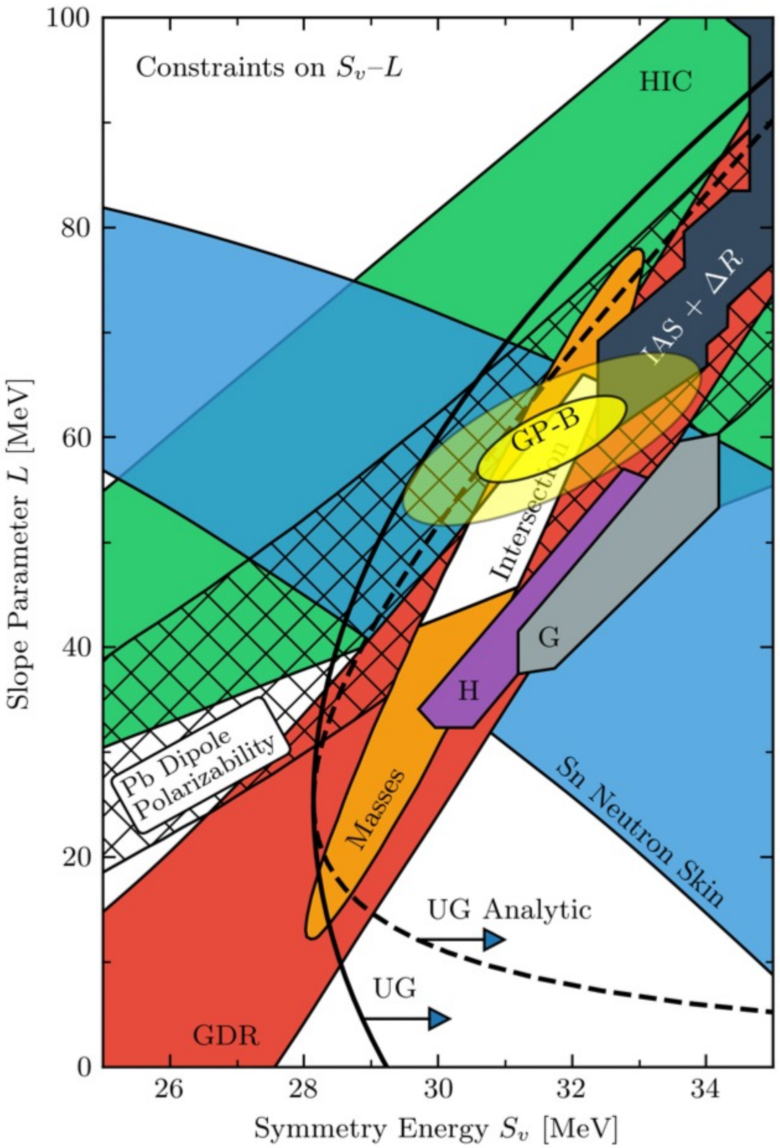
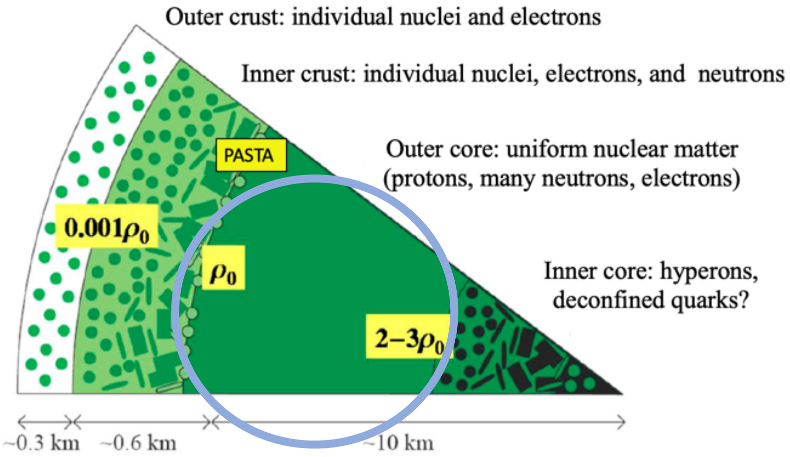


Bridge between Neutron stars and Nuclei



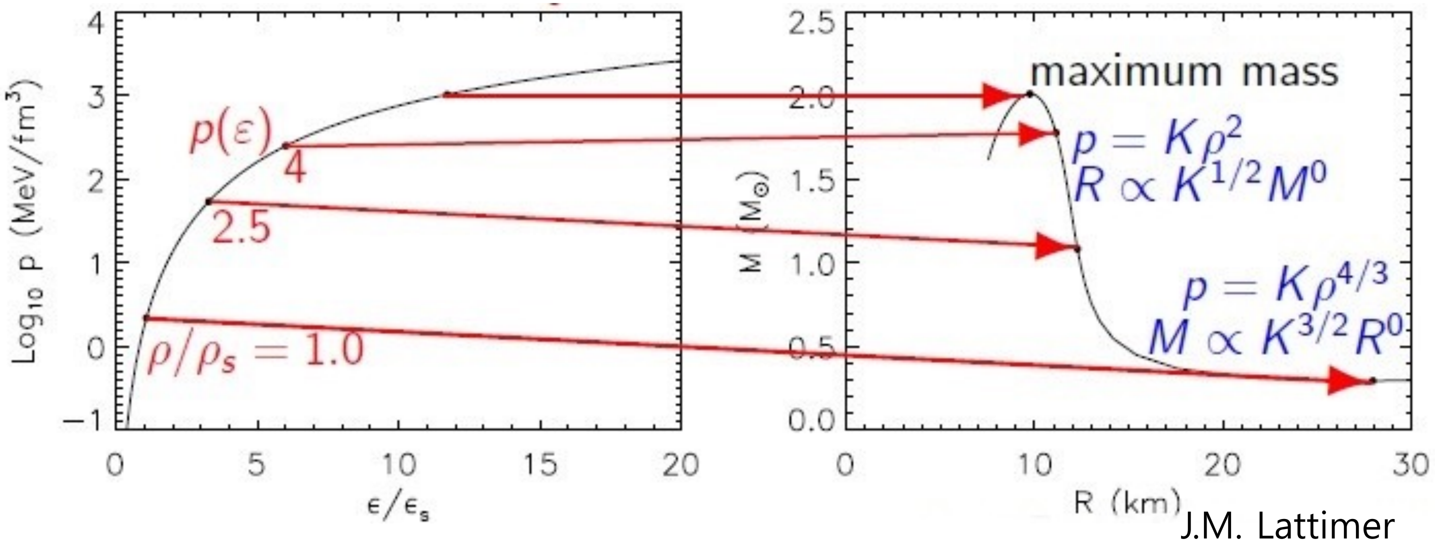
Strong force + Gravity

- Skyrme approach
- RMF approach
- etc…

Tolman–Oppenheimer–Volkoff (TOV) equation

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(Mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2GM/c^2)}, \qquad \frac{dm}{dr} = 4\pi\varepsilon r^2/c^2,$$

Energy density ~ Pressure → Radius ~ Mass

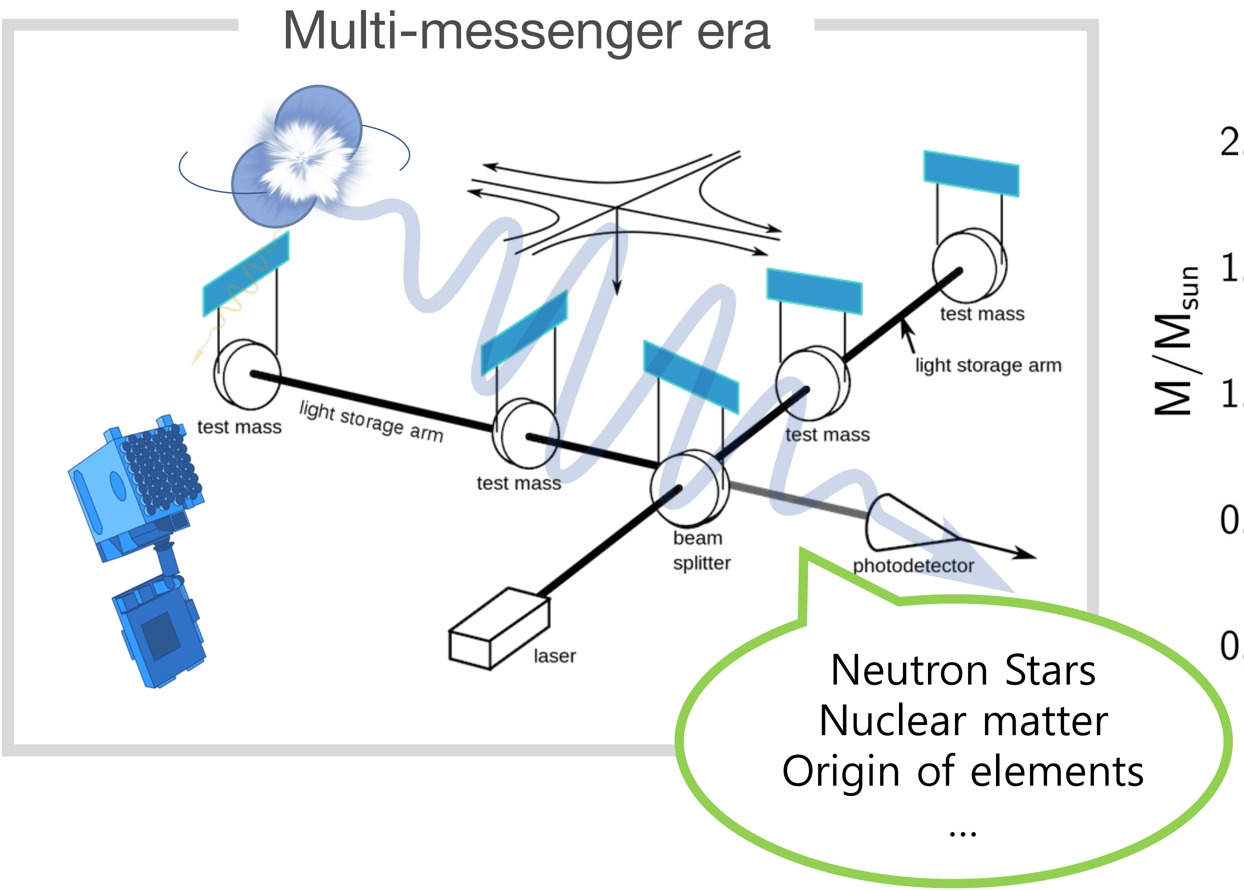


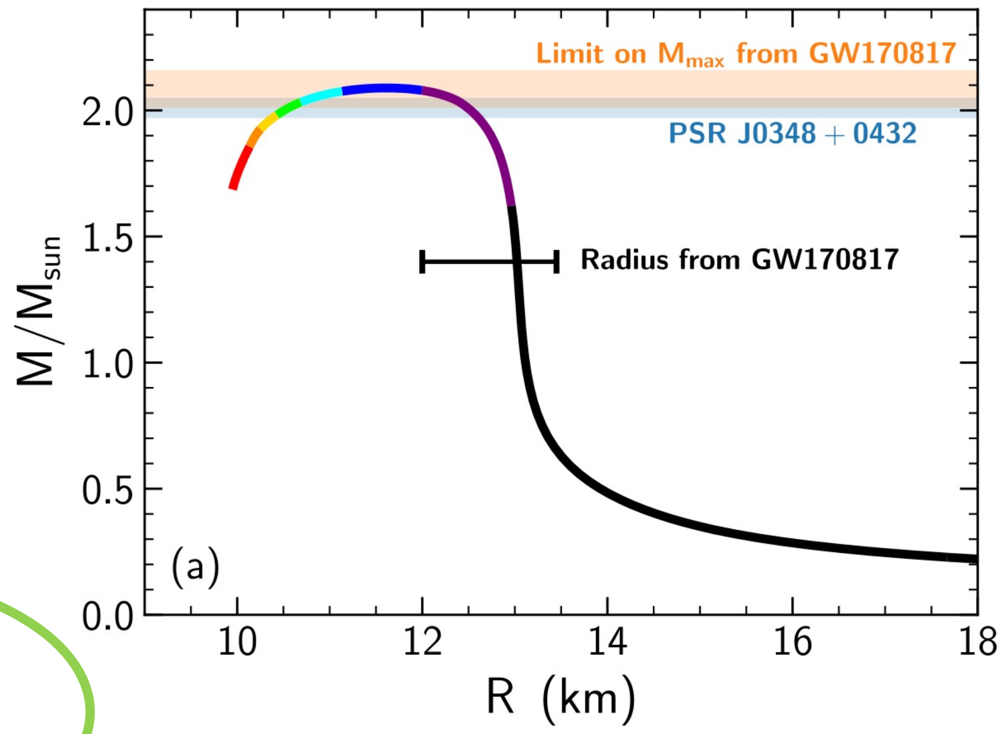
J.M. Lattimer

ρ_0, S_0, K_0, J …

Neutron Star Collision

- One of the sources of heavy elements
- And the source of gravitational wave, such as GW170817





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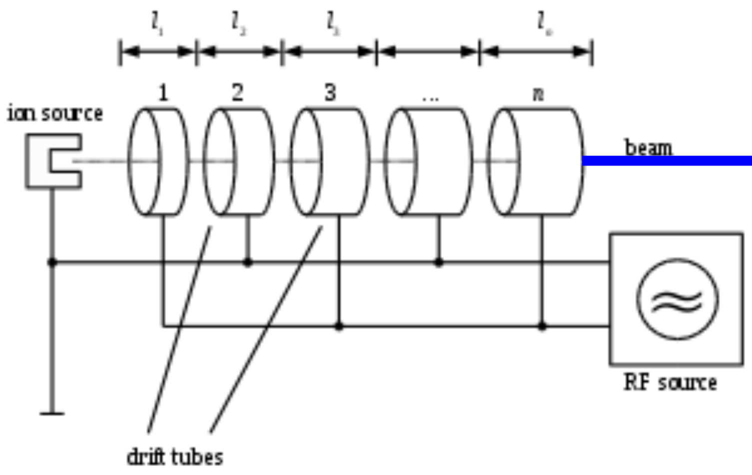
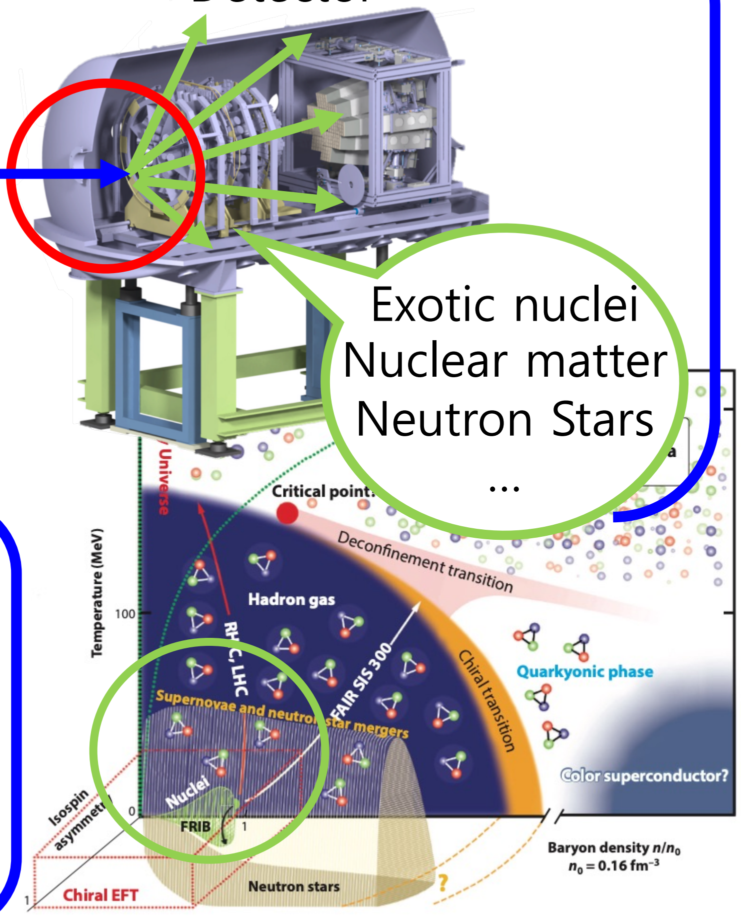
Mini-Neutron Star Collision on Lab

Heavy Ion Collisions (Experiment) → Information

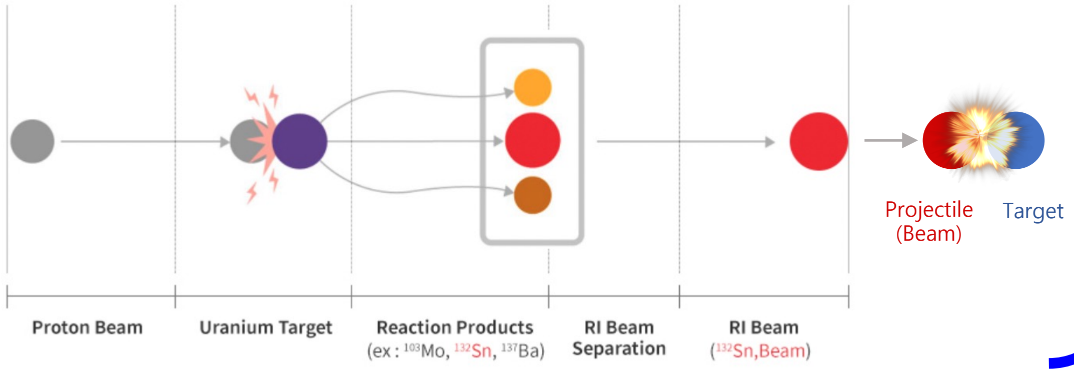
Accelerator



Detector



Rare Isotope(RI) beam facilities, FRIB, FAIR & RAON



Mini-Neutron Star Collision on Laptop

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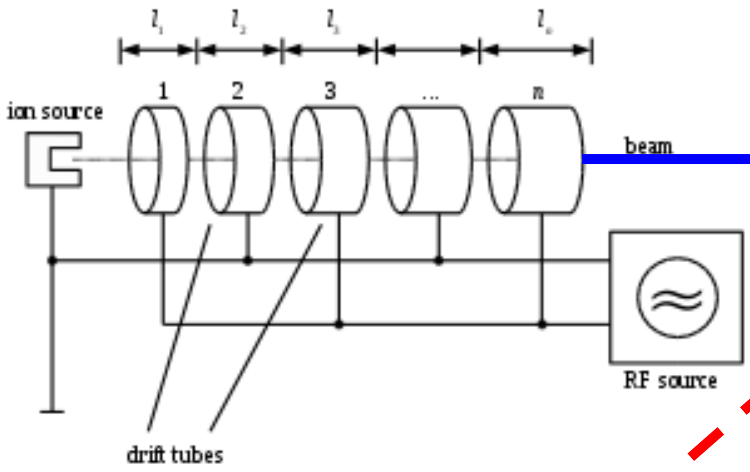
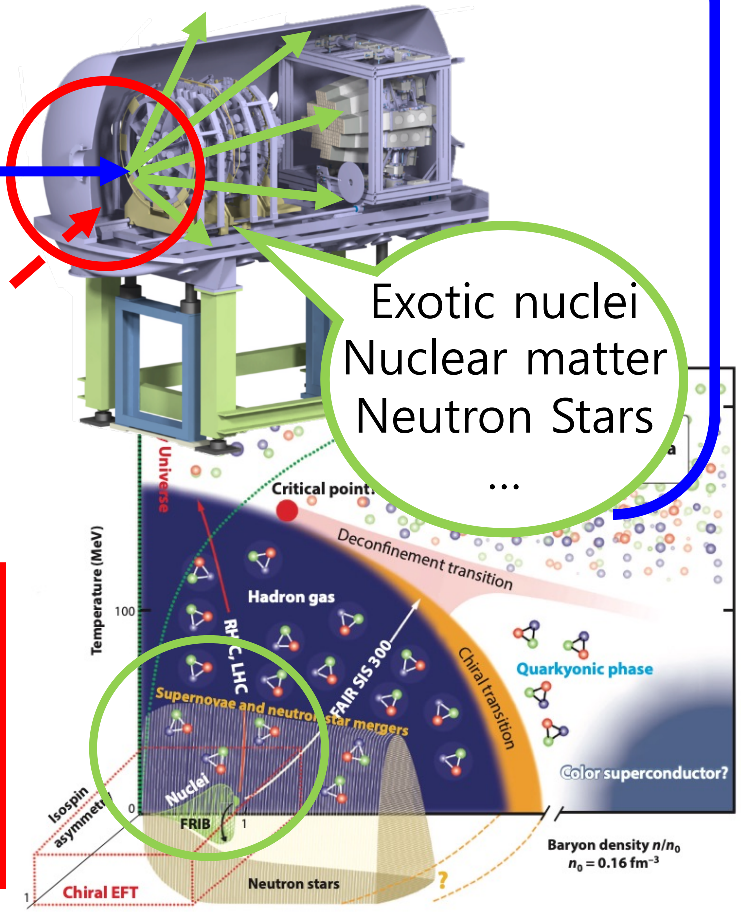


Detector



With numerical model (Theory)

Transport model

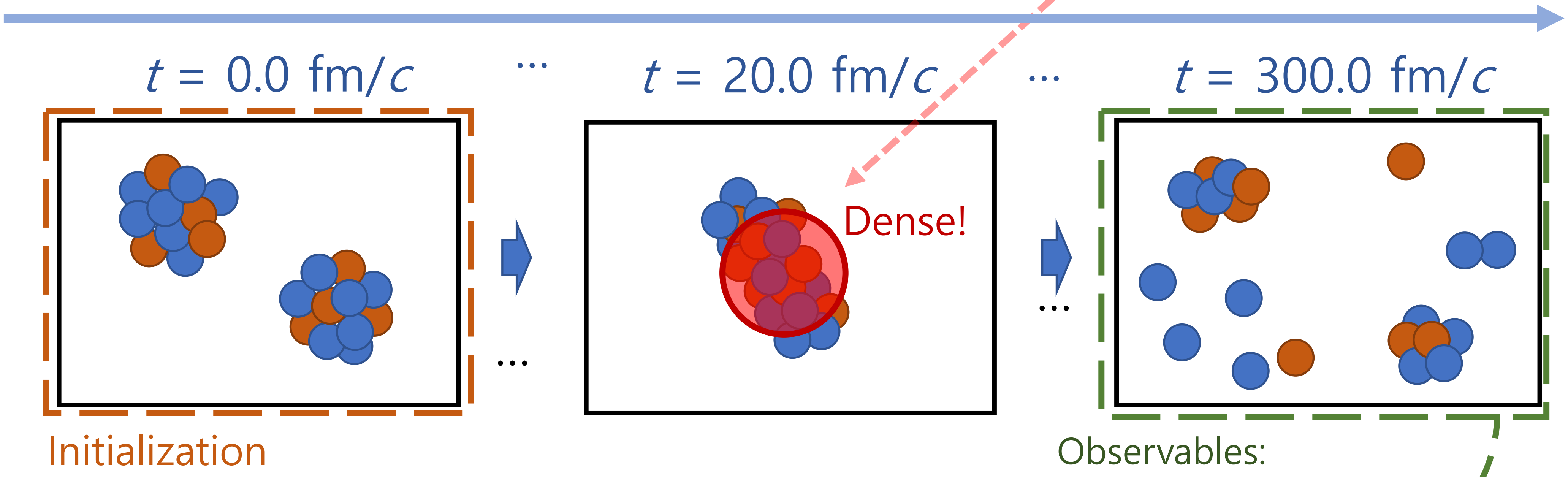
Transport model

Transport model is Theoretical model to allow us study dense matter

$\rho_B > \rho_0$

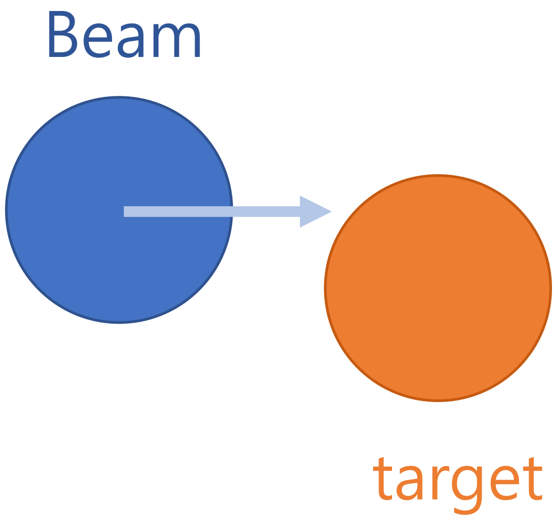
- Semi-classical method
- Hadron degree of freedom

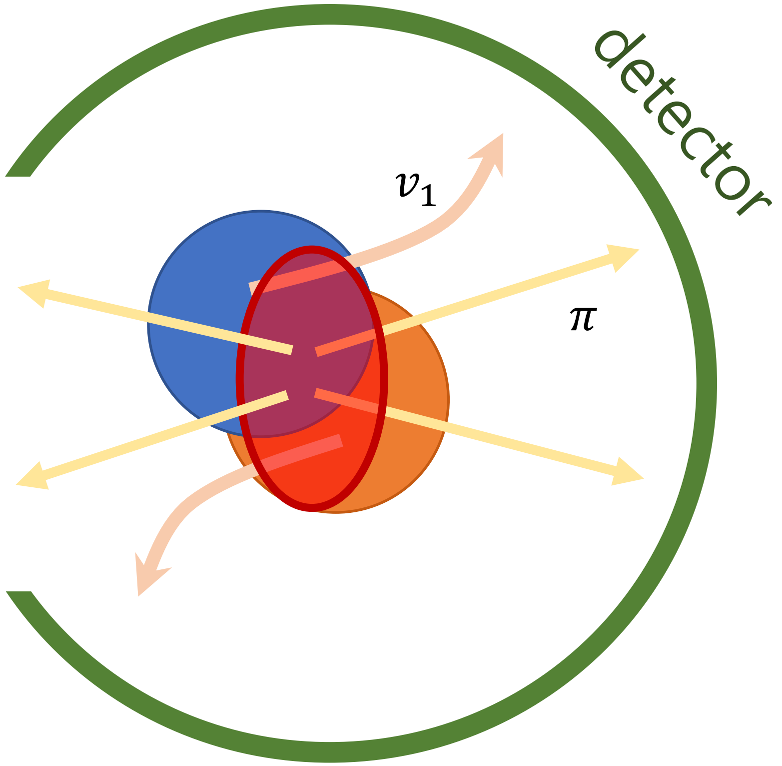
Full time evolution of Dynamics in Heavy Ion Collision!!



Observables:
Flow (v_1, v_2), Yield,
Ratio (p/n, π^+/π^-, …),
Fragment …

HIC observables



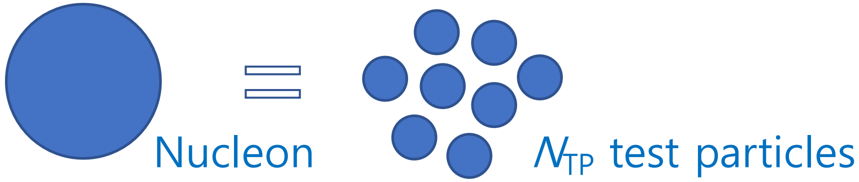


$$\frac{dN}{u_t du_t dy d\phi} = v_0[1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi)],$$

$$v_1 = \left\langle \frac{p_x}{p_t} \right\rangle = \langle \cos(\phi) \rangle,\ v_2 = \left\langle \left(\frac{p_x}{p_t}\right)^2 - \left(\frac{p_x}{p_t}\right)^2 \right\rangle = \langle \cos(2\phi) \rangle$$

Two type of Transport model

Boltzmann-Uehling-Uhlenbeck (BUU)



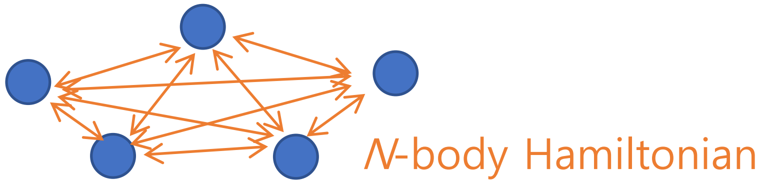
$$f(\vec{x},\vec{p}) = \frac{(2\pi)^3}{N_{TP}} \sum_{i=1}^{AN_{TP}} g_x(\vec{x}-\vec{x_i}) g_p(\vec{p}-\vec{p_i})$$

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\cdot\nabla_r - \nabla_r U \cdot \nabla_p\right) f(\vec{r},\vec{p};t) = I_{coll}[f;\sigma_{12}]$$

BLOB, GiBUU, pBUU, SMASH and **DJBUU**

Quantum Molecular Dynamics (QMD)

$$f(\vec{x},\vec{p}) = \exp[-\frac{1}{2\sigma_r^2}(\vec{r}-\vec{R})^2 + \left(-\frac{2\sigma_r^2}{\hbar^2}\right)(\vec{p}-\vec{P})^2]$$



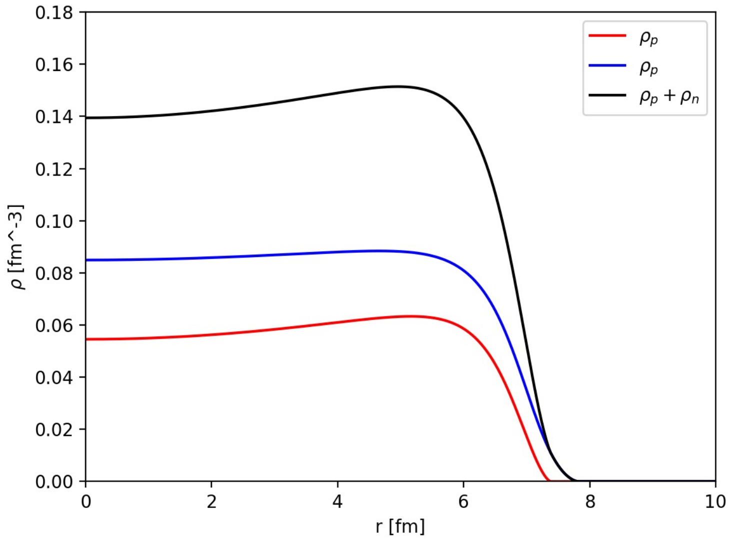
$$H(\vec{r}_n,\vec{p}_n) = \sum_{i=1}^{A} \frac{\vec{p}_i^2}{2m_i} + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|)$$

AMD, UrQMD, CoMD, ImQMD and **SQMD** …

We've developed two Transport model, **DJBUU** and **SQMD**
To study HIC experiments that will be conducted in RAON

Initialization

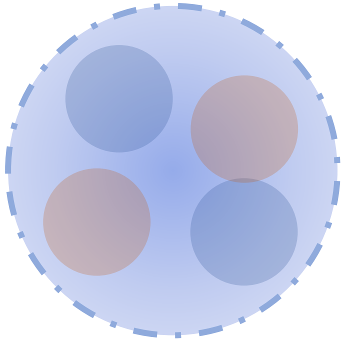
- Wood-Saxon, $\rho(r) = \frac{\rho_0}{1+e(r-R)/d}$
- Relativistic Thomas Fermi (RTF)



Density profile -> Position
Fermi momentum -> momentum

Test particles method,

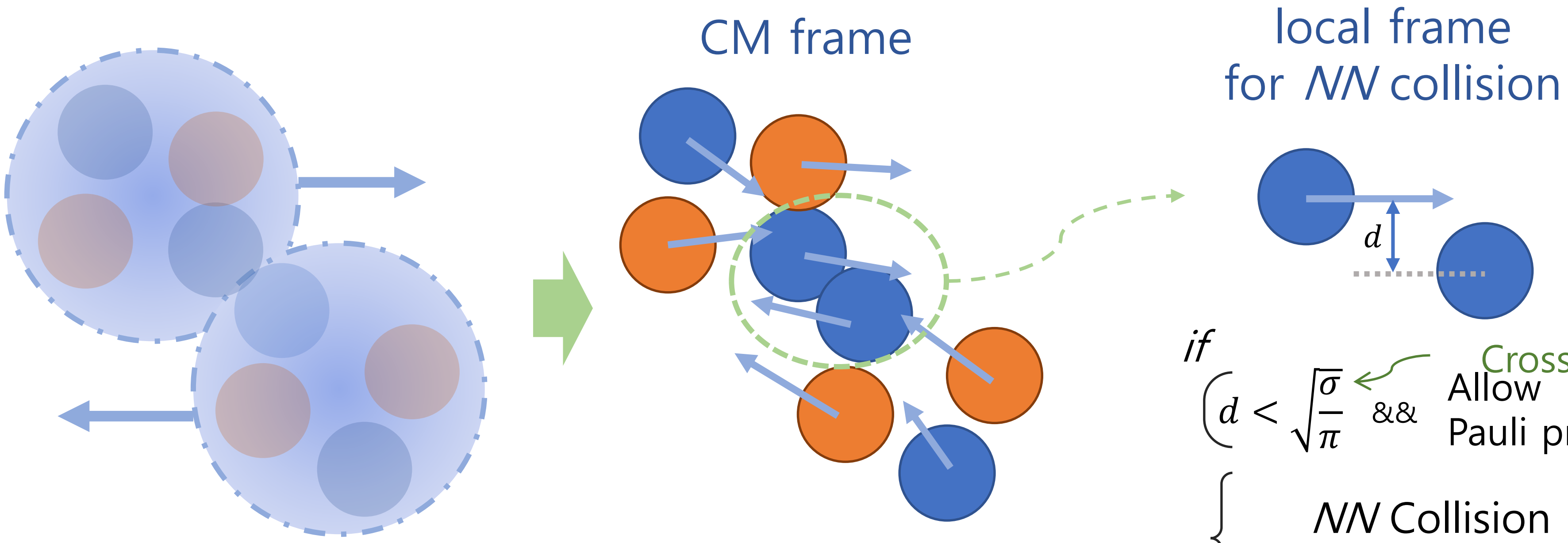
$$g(\vec{u}) = g(u) = N_{m,n}\left[1 - \left(\frac{u}{a_{cut}}\right)^m\right]^n$$



- RCHB (Bubble), DRHBc (deformation)
  Density profile of Nucleus → HIC observable, such as v_1

Nucleon-Nucleon collisions

Two Nuclei system → Many Nucleons system & Hard collision



if $\left(d < \sqrt{\frac{\sigma}{\pi}} \right.$ && Allow Pauli principle $\left. \right)$ ← Cross-section

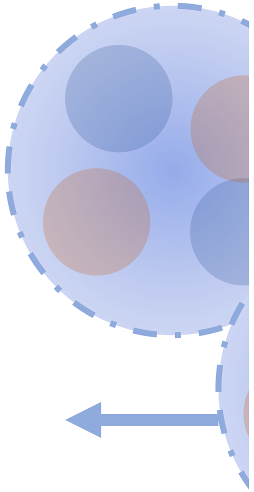
{ *NN* Collision occur!
like Billiard balls Collision }

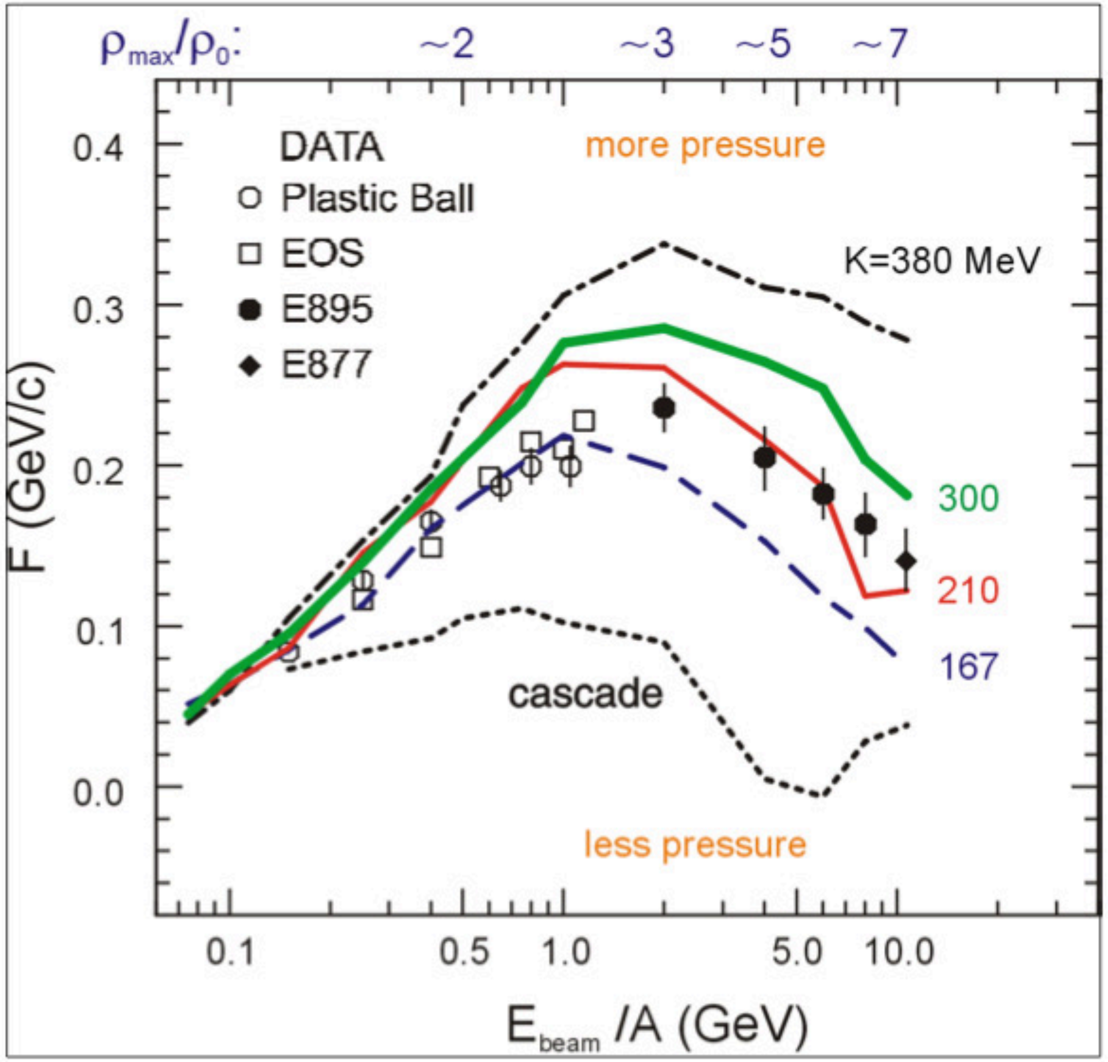
NN collisions & ~~free propagation (Cascade mode)~~ not enough

& Propagation with potential

Nucleon-Nucleon collisions

Two Nucle





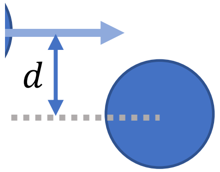
Science 298, 1592 (2002)

NN collisi

d collision

al frame

N collision



Cross-section

Allow

&&

Pauli principle

N Collision occur!

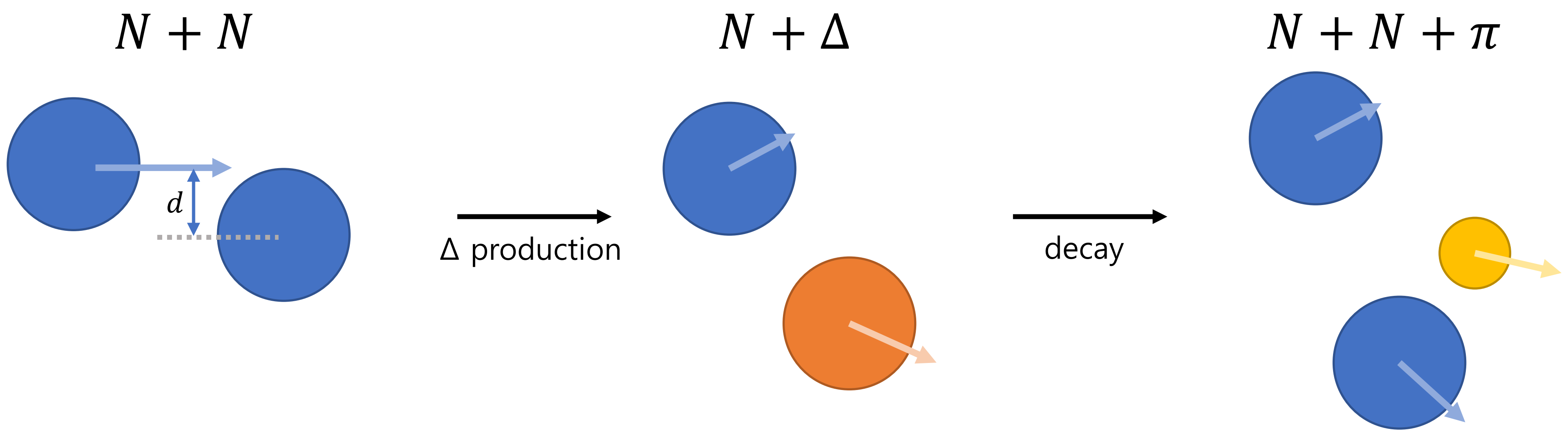
Billiard balls Collision

t enough

h potential

Nucleon-Nucleon collisions

Pion production in Heavy Ion Collision in intermediate energy reign

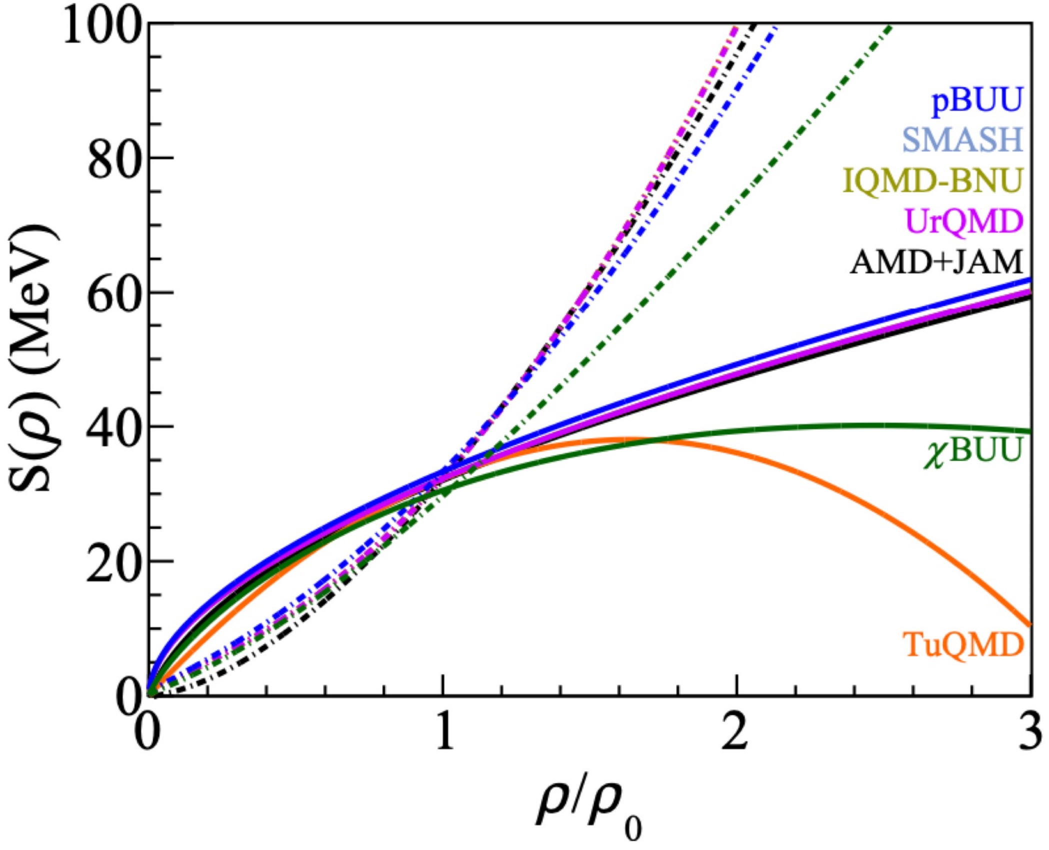


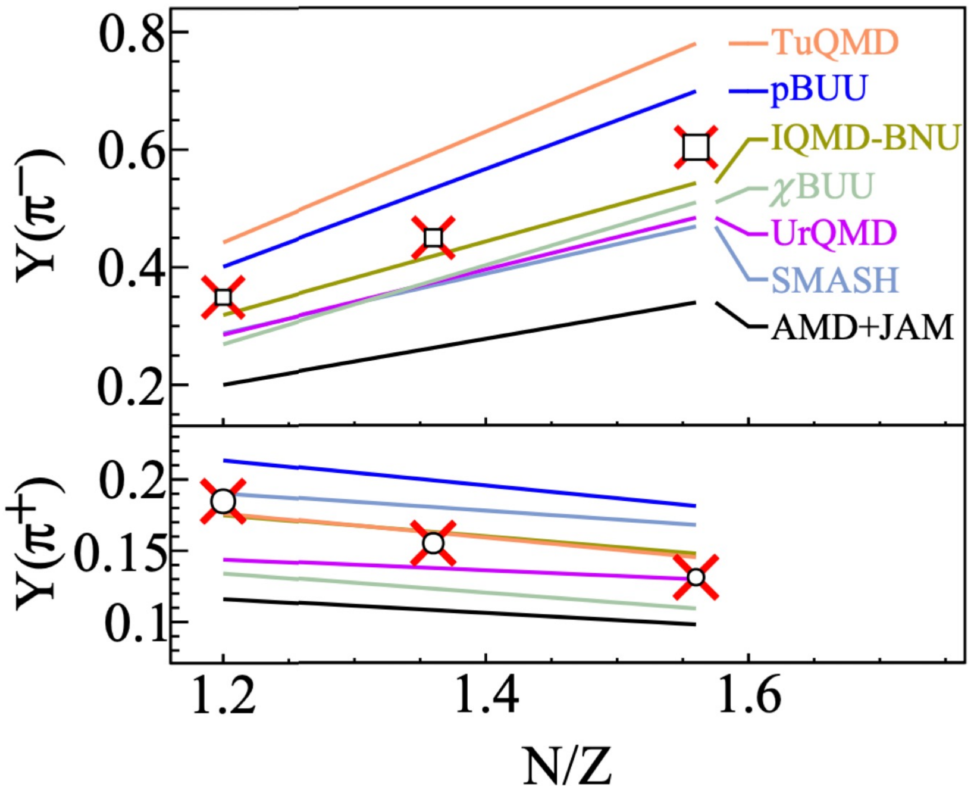
Isospin-dependent Cross-section

$$\sigma_{NN\to N\Delta}(\rho_B) = \sigma_{NN\to N\Delta}(0)\times \exp\left(C\frac{\rho_B}{\rho_0}\right)\left(\frac{N}{Z}\right)_{sys}^{x^{\pm,0}}$$

Nucleon-Nucleon collisions

Pion production in Heavy Ion Collision in intermediate energy reign





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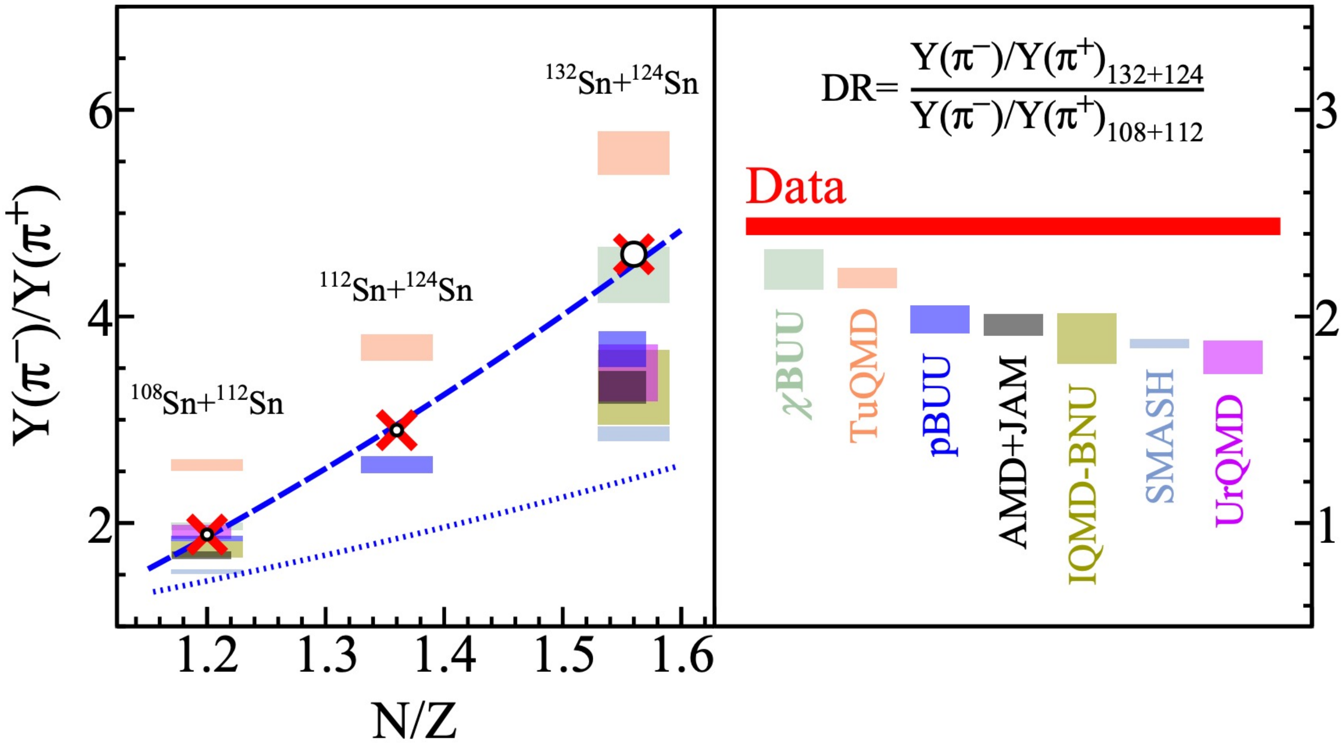
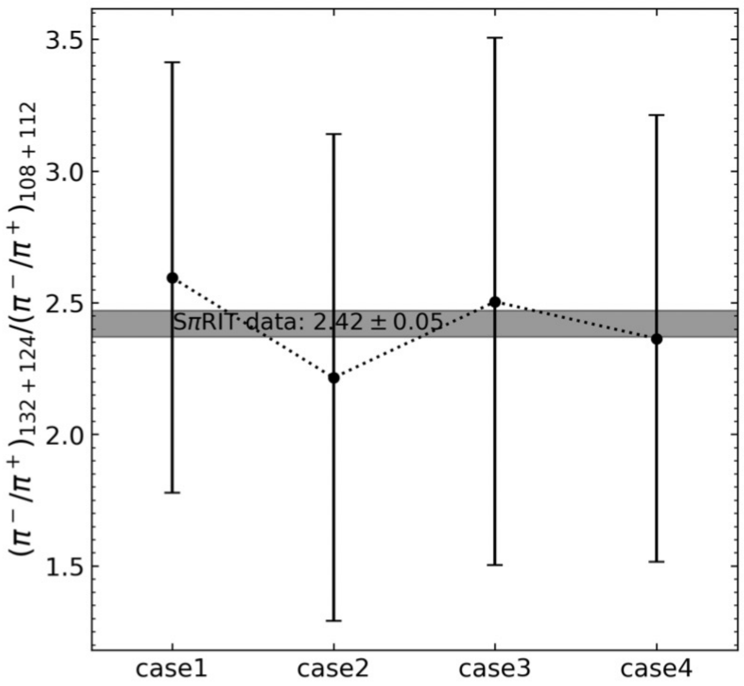
Pion production in Heavy Ion Collision in intermediate energy reign



$N + N + \pi$

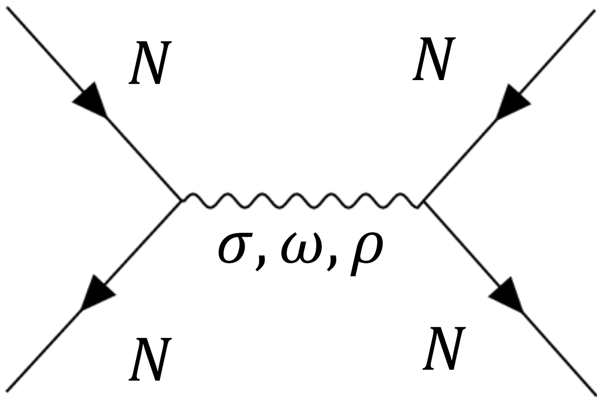


(c) Pion double ratios

Propagation with potential

In DJBUU

Relativistic Mean-Field (RMF) Theory
(ex. QHD, modified Walecka model)



σ : Scalar-Isoscalar - Attractive
ω : Vector-Isoscalar - Repulsive
ρ : Vector-Isovector - Repulsive

In SQMD

Non-Relativistic, phenomenological
Skyrme parameterization

$$U_{tot} = \frac{\alpha}{2\rho_0}\sum_{i,j\neq i}\rho_{ij} + \frac{\beta}{\gamma+1}\sum_i\left(\sum_{j\neq i}\frac{\rho_{ij}}{\rho_0}\right)^{\gamma} + \frac{g_{surf}}{2\rho_0}\sum_{i,j\neq i}\nabla^2_{r_i}(\rho_{ij}) + \frac{g_{sym}}{2\rho_0}\sum_{i,j\neq i}[2\delta_{\tau_i\tau_j}-1]\rho_{ij} + \frac{e^2}{2}\sum_{\substack{i,j\neq i,\\ (i,j \text{ for protons})}}\frac{1}{|\vec{r}_i-\vec{r}_j|}\mathrm{erf}\left(\frac{|\vec{r}_i-\vec{r}_j|}{2\sigma_r}\right),$$

Relative Mean Field(RMF) theory

Lagrangian density

$$\mathcal{L} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - (m_N + g_\sigma\sigma) - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\,\vec{\tau}\cdot\vec{\rho}^\mu - \frac{e}{2}\gamma_\mu(1+\tau^3)A^\mu\right]\psi$$
$$+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - U(\sigma) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu$$
$$-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where,

$$U(\sigma) = \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4,\ \Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu,\ R_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu,\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Table 1. Parameters in the relativistic mean-field model with $f_i = (g_i^2/m_i^2)$ [fm^2], $A \equiv a/g_\sigma^3$ [fm^{-1}], $B \equiv b/g_\sigma^4$, and vacuum masses of nucleon and all mesons in GeV unit.

f_σ	f_ω	f_ρ	A	B	m_N	m_σ	m_ω	m_ρ
10.33	5.42	0.95	0.033	−0.0048	0.938	0.5082	0.783	0.763

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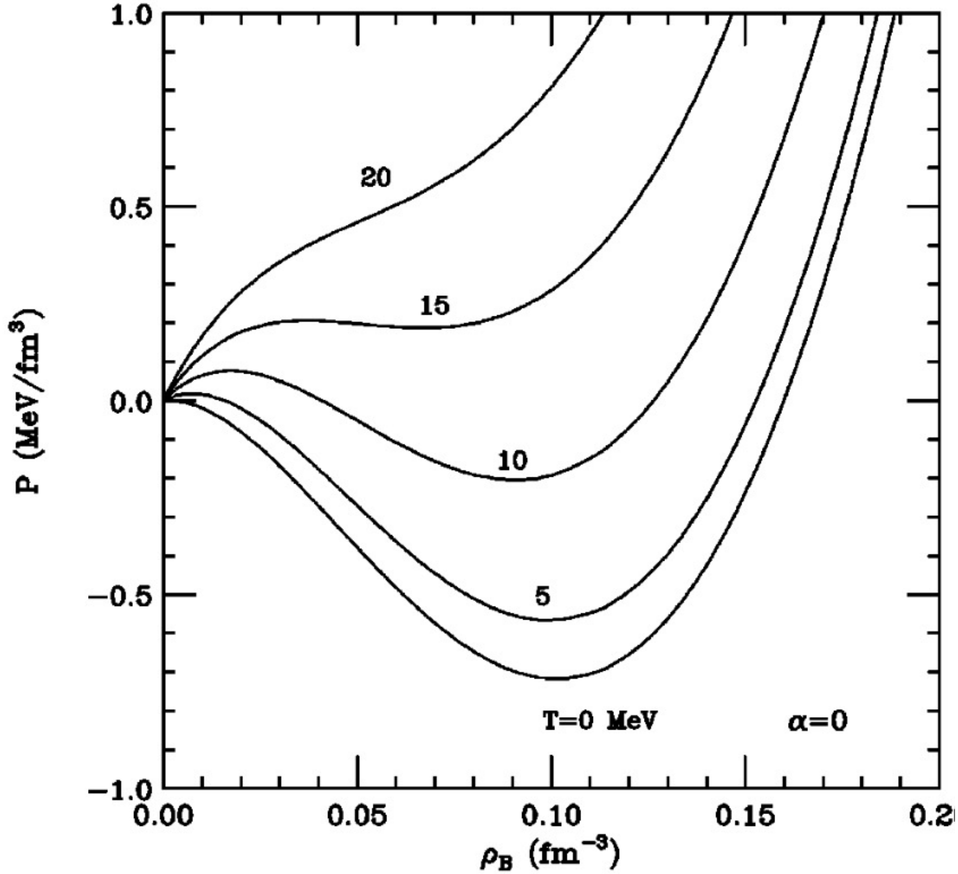
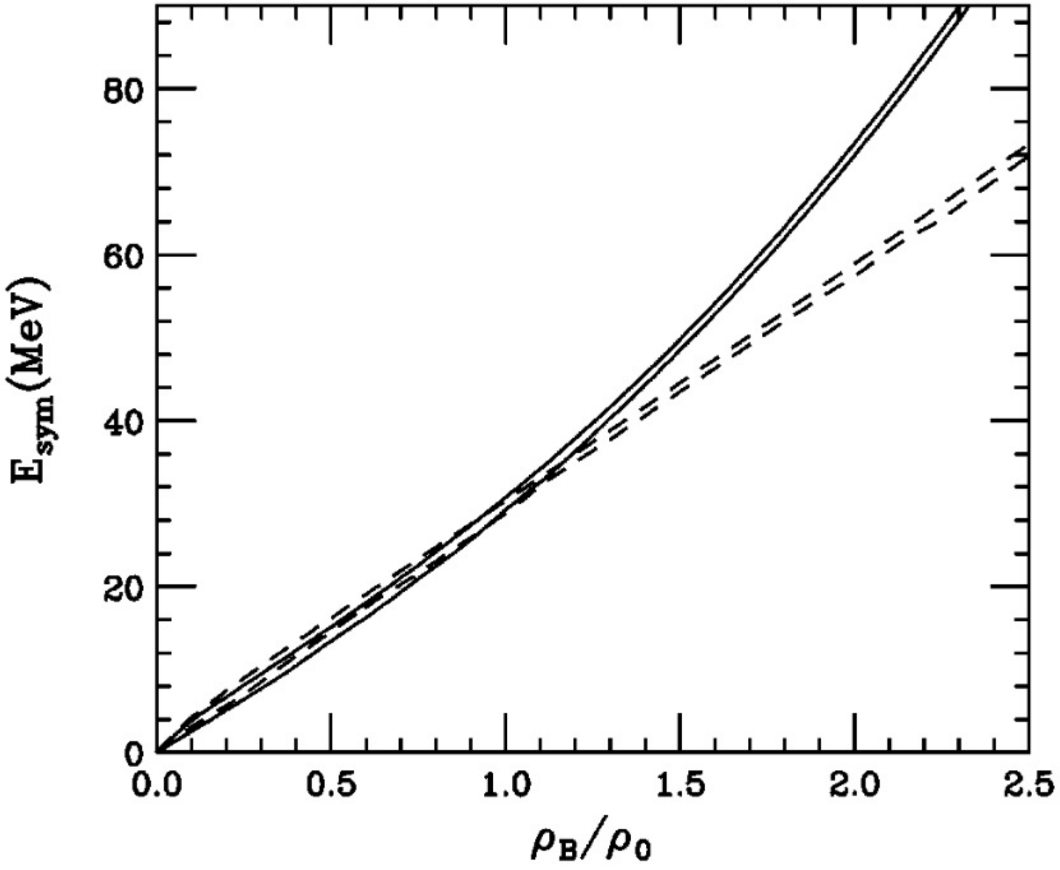
Relative Mean Field(RMF) theory

Lagrangian density





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Relative Mean Field(RMF) theory

To get nuclear potential in DJBUU, We use Relativistic Mean-Field (RMF) Theory and Quantum Hadron Dynamics (QHD), so called Walecka model.

Lagrangian density

$$\mathcal{L} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - (m_N + g_\sigma\sigma) - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\,\vec{\tau}\cdot\vec{\rho}^\mu - \frac{e}{2}\gamma_\mu(1+\tau^3)A^\mu\right]\psi$$
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$$-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Euler-Lagrange equation

$$\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu q)} - \frac{\partial\mathcal{L}}{\partial q} = 0$$

mean-field approximation

$$\sigma \rightarrow <\sigma> \equiv \sigma_0$$
$$\omega^\mu \rightarrow <\omega^\mu> \equiv \delta^{\mu0}\omega_0$$
$$\rho^\mu \rightarrow <\vec{\rho}^\mu> \equiv \delta^{\mu0}\vec{\rho}_0$$

Relative Mean Field(RMF) theory

$$\mathcal{L} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - (m_N + g_\sigma\sigma) - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\,\vec{\tau}\cdot\vec{\rho}^\mu - \frac{e}{2}\gamma_\mu(1+\tau^3)A^\mu\right]\psi$$
$$+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - U(\sigma) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu$$
$$-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Euler–Lagrange equation for $\bar{\psi}$

$$\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})} - \frac{\partial\mathcal{L}}{\partial\bar{\psi}} = 0 \rightarrow \frac{\partial\mathcal{L}}{\partial\bar{\psi}} = 0$$

$$\left[i\gamma_\mu\partial^\mu - (m_N + g_\sigma\sigma) - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\,\vec{\tau}\cdot\vec{\rho}^\mu - \frac{e}{2}\gamma_\mu(1+\tau^3)A^\mu\right]\psi = 0$$

$$\left(i\gamma_\mu(\partial^\mu - V^\mu) - M^*\right)\psi = 0$$

Relative Mean Field(RMF) theory

Meson equation

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \sigma)} - \frac{\partial \mathcal{L}}{\partial \sigma} = 0 \ \rightarrow \partial_\mu \partial^\mu \sigma + m_\sigma^2 \sigma + \frac{\partial U(\sigma)}{\partial \sigma} + g_\sigma \bar{\psi}\psi$$

$$\partial_\mu \sigma \partial^\mu \sigma + m_\sigma^2 \sigma + g_2 \sigma^2 + g_3 \sigma^3 = -g_\sigma \bar{\psi}\psi$$

Mean field approximation: $\sigma \rightarrow \langle\sigma\rangle \equiv \sigma$

$$\partial_\mu \partial^\mu \langle\sigma\rangle + m_\sigma^2 \langle\sigma\rangle + g_2 \langle\sigma\rangle^2 + g_3 \langle\sigma\rangle^3 = -g_\sigma \langle\bar{\psi}\psi\rangle$$

Same way for ω, ρ

$$\partial_\mu \partial^\mu \langle\omega^\nu\rangle + m_\sigma^2 \langle\omega^\nu\rangle = g_\omega \langle\bar{\psi}\gamma^\nu\psi\rangle$$

$$\partial_\mu \partial^\mu \langle\rho_3^\nu\rangle + m_\sigma^2 \langle\rho_3^\nu\rangle = g_\rho \langle\bar{\psi}\gamma^\nu\tau_3\psi\rangle$$

Relative Mean Field(RMF) theory

Mean field approximation:

$\langle\sigma\rangle \equiv \sigma, \qquad \langle\omega^\nu\rangle \equiv \omega^0, \qquad \langle\rho_3^\nu\rangle \equiv \rho_3^0$

$$\partial_\mu\partial^\mu\sigma + m_\sigma^2\sigma + g_2\sigma^2 + g_3\sigma^3 = -g_\sigma\rho_s$$

$$\partial_\mu\partial^\mu\omega^0 + m_\sigma^2\omega^0 = g_\omega j_B^\mu$$

$$\partial_\mu\partial^\mu\rho_3^0 + m_\sigma^2\rho_3^0 = g_\rho j_{B,I}^\mu$$

where,

$$\langle\bar{\psi}\psi\rangle = \rho_s = \frac{g}{(2\pi)^3}\sum\int\frac{m^*}{p^{*0}}dp^3 f(\vec{x},\vec{p})$$

$$\langle\bar{\psi}\gamma^\nu\psi\rangle = j_B^\mu = \frac{g}{(2\pi)^3}\sum\int\frac{p^{*\mu}}{p^{*0}}dp^3 f(\vec{x},\vec{p})$$

$$\langle\bar{\psi}\gamma^\nu\tau_3\psi\rangle = j_{B,I}^\mu = \frac{g}{(2\pi)^3}\sum\int\frac{p^{*\mu}}{p^{*0}}dp^3 f(\vec{x},\vec{p})$$

In Uniform nuclear matter, $\partial_\mu\partial^\mu\sigma = \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\sigma = 0$, same for ω, ρ

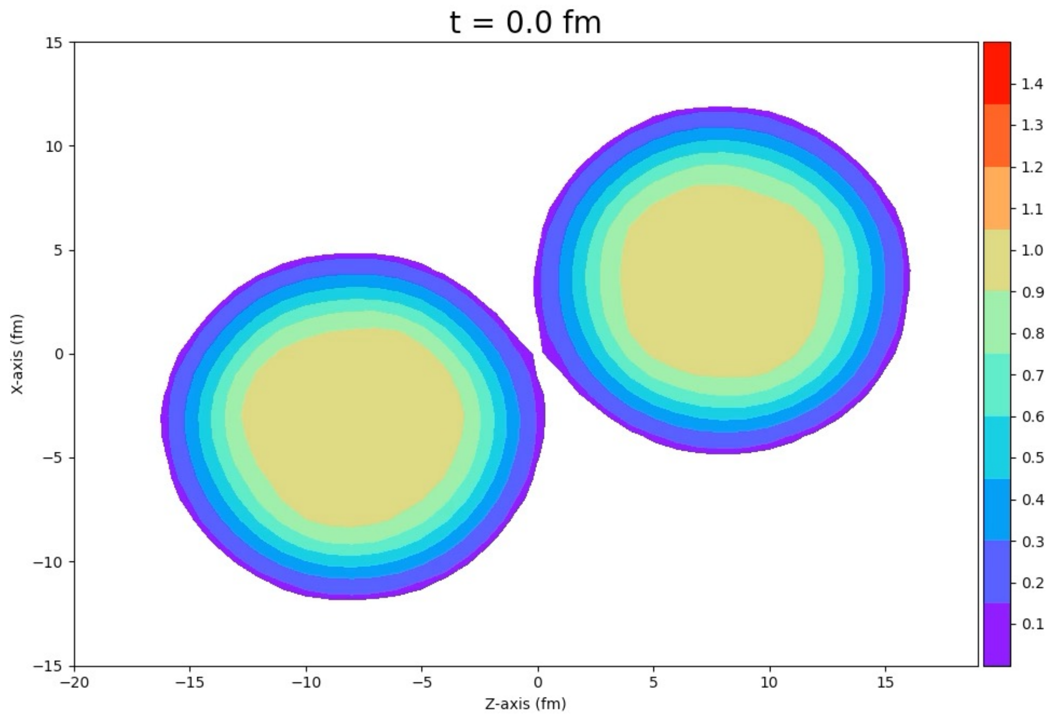
$$m_\sigma^2\sigma + g_2\sigma^2 + g_3\sigma^3 = -g_\sigma\rho_s = -\frac{g_\sigma}{(2\pi)^3}\sum\int\frac{m^*}{p^{*0}}dp^3 f(\vec{x},\vec{p})$$

$$m_\sigma^2\omega^0 = g_\omega\rho_B = \frac{g_\omega}{(2\pi)^3}\sum\int dp^3 f(\vec{x},\vec{p})$$

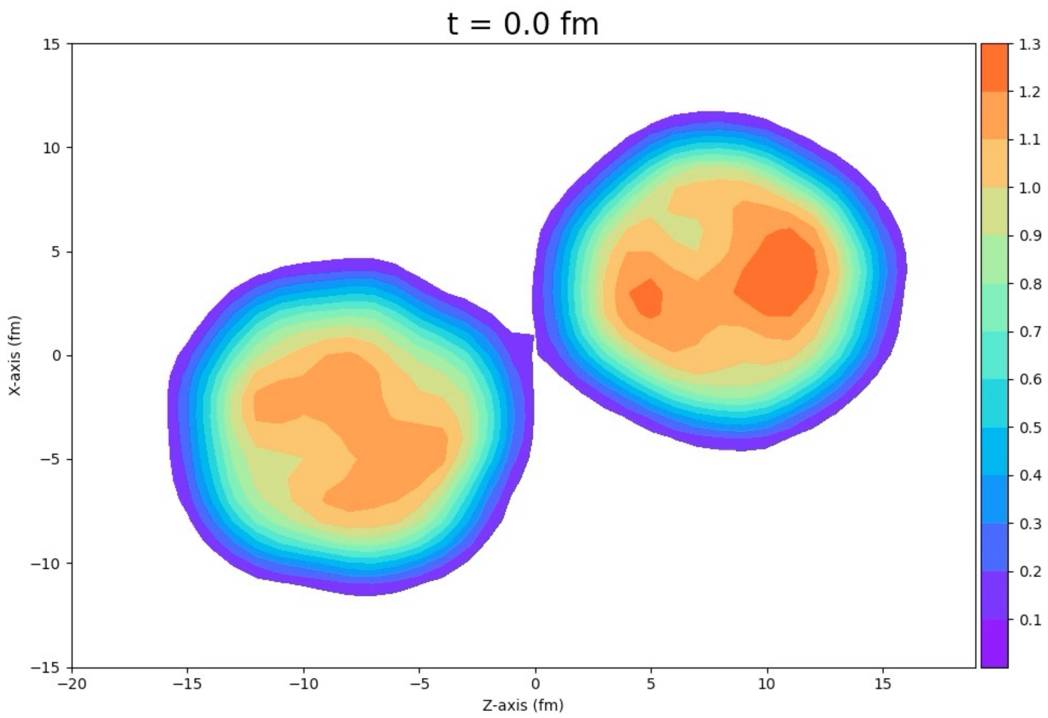
$$m_\sigma^2\rho_3^0 = g_\rho\rho_{B,I} = \frac{g_\rho}{(2\pi)^3}\sum\int dp^3\tau_3 f(\vec{x},\vec{p})$$

Full time evolution

^{197}Au+^{197}Au, E_{beam}=100 A MeV, b = 7 fm



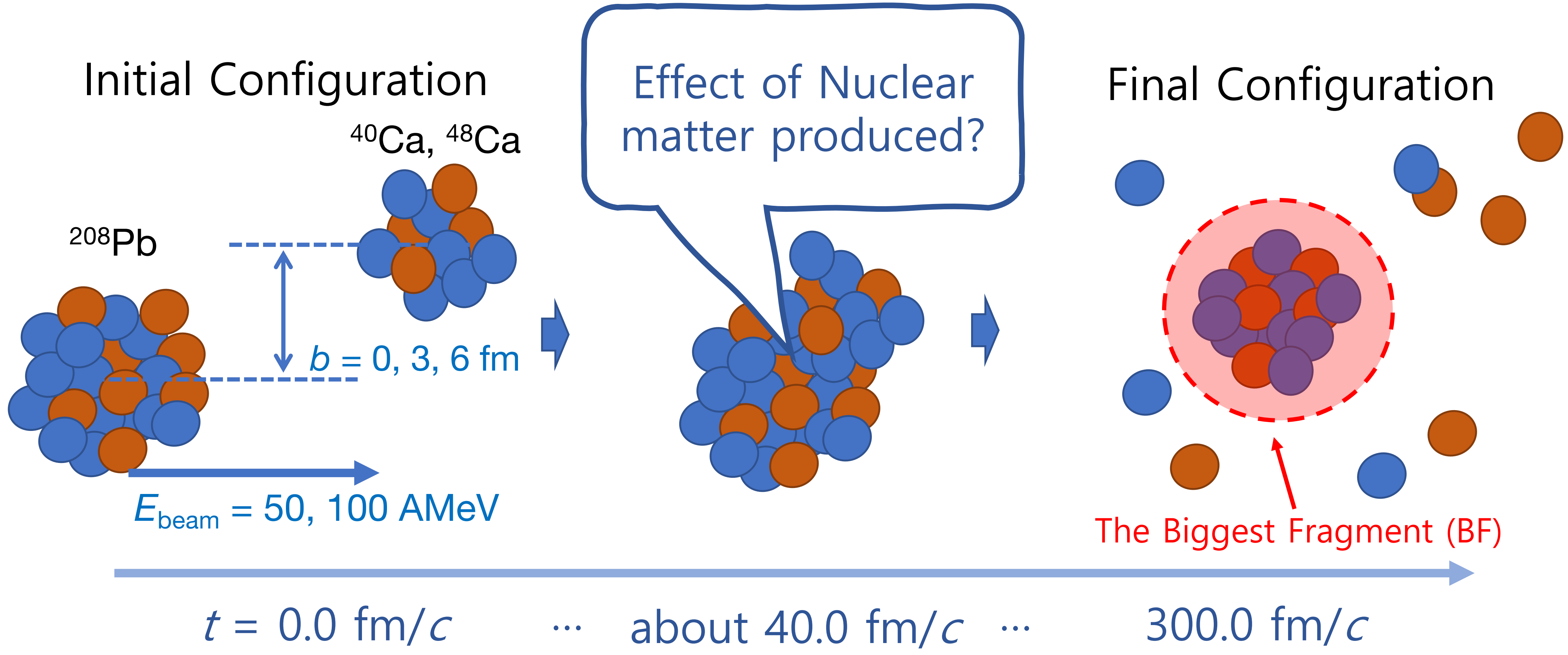
1 event, N_{TP} = 50
corresponding to 50 events (DJBUU work)



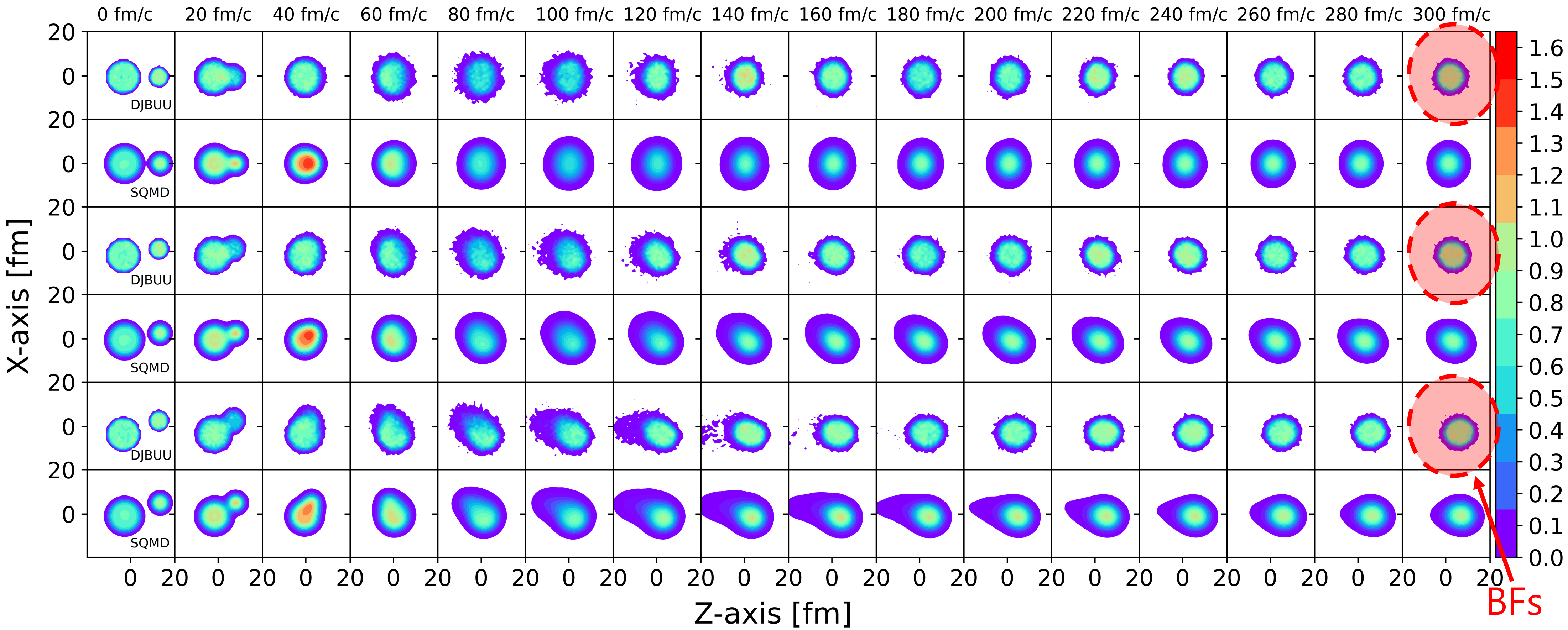
1 event (SQMD work)

Comparative study

Comparative study : Fragment



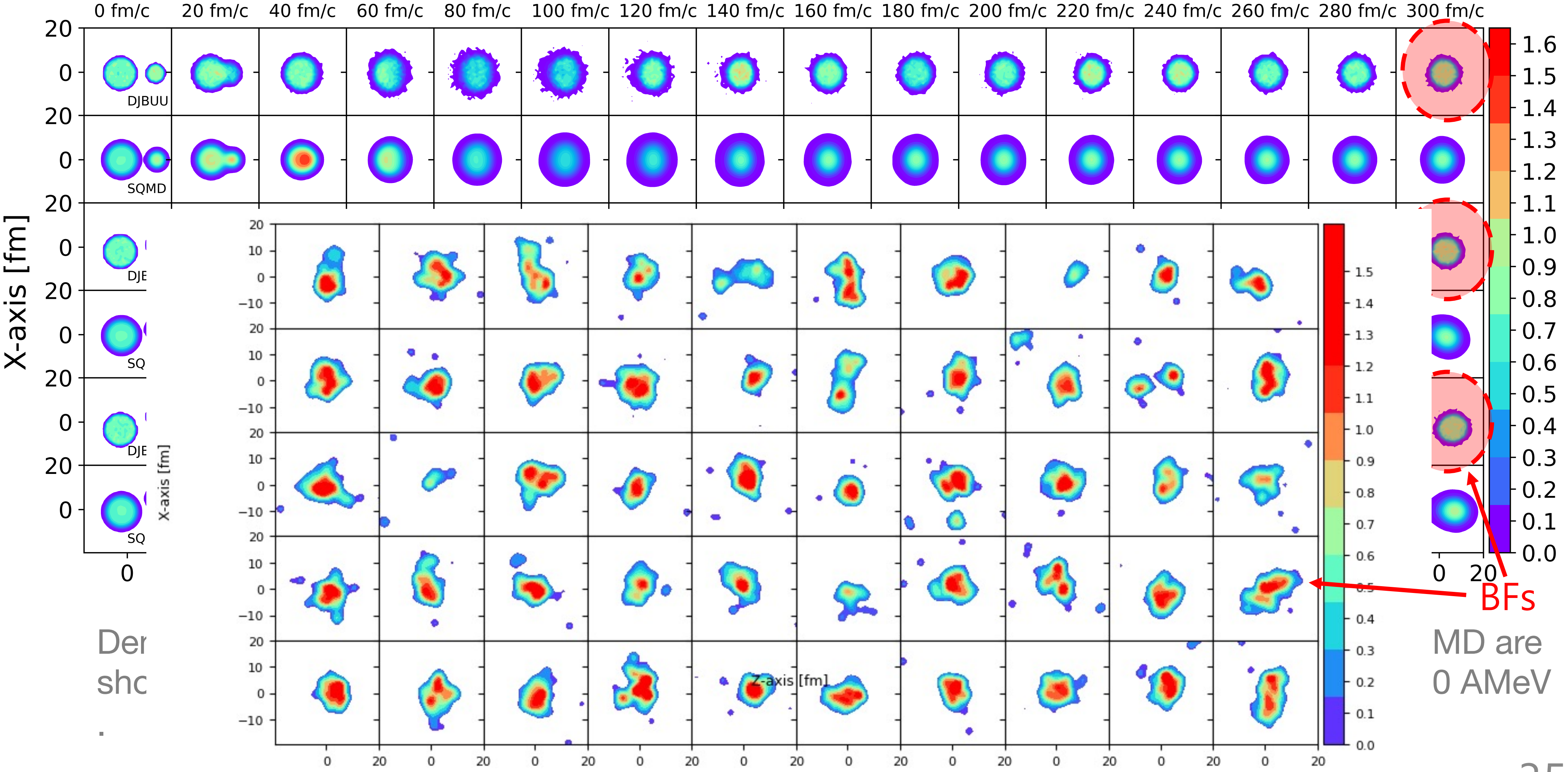
Comparative study : Fragment



Density distribution in the collision plane. For comparison, the results of DJBUU and SQMD are shown alternatively. From top to bottom, the systems are 208 Pb + 40 Ca at E beam = 50 AMeV

.

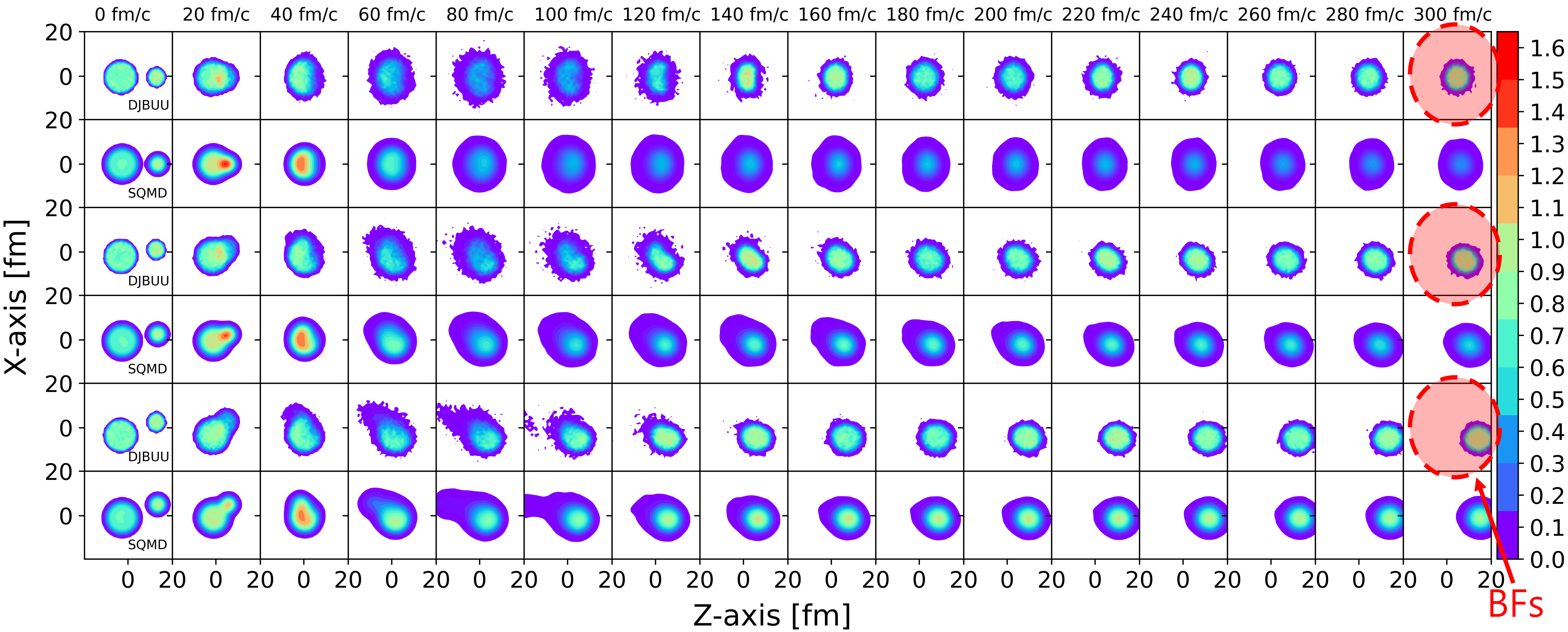
Comparative study : Fragment



Der
shc

MD are
0 AMeV

Comparative study : Fragment



Density distribution in the collision plane. For comparison, the results of DJBUU and SQMD are shown alternatively. From top to bottom, the systems are 208 Pb + 40 Ca at E beam = 100 AMeV.

Comparative study : Fragment

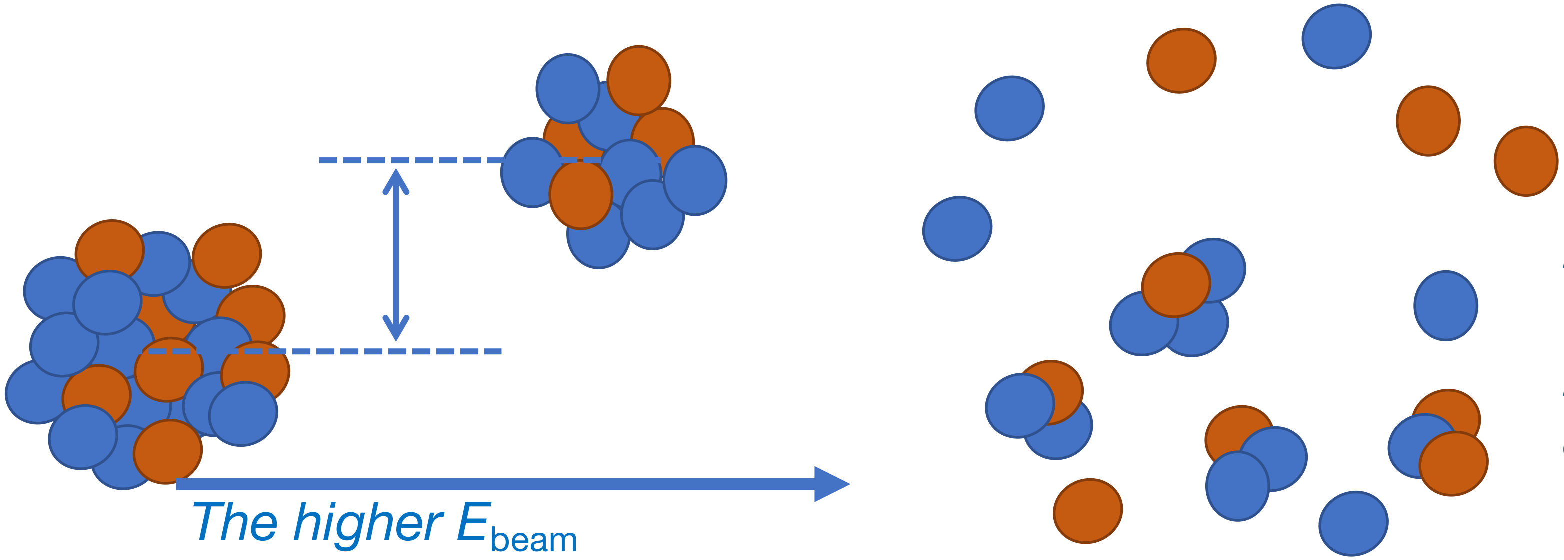
The higher beam energy

The smaller impact parameter

Target	E_{beam} (AMeV)	b (fm)	DJBUU	SQMD
^{40}Ca	50	0	$^{163}_{73}Ta$, $^{162}_{73}Ta$, $^{164}_{73}Ta$, $^{163}_{74}W$	$^{163}_{69}Tm$, $^{173}_{74}W$, $^{169}_{72}Hf$
		3	$^{163}_{73}Ta$, $^{165}_{74}W$, $^{164}_{73}Ta$,	$^{169}_{72}Hf$, $^{173}_{74}W$, $^{172}_{74}W$
		6	$^{167}_{74}W$, $^{169}_{75}Re$, $^{165}_{73}Ta$, $^{168}_{75}Re$	$^{168}_{72}Hf$, $^{164}_{70}Yb$, $^{169}_{72}Hf$
	100	0	$^{123}_{56}Ba$, $^{121}_{55}Cs$, $^{124}_{57}La$, $^{122}_{56}Ba$, $^{124}_{56}Ba$	$^{78}_{33}As$, $^{114}_{50}Sn$, $^{124}_{54}Xe$
		3	$^{130}_{59}Pr$, $^{130}_{58}Ce$, $^{128}_{57}La$, $^{128}_{58}Ce$, $^{129}_{58}Ce$, $^{127}_{58}Ce$, $^{127}_{57}La$	$^{125}_{53}I$, $^{128}_{56}Ba$, $^{132}_{57}La$
		6	$^{145}_{64}Gd$, $^{144}_{64}Gd$, $^{146}_{65}Tb$, $^{147}_{65}Tb$	$^{151}_{64}Gd$, $^{149}_{63}Eu$, $^{154}_{66}Dy$
^{48}Ca	50	0	$^{161}_{72}Hf$, $^{162}_{72}Hf$, $^{160}_{71}Lu$, $^{159}_{71}Lu$	$^{167}_{70}Yb$, $^{167}_{71}Lu$, $^{170}_{71}Lu$
		3	$^{162}_{72}Hf$, $^{164}_{73}Ta$	$^{165}_{70}Yb$, $^{167}_{70}Yb$, $^{167}_{71}Lu$
		6	$^{164}_{72}Hf$, $^{163}_{72}Hf$, $^{166}_{73}Ta$, $^{165}_{72}Hf$	$^{165}_{69}Tm$, $^{159}_{68}Er$, $^{164}_{69}Tm$
	100	0	$^{113}_{51}Sb$, $^{115}_{52}Te$, $^{114}_{51}Sb$, $^{116}_{52}Te$, $^{112}_{51}Sb$	$^{58}_{25}Mn$, $^{74}_{32}Ge$, $^{107}_{48}Pd$
		3	$^{121}_{54}Xe$, $^{122}_{55}Cs$, $^{120}_{54}Xe$, $^{123}_{55}Cs$, $^{121}_{55}Cs$	$^{120}_{52}Te$, $^{106}_{45}Rh$, $^{113}_{48}Cd$
		6	$^{140}_{62}Sm$, $^{139}_{62}Sm$, $^{138}_{61}Pm$, $^{137}_{61}Pm$, $^{137}_{60}Nd$	$^{147}_{62}Sm$, $^{153}_{64}Gd$, $^{148}_{62}Sm$

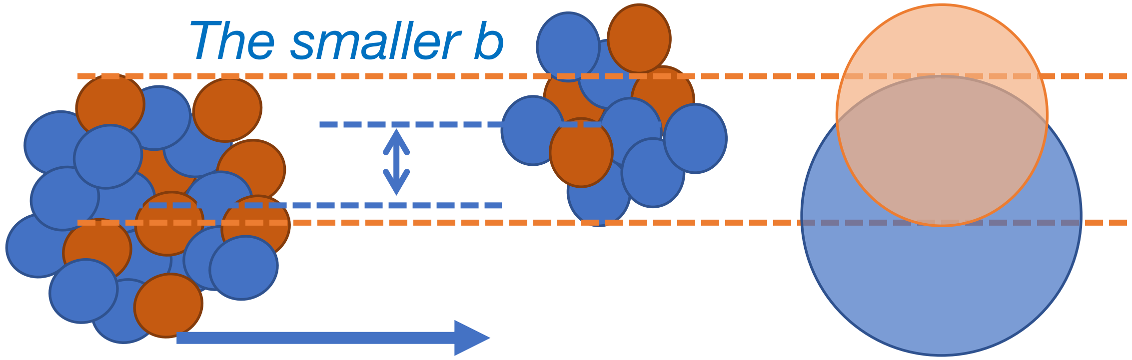
The BFs in DJBUU and SQMD; the BFs from the ten runs of DJBUU and the most abundantly produced three BFs from SQMD runs

Comparative study : Fragment



Break into smaller pieces

More pronounced different definition of BFs



The more participants
EOS dependence ↑

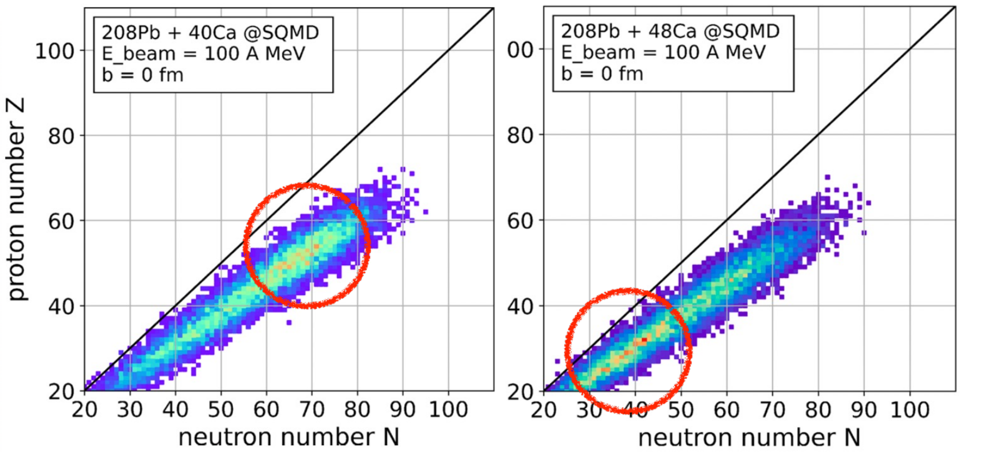
SQMD-Skyrme paraterization (K_0 = 236 MeV)
DJBUU-RMF approach (K_0 = 240 MeV)

Comparative study : Fragment

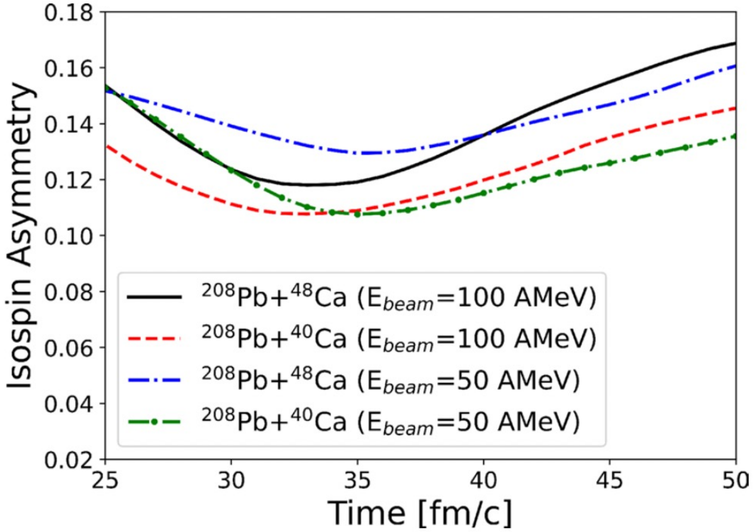
Target	E_{beam} (AMeV)	b (fm)	DJBUU	SQMD
^{40}Ca	50	0	${}^{163}_{73}$Ta, ${}^{162}_{73}$Ta, ${}^{164}_{73}$Ta, ${}^{163}_{74}$W	${}^{163}_{69}$Tm, ${}^{173}_{74}$W, ${}^{169}_{72}$Hf
		3	${}^{163}_{73}$Ta, ${}^{165}_{74}$W, ${}^{164}_{73}$Ta,	${}^{169}_{72}$Hf, ${}^{173}_{74}$W, ${}^{172}_{74}$W
		6	${}^{167}_{74}$W, ${}^{169}_{75}$Re, ${}^{165}_{73}$Ta, ${}^{168}_{75}$Re	${}^{168}_{72}$Hf, ${}^{164}_{70}$Yb, ${}^{169}_{72}$Hf
	100	0	${}^{123}_{56}$Ba, ${}^{121}_{55}$Cs, ${}^{124}_{57}$La, ${}^{122}_{56}$Ba, ${}^{124}_{56}$Ba	${}^{78}_{33}$As, ${}^{114}_{50}$Sn, ${}^{124}_{54}$Xe
		3	${}^{130}_{59}$Pr, ${}^{130}_{58}$Ce, ${}^{128}_{57}$La, ${}^{128}_{58}$Ce, ${}^{129}_{58}$Ce, ${}^{127}_{58}$Ce, ${}^{127}_{57}$La	${}^{125}_{53}$I, ${}^{128}_{56}$Ba, ${}^{132}_{57}$La
		6	${}^{145}_{64}$Gd, ${}^{144}_{64}$Gd, ${}^{146}_{65}$Tb, ${}^{147}_{65}$Tb	${}^{151}_{64}$Gd, ${}^{149}_{63}$Eu, ${}^{154}_{66}$Dy
^{48}Ca	50	0	${}^{161}_{72}$Hf, ${}^{162}_{72}$Hf, ${}^{160}_{71}$Lu, ${}^{159}_{71}$Lu	${}^{167}_{70}$Yb, ${}^{167}_{71}$Lu, ${}^{170}_{71}$Lu
		3	${}^{162}_{72}$Hf, ${}^{164}_{73}$Ta	${}^{165}_{70}$Yb, ${}^{167}_{70}$Yb, ${}^{167}_{71}$Lu
		6	${}^{164}_{72}$Hf, ${}^{163}_{72}$Hf, ${}^{166}_{73}$Ta, ${}^{165}_{72}$Hf	${}^{165}_{69}$Tm, ${}^{159}_{68}$Er, ${}^{164}_{69}$Tm
	100	0	${}^{113}_{51}$Sb, ${}^{115}_{52}$Te, ${}^{114}_{51}$Sb, ${}^{116}_{52}$Te, ${}^{112}_{51}$Sb	${}^{58}_{25}$Mn, ${}^{74}_{32}$Ge, ${}^{107}_{48}$Pd
		3	${}^{121}_{54}$Xe, ${}^{122}_{55}$Cs, ${}^{120}_{54}$Xe, ${}^{123}_{55}$Cs, ${}^{121}_{55}$Cs	${}^{120}_{52}$Te, ${}^{106}_{45}$Rh, ${}^{113}_{48}$Cd
		6	${}^{140}_{62}$Sm, ${}^{139}_{62}$Sm, ${}^{138}_{61}$Pm, ${}^{137}_{61}$Pm, ${}^{137}_{60}$Nd	${}^{147}_{62}$Sm, ${}^{153}_{64}$Gd, ${}^{148}_{62}$Sm

The BFs in DJBUU and SQMD; the BFs from the ten runs of DJBUU and the most abundantly produced three BFs from SQMD runs

the symmetry energy pushes out the neutrons and so disturbs the formation of large fragments.



Proton and neutron distributions of the BFs in SQMD



Surface term

Restoring Surface term

Ignore time derivative term ~~and spatial derivative term~~

We need to solve Numerically And Efficiently →

$$-\nabla^2\sigma + m^2\sigma + g_2\sigma^2 + g_3\sigma^3 = -g_\sigma\rho_s$$

We found Analytic solution for ω, $\boldsymbol{\rho}$ →

$$-\nabla^2\omega^0 + m^2\omega^0 = g_\omega\rho_B$$

$$-\nabla^2\rho^0 + m^2\rho^0 = g_\omega\rho_I$$

$$-\nabla^2 A^0 = e\rho_q$$

$$\rho_s = \frac{g}{(2\pi)^3}\sum\int m^* dp^3 f(\vec{x},\vec{p})$$

- Found Analytic solution for omega, rho fields.
- Couldn't find Analytic solution for sigma fields. → Numerical method, (Jacobi method)

Restoring Surface term

$$-\nabla^2 A^0 = e\rho_q$$

$$-\nabla^2 \omega^0 + m^2 \omega^0 = g_\omega \rho_B$$

- We have solved Poisson equations
- for Coulomb interaction, with Green's function

3.2 Coulomb Integral

$$\phi_i(\mathbf{x}) = \int d^3x' \frac{1}{4\pi|\mathbf{x}-\mathbf{x}'|} W(\mathbf{x}'-\mathbf{x}_i)$$

...

$$\phi_i(\mathbf{x}) = \theta(a>s)\frac{315}{64\pi a^3}\left(\frac{s^8}{72a^6} - \frac{s^6}{14a^4} + \frac{3s^4}{20a^2} + \frac{a^2}{8} - \frac{s^2}{6}\right) + \theta(s>a)\frac{1}{4\pi s}$$

Courtesy of Prof. Jeon

3.3 Yukawa Integral

$$\phi_i(\mathbf{x}) = \int d^3x' \frac{e^{-m|\mathbf{x}-\mathbf{x}'|}}{4\pi|\mathbf{x}-\mathbf{x}'|} W(\mathbf{x}'-\mathbf{x}_i) \quad (43)$$

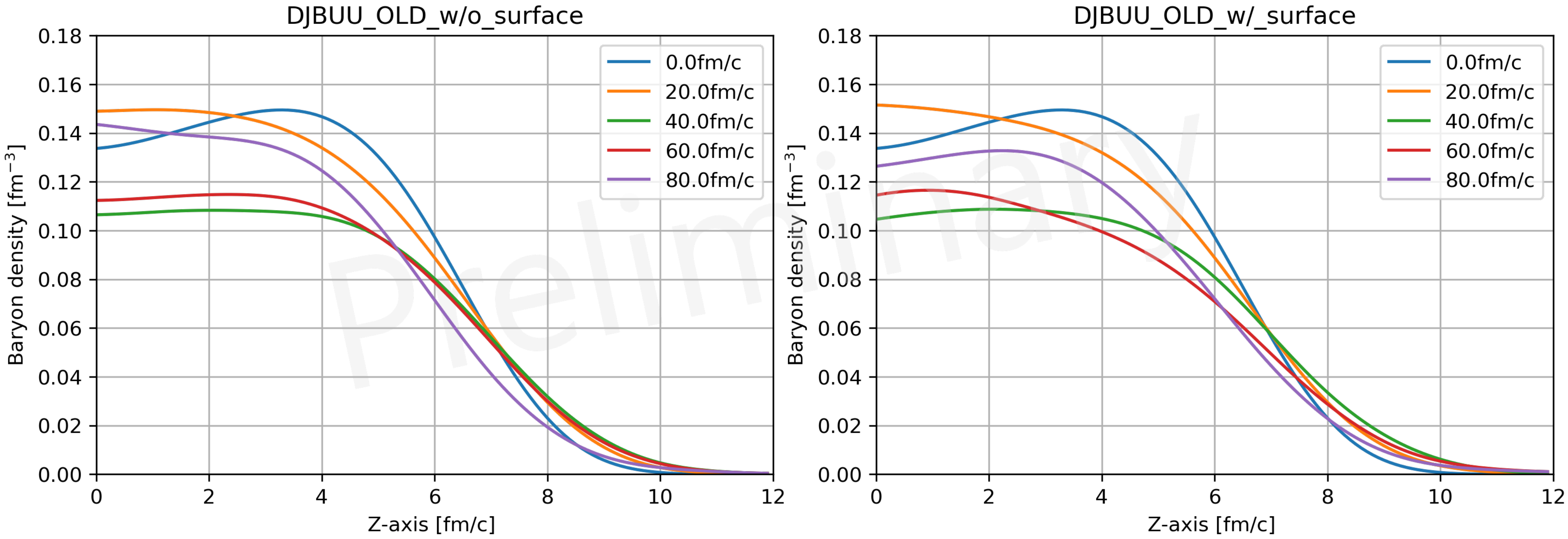
...

Defining $ma = \tilde{a}$ and $ms = \tilde{s}$,

$$\begin{aligned}\phi_i(\mathbf{x}) &= \theta(s<a)\frac{315m}{64\pi\tilde{a}^9\tilde{s}} \\ &\quad \Big(48(\tilde{a}(\tilde{a}(\tilde{a}(\tilde{a}+10)+45)+105)+105)\sinh(\tilde{s})e^{-\tilde{a}} \\ &\quad - \tilde{s}\left(-\tilde{a}^6 + 3\tilde{a}^4\left(\tilde{s}^2+6\right) - 3\tilde{a}^2\left(\tilde{s}^4+20\tilde{s}^2+120\right) + 840\left(\tilde{s}^2+6\right) + \tilde{s}^4\left(\tilde{s}^2+42\right)\right)\Big) \\ &\quad + \theta(s>a)\frac{945me^{-\tilde{s}}}{4\pi\tilde{a}^9\tilde{s}}\left(\left(\tilde{a}^4+45\tilde{a}^2+105\right)\sinh(\tilde{a}) - 5\tilde{a}\left(2\tilde{a}^2+21\right)\cosh(\tilde{a})\right) \quad (54)\end{aligned}$$

Stability (w/o and w/ Surface term)

^{197}Au + ^{197}Au, E_{beam} = 50 A MeV, b = 40.0 fm

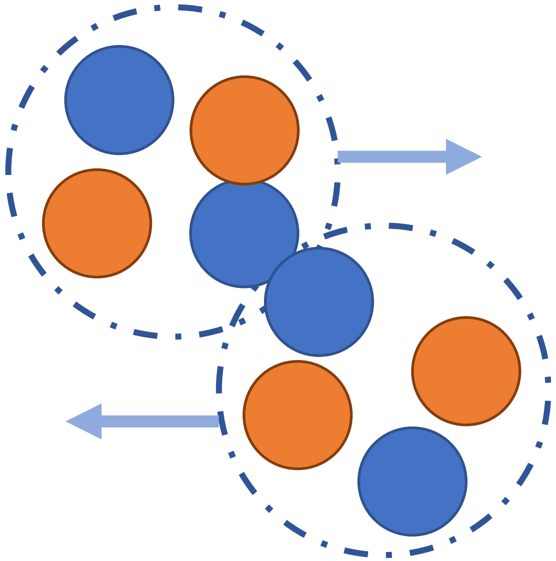


^{197}Au Stability in the DJBUU simulation w/o surface term (left), w/ surface term (right)

QMC model

Adopting Quark-Meson Coupling model

Quantum Hadron Dynamics (QHD)
Such as Walecka model



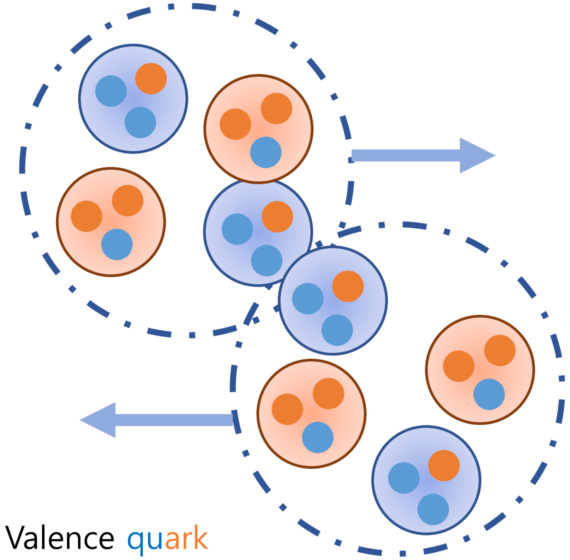
Nucleon-meson coupling

$$g_\sigma^N, g_\omega^N, g_\rho^N$$

Effective mass

$$m^* = m - g\sigma$$

Quark-Meson Coupling (QMC)



Valence quark in Bag

Quark-meson coupling

$$g_\sigma^{u,d}, g_\sigma^{s} \to g_\sigma^{N,Y} \ \dots$$

Scalar polarization

$$m^* = m - g\sigma + \frac{d}{2}(g\sigma)^2$$

Adopting Quark-Meson Coupling model

Lagrangian for quark

$$\mathcal{L} = [\bar{\psi}(i\partial - m_q - g_\sigma^q \sigma - g_\omega^q \omega - g_\rho^q \tau \cdot \rho)\psi - B]\theta_V - \frac{1}{2}\bar{\psi}\psi\theta_S$$

Lagrangian for nucleon

$$\mathcal{L} = \bar{\psi}[i\partial - \boxed{(m_N - g_\sigma(\sigma)\sigma)} - g_\omega \omega - g_\rho \tau \cdot \rho]\psi$$

$$m_N - g\sigma \rightarrow m_N - (g\sigma - \frac{a_N}{2}(g\sigma)^2)$$

Simple parameterization

$$g_\sigma(\sigma) = g_{\sigma=0} - \frac{a_N}{2} g_{\sigma=0}^2 \sigma$$

$$C_N(\sigma) = 1 - a_N g_{\sigma=0}\sigma$$

Meson eqs. in QHD

$$-\nabla^2\sigma + m^2\sigma + g_2\sigma^2 + g_3\sigma^3 = -g_\sigma\rho_s$$

$$-\nabla^2\omega^0 + m^2\omega^0 = g_\omega\rho_B$$

$$-\nabla^2\rho^0 + m^2\rho^0 = g_\omega\rho_I$$

Meson eqs. in QMC

$$\omega = \frac{g_\omega}{m_\omega^2}\rho_B \equiv \frac{g_\omega}{m_\omega^2}\frac{4}{(2\pi)^3}\int d^3k\, \theta(k_F - |\vec{k}|),$$

$$\sigma = \frac{g_\sigma^N}{m_\sigma^2}C_N(\sigma)\rho_s \equiv \frac{g_\sigma^N}{m_\sigma^2}C_N(\sigma)\frac{4}{(2\pi)^3}\int d^3k\, \theta(k_F - |\vec{k}|)\frac{m_N^*(\sigma)}{\sqrt{m_N^{*2}(\sigma) + \vec{k}^2}},$$

PTEP, 2022 043D02

Adopting Quark-Meson Coupling model

$$m_B^* \simeq m_B - \frac{n_q}{3} g_\sigma^N \left[1 - \frac{a_B}{2}(g_\sigma^N \sigma)\right]\sigma = m_B - \frac{n_q}{3}\left[(g_\sigma^N \sigma) - \frac{a_B}{2}(g_\sigma^N \sigma)^2\right]$$

scalar polarizability

$$(B = N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Lambda_c, \Sigma_c, \Xi_c, \Lambda_b, \Sigma_b, \Xi_b),$$

$$\sigma = \frac{g_\sigma^N}{m_\sigma^2} C_N(\sigma)\rho_s \equiv \frac{g_\sigma^N}{m_\sigma^2} C_N(\sigma) \frac{4}{(2\pi)^3} \int d^3k\, \theta(k_F - |\vec{k}|) \frac{m_N^*(\sigma)}{\sqrt{m_N^{*2}(\sigma) + \vec{k}^2}},$$

PTEP, 2022 043D02

Density Parameterization, Prof. Tsushima

$(\rho_0 = 0.15\,\text{fm}^{-3})$ as

$$(g_\sigma^N \sigma)(x) = \begin{cases} 1.60828 - 23.9107\sqrt{x} + 350.631x \\ \qquad -144.309x\sqrt{x} + 19.4750x^2 & (x > 0), \\ 0 & (x = 0), \end{cases} \tag{20}$$

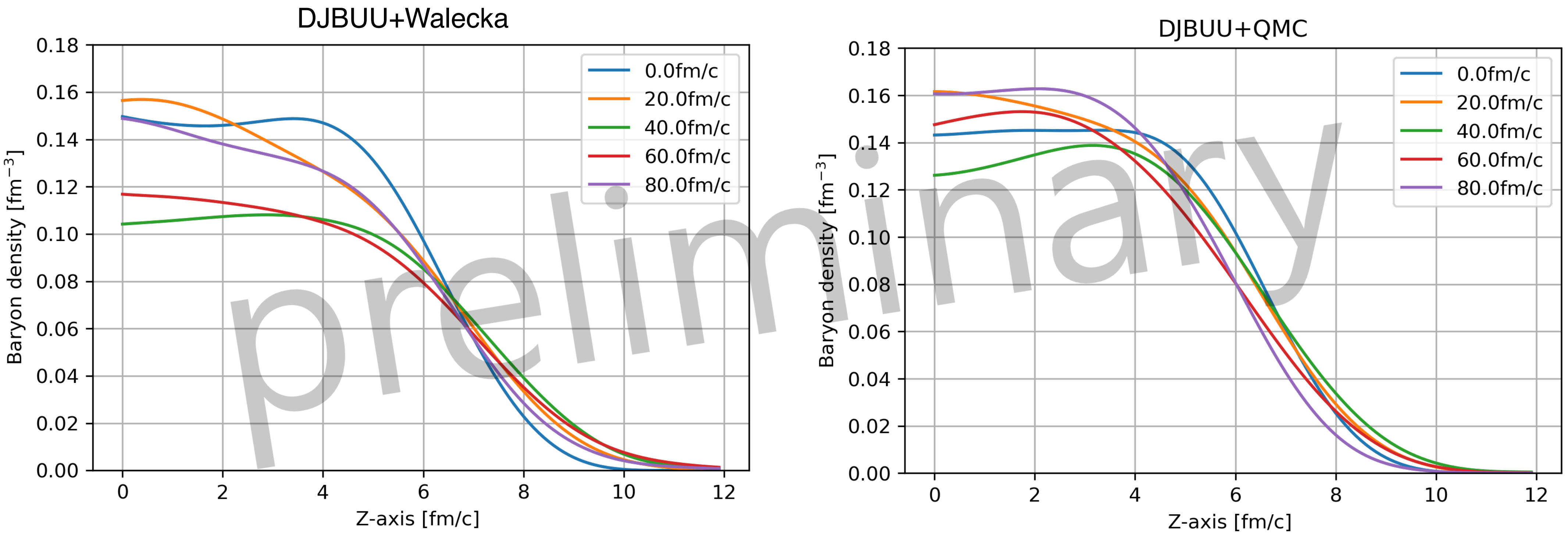
$$V_\omega^B(x) = b_B x,$$

For completeness, we also give the Lorentz-vector-isovector mean field potential (in MeV) as a function of $y \equiv \rho_3/\rho_0 = (\rho_p - \rho_n)/\rho_0$ with the isospin-third component of the hadron h, I_3^h,

$$I_3^h V_\rho^h(y) = I_3^h \times 84.61y, \tag{24}$$

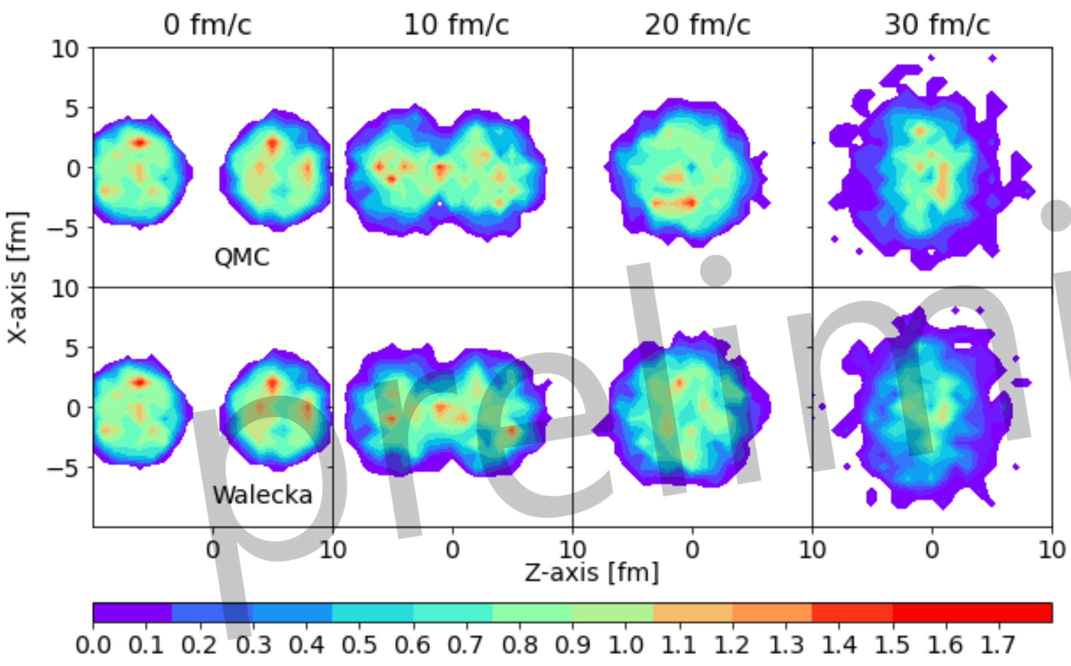
Adopting Quark-Meson Coupling model

Check stability ^{197}Au+ ^{197}Au, E_{beam} = 50 A MeV

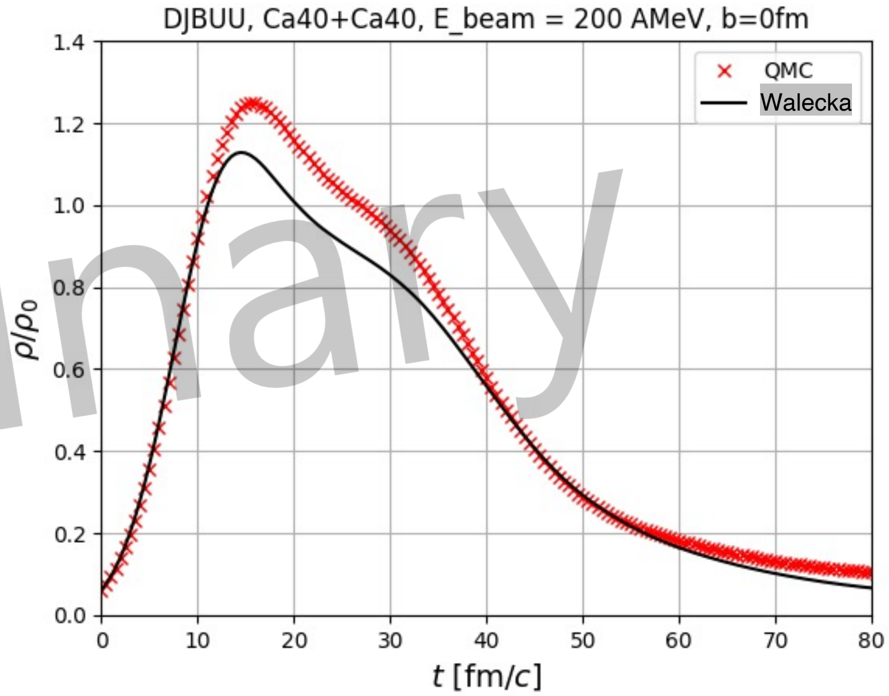


Adopting Quark-Meson Coupling model

$^{40}Ca + {}^{40}Ca$, E_{beam} = 200 A MeV, b = 0 fm



Contour QMC (top), QHD-Walecka (bottom)



Baryon density at center of mass

- Central density with QMC is higher than one with QHD

Adopting Quark-Meson Coupling model

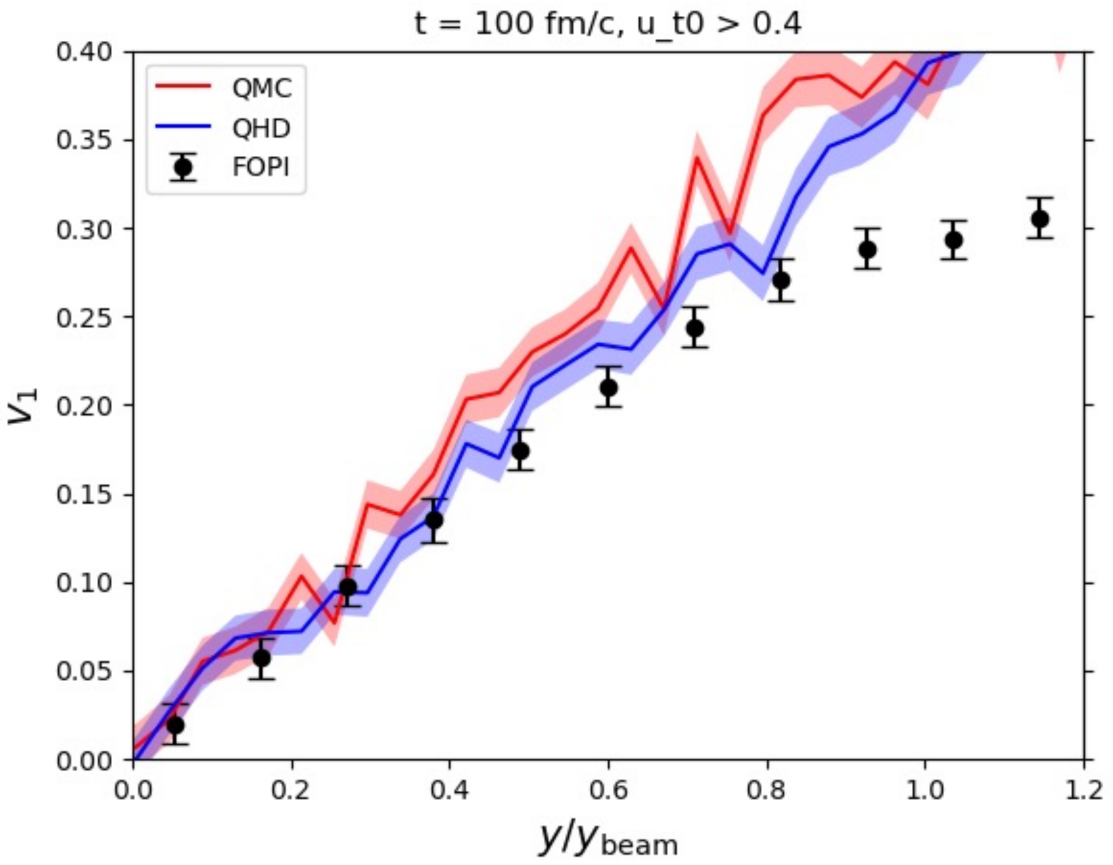
^{197}Au+ ^{197}Au, $E_{\rm beam}$ = 400 A MeV, 0.25<b_0<0.45

$b_0 = 1.15 \times (A_{\rm projectile}^{1/3} + A_{\rm target}^{1/3})$

$b_0 = 0.25 \rightarrow b = 3.346$ fm

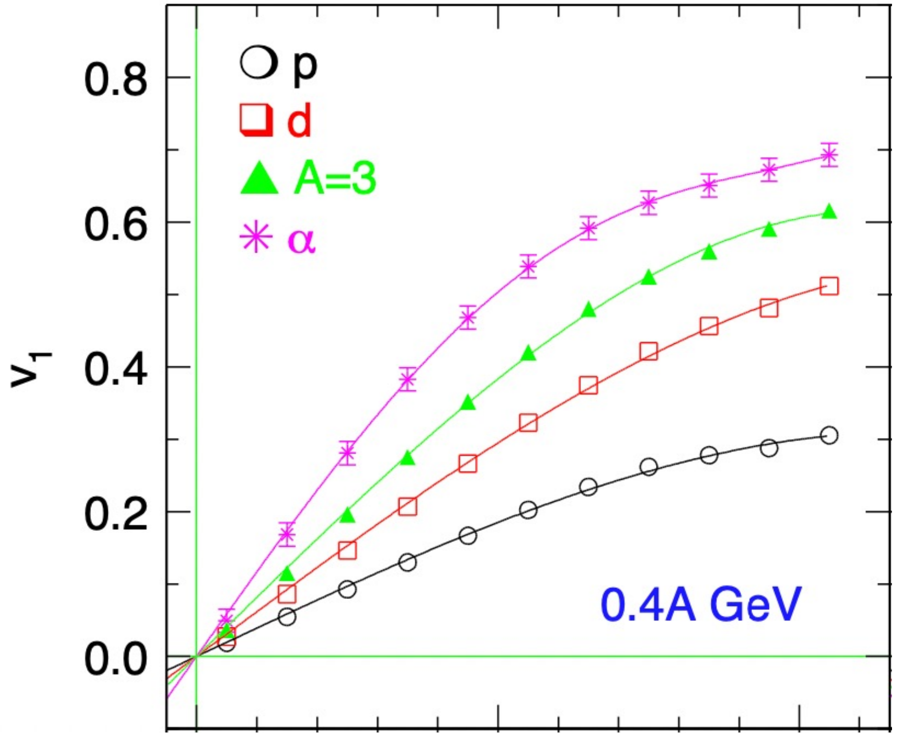
$b_0 = 0.35 \rightarrow b = 4.684$ fm

$b_0 = 0.45 \rightarrow b = 6.022$ fm



Adopting Quark-Meson Coupling model

^{197}Au+ ^{197}Au, E_{beam} = 400 A MeV, 0.25<b_0<0.45



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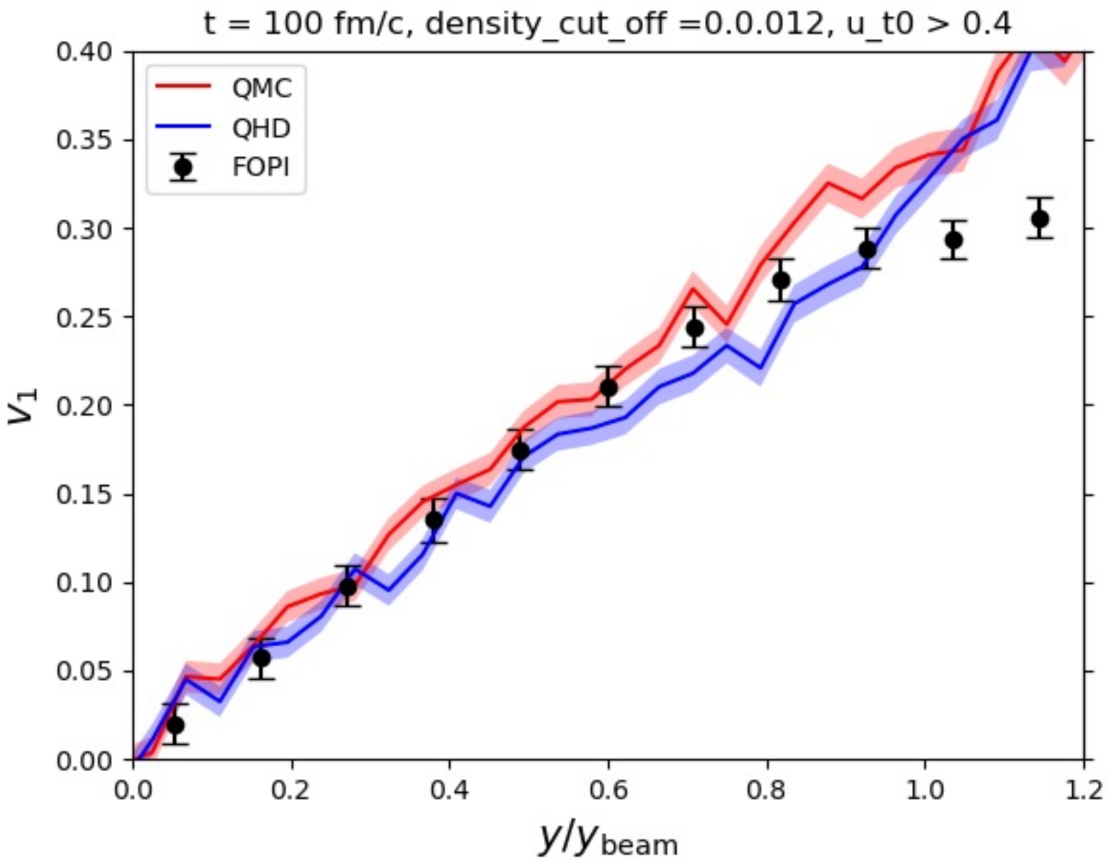

Thank you for your attention

Any questions?

Acknowledgement
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BACK UP SLICES